

## Casimir Forces between Thermally Activated Nanocomposites

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### ABSTRACT

We present a theoretical study of the modification of Casimir forces between nanocomposite slabs that exhibit a metal-dielectric transition. In particular, we consider slabs made of VO<sub>2</sub> precipitates in sapphire, whose effective dielectric function is calculated within a mean field approximation. The results for the Casimir force as a function of the separation of the slabs, show that at a fixed separation the magnitude of the force changes as temperature increases from 300 K to 355 K. The possible applications of these results to Casimir devices is discussed.

### INTRODUCTION

In 1948 Casimir [1] showed that two parallel plates separated by a distance  $a$  and made of a perfect conductor will attract each other with a force per unit area given by

$$F = -\frac{\pi^2 \hbar c}{240a^4}. \quad (1)$$

This force is attributed to the quantum vacuum fluctuations of the electromagnetic field. Indeed, Casimir forces appear whenever the mode distribution of a fluctuating field is modified by the presence of boundaries [2]. Although Casimir forces are small (0.13 dynes for 1 mm<sup>2</sup> plates separated by one micron) they have been measured through a series of ingenious experiments. Lamoreaux [3] reported an agreement with theory at the level of 5% using an electromechanical system based on a torsion balance. More recent experiments performed by Mohideen with atomic force microscopes achieved precisions close to 1% [4]. In another experiment, a micromachined torsional device was employed to measure the Casimir attraction between a plate and a spherical metallic surface [5]. The original formulation of Casimir was for perfect conductors motivating Lifshitz to propose in 1956 a theory for vacuum forces between semi-infinite dielectric media [6]. The corresponding theory for finite dielectric systems has been developed in last few years [7].

With the advent of novel experimental techniques associated to the development of micro electromechanical systems (MEMS), and instruments such as the atomic force microscope (AFM) different proposals related with the technological uses of the Casimir forces have been investigated. For example, the deflection of a thin microfabricated rectangular strip due to Casimir forces was calculated by Serry *et al.* [8]. According to their results, the strength of these forces is high enough as to buckle the strip and limit the operation of MEMS. Maclay [9] has also suggested to build MEMS devices in order to study the properties and energy balance of MEMS when static or vibrating membranes are

placed on the top of open rectangular cavities. The proposed experimental configuration consists of an array of open rectangular metallic cavities. A top plate suspended by micromechanical springs may be used to measure the sign and magnitude of the Casimir interaction between the plate and the cavity array.

In a recent paper [10], we discussed the possibility of controlling the strength of Casimir forces using heterostructures made of materials with different dielectric properties such as metals and semiconductors. This kind of structures would be also useful in the building of Casimir engines in which part of the energy cycle could be driven by the Casimir interactions. Such a cycle has been proposed by Pinto [11] in order to design a vacuum energy transducer, using optically active elements. He estimated that the power per unit area of this Casimir engine could be as high as  $1 \text{ kW/m}^2$ . Based on these ideas, in this work we study a system made of nanocomposite slabs that exhibit a metal-dielectric transition. In particular, we consider slabs made of  $\text{VO}_2$  precipitates in sapphire [12], whose effective dielectric function is calculated within a mean field approximation. These nanocomposites undergo a first order phase transition which changes their dielectric response from semiconducting to metallic. This behavior would allow to build devices in which the Casimir forces could be modified not only by changing the separation between the slabs, but also by temperature variations as we show in this paper.

## THEORY

Consider two parallel slabs made of a dielectric of thickness  $d$  and separated by a distance  $a$ . The Casimir force between dielectric media considering only wave vectors perpendicular to the slabs is given by [7]

$$F = \frac{2\hbar}{\pi c} \text{Re} \int_0^\infty d\omega \omega \left[ 1 - \frac{1 - |r(\omega)|^2}{|1 - r^2(\omega)e^{2i\omega a/c}|^2} \right] \quad (2)$$

where the  $r(\omega)$  is the frequency-dependent reflection coefficient. To calculate  $r(\omega)$  the dielectric function of the slabs is needed. For a nanocomposite slab made of a host material with a dielectric function  $\epsilon_h(\omega)$  and inclusions with dielectric function  $\epsilon_i(\omega)$ , the effective dielectric response  $\epsilon_M(\omega)$  can be calculated within a mean field approximation as [13]

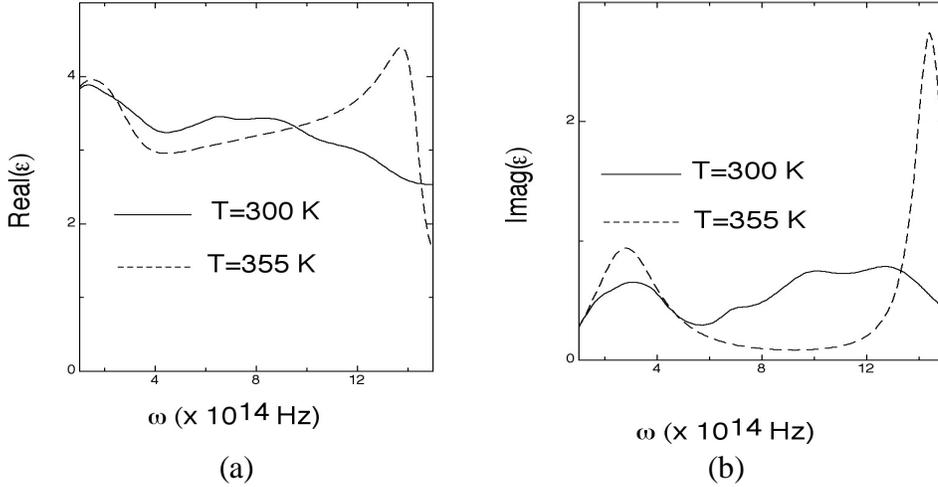
$$\frac{\epsilon_M}{\epsilon_h} = 1 + 3f\alpha + f^2\alpha^2 \left[ 3 + 2 \log \frac{8 + \alpha}{8 - 2\alpha} \right] \quad (3)$$

where  $f$  is the volume fraction of the inclusions and  $\alpha$  is the effective polarizability given by the Maxwell Garnett theory. This expression for the effective dielectric function includes higher order corrections in the volume fraction and is a first approximation beyond

single particle mean field theories. To model the dielectric function of the VO<sub>2</sub> precipitates embedded in sapphire we considered a Lorentz model for both  $\epsilon_i(\omega)$  (sapphire) and  $\epsilon_i(\omega)$  (VO<sub>2</sub>). The parameters appearing in Lorentz formula for these materials have been taken from Ref.[14]. In the case of VO<sub>2</sub>, these parameters change as a function of temperature due to the phase transition.

## RESULTS

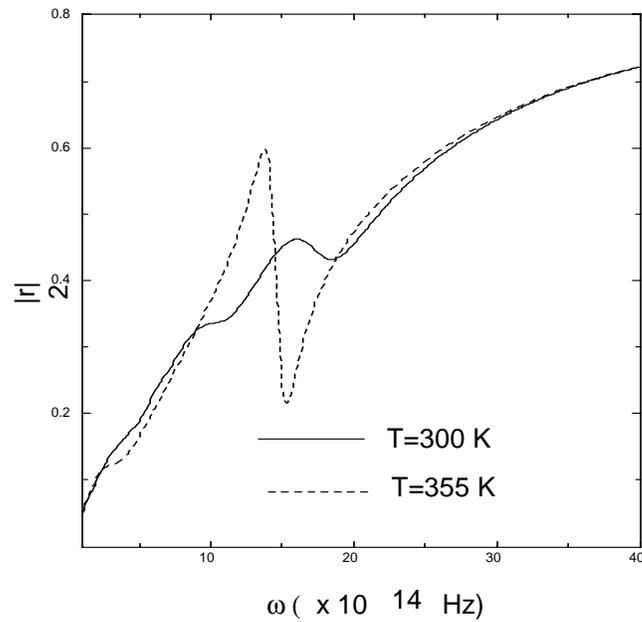
The system we study consists of two parallel planar nanocomposite slabs separated a distance  $a$  and with a thickness of  $d=0.1$  microns. The volume fraction of the VO<sub>2</sub> precipitates was arbitrarily set at 20 %. In figures (1a) and (1b), we present the real and imaginary parts of the dielectric function as calculated from Eq.(3). The dielectric function is shown for two different temperatures,  $T_{sc} = 300$  K, and ,  $T_m = 355$  K. The first temperature corresponds to the semiconducting state, and the second to the metallic one.



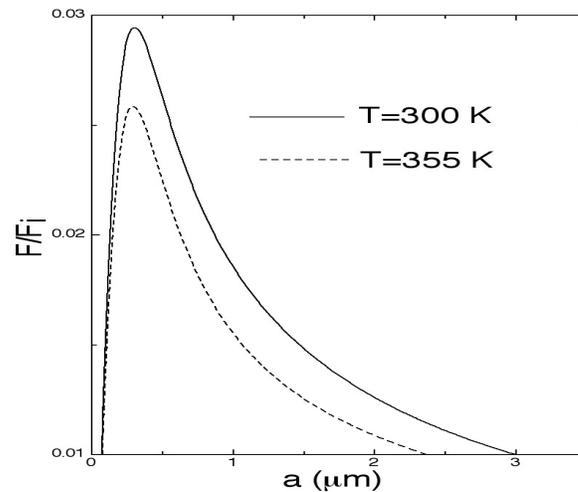
**Figure 1.** (a) Frequency dependence of the real part of the effective dielectric function for the semiconducting ( $T_{sc} = 300$  K) and metallic phases ( $T_m = 355$  K). (b) Frequency dependence of the imaginary part of the effective dielectric function for the semiconducting ( $T_{sc} = 300$  K) and metallic phases ( $T_m = 355$  K).

In figure (2) we present the calculated reflectance of a nanocomposite slab for the semiconducting and metallic phases based on our results for  $\epsilon_M(\omega)$ . The metallic case shows a significant deviation with respect to the semiconducting one at an angular frequency of  $1.5 \times 10^{15}$  Hz. The changes in  $r(\omega)$  as a function of temperature should be reflected in the calculation of the Casimir force. This is indeed the case, as shown in figure (3) where we plot the force as a function of the separation  $a$ . The force is normalized to the ideal case. The ideal case is when the slabs are made of a perfect conductor. As shown in the figure, after a separation of roughly  $0.3 \mu\text{m}$  the magnitude of the force is different. This is understood from Eq. (2) since at a given separation of the slabs the modes that will

contribute more to the force are those with a wavelength that fit within the slabs. This is, modes with a frequency of  $\omega = \pi/a$ . The region of small separations ( $a < 0.2 \mu\text{m}$ ) the force is the same at the two temperatures since these separations correspond to frequencies at which the reflectivity of the material is very small and the mode density within the slabs is similar to that of vacuum resulting in a negligible Casimir force.



**Figure 2.** Frequency dependence of the reflectance of the nanocomposite slab at  $T_{sc}$ , and  $T_m$ . In the frequency region around  $1.5 \times 10^{15}$  Hz the reflectance differs at the two temperatures.



**Figure 3.** Casimir force as a function of the nanocomposite slab separation  $a$ . The dashed curve corresponds to the metallic phase, while the continuous one corresponds to the semiconducting phase. The force is normalized with respect to the perfectly conducting case given by equation (1).

## CONCLUSIONS

We have explored the possibility of using the dielectric properties of nanocomposite slabs that exhibit a metal-dielectric transition to modulate the Casimir forces. In this work, we considered slabs made of  $\text{VO}_2$  precipitates in sapphire. The results for the Casimir force as a function of the separation of the slabs show that at a fixed separation the magnitude of the force changes as temperature increases from 300 K to 355 K. This effect could be used in principle, as part of a thermodynamic cycle of a micromachined motor similar to the one proposed by Pinto [11] but working at ambient temperature.

## ACKNOWLEDGMENTS.

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