Light Source Design – Part 3: Beam Stability Requirements

Wednesday 24 Nov 2010 at 09:00 (01h00')

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Co-authors :

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Photon Beam Stability Requirements for Light Source Users and How They Translate to Electron Beam Parameters

- an elementary analysis -

R. Hettel, SLAC

Mexican Light Source Workshop

November 22-24,2010

Photon Stability Requirements How They Translate to Electron Beam Parameters

- User requirements
- SR sensitivity to electron parameters
- Electron beam properties
- Photon-electron relationships
- Stability time scales
- Derivation of basic stability requirements
- Conclusions and example of “integrated” solution to instability on beam line

Beam Stability for SR Experiments

Beam Stability for SR Experiments

Users want stability of:

- flux (and coherent flux) after apertures **< 0.1% (0.01% for some dichroism)**
- steering accuracy on small samples **< few % photon beam dimensions**

  pointing accuracy
- e- trajectory in source magnets **< few % e- beam dimensions**

  emission pattern, off-axis
  energy pattern, polarization, etc.
- photon energy and energy spread **< 10^{-4} resolution**
- timing **< 10% of critical time scale**

  pump-probe, etc.
- beam lifetime **hours**

Beam stability characterized in 6-D phase space: (x, x', y, y', E, t)

Beam Stability Criteria

Beam stability requirements depend on:

- beam line optical configuration and apertures
- sample size
- measurement technique and instrumentation
- data acquisition time scale
- data averaging and processing methods

Stability is relative:

Beam Stability Criteria

Beam stability requirements depend on:

- beam line optical configuration and apertures
- sample size
- measurement technique and instrumentation
- data acquisition time scale
- data averaging and processing methods

Stability is relative:

- flux constancy with respect to apertures within the 6-D acceptance phase space of the experiment

While stability requirements vary, generic requirements can be estimated from criteria common to many experiments

Beam Stability Criteria – cont.

In the "old days", types of photon beam instability could be divided into 2 categories:

- those associated with beam line optical components and experimental apparatus

the beam line staff's problem!

- those associated with the electron beam

the accelerator staff's problem!

In the "new days" with very low emittance rings and high performance beam lines:

Beam Stability Criteria – cont.

In the "old days", types of photon beam instability could be divided into 2 categories:

- those associated with beam line optical components and experimental apparatus

the beam line staff's problem!

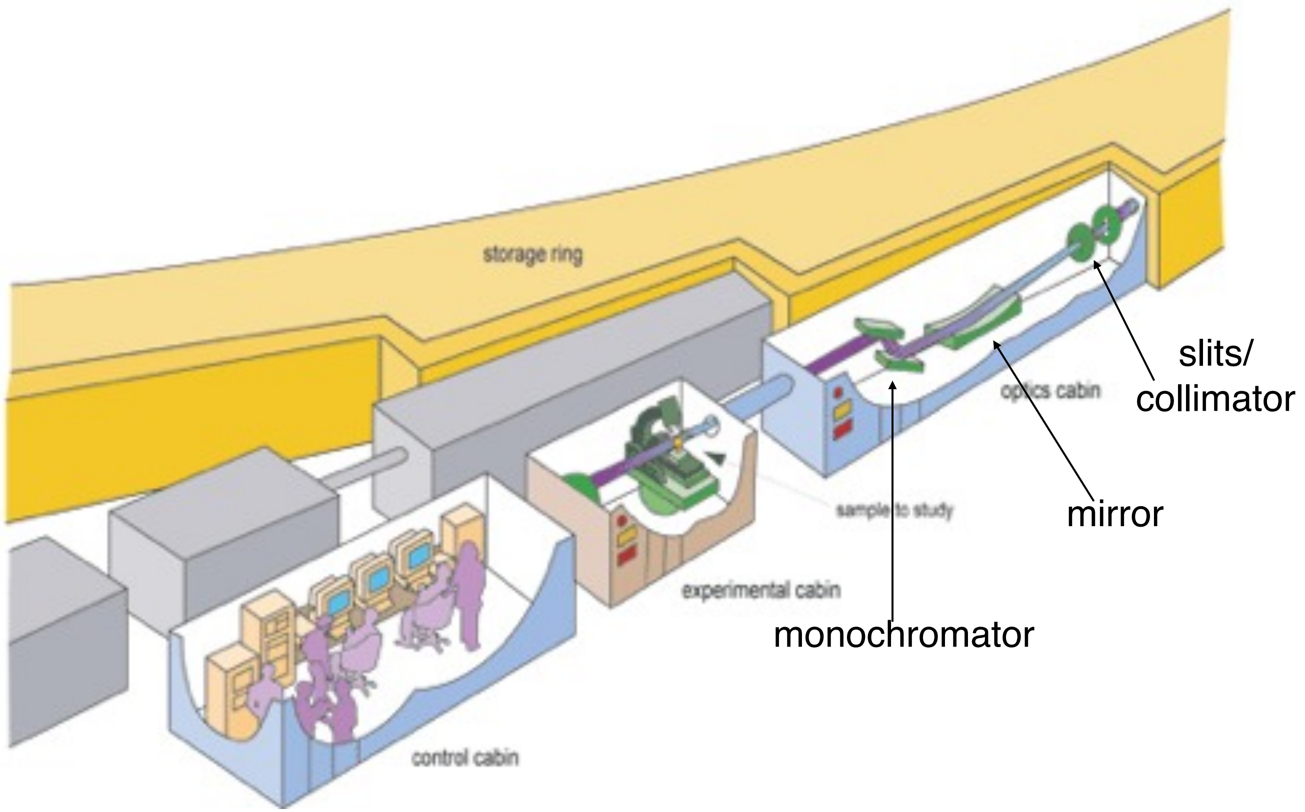
- those associated with the electron beam

the accelerator staff's problem!

In the "new days" with very low emittance rings and high performance beam lines:

noise problems require integrated solutions involving both beam line and accelerator staff

SR Generic Beam Line



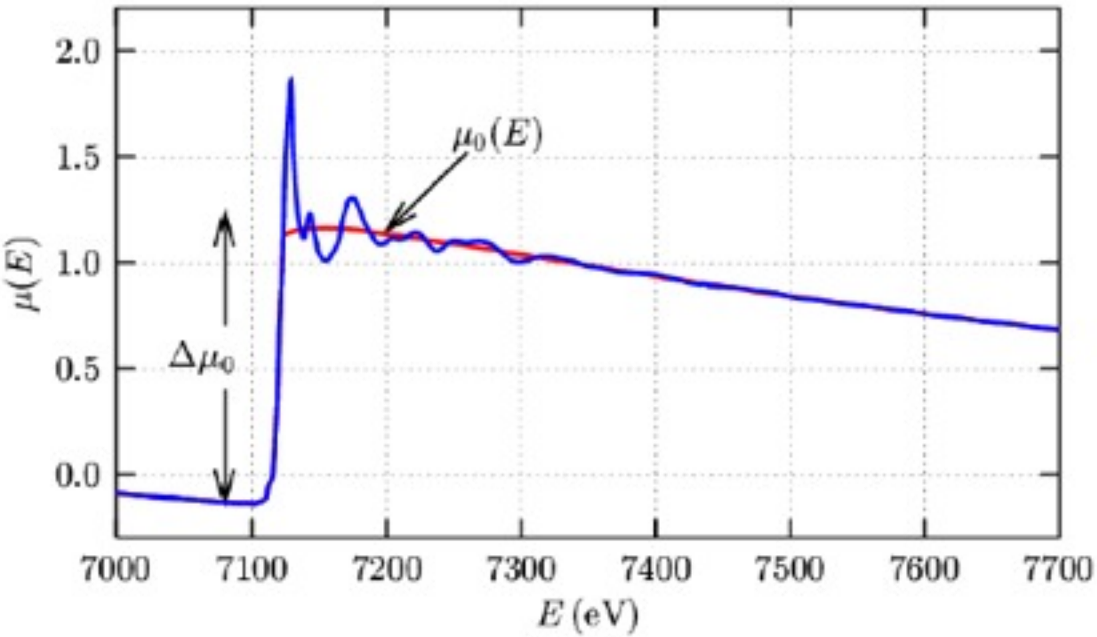
could be more apertures (slits, etc) than shown

XAFS Measurement

We're interested in the energy-dependent oscillations in $\mu(\mathrm{E})$, as these will tell us something about the neighboring atoms, so define the EXAFS as:

$$\chi(\mathbf{E}) = \frac{\mu(\mathbf{E}) - \mu_0(\mathbf{E})}{\Delta\mu_0(\mathbf{E}_0)}$$

We subtract off the smooth *"bare atom" background* $\mu_0(\mathrm{E})$, and divide by the *"edge step"* $\Delta\mu_0(\mathrm{E}_0)$ to give the oscillations normalized to 1 absorption event:



from M. Newville, CARS, U. Chicago, 2002

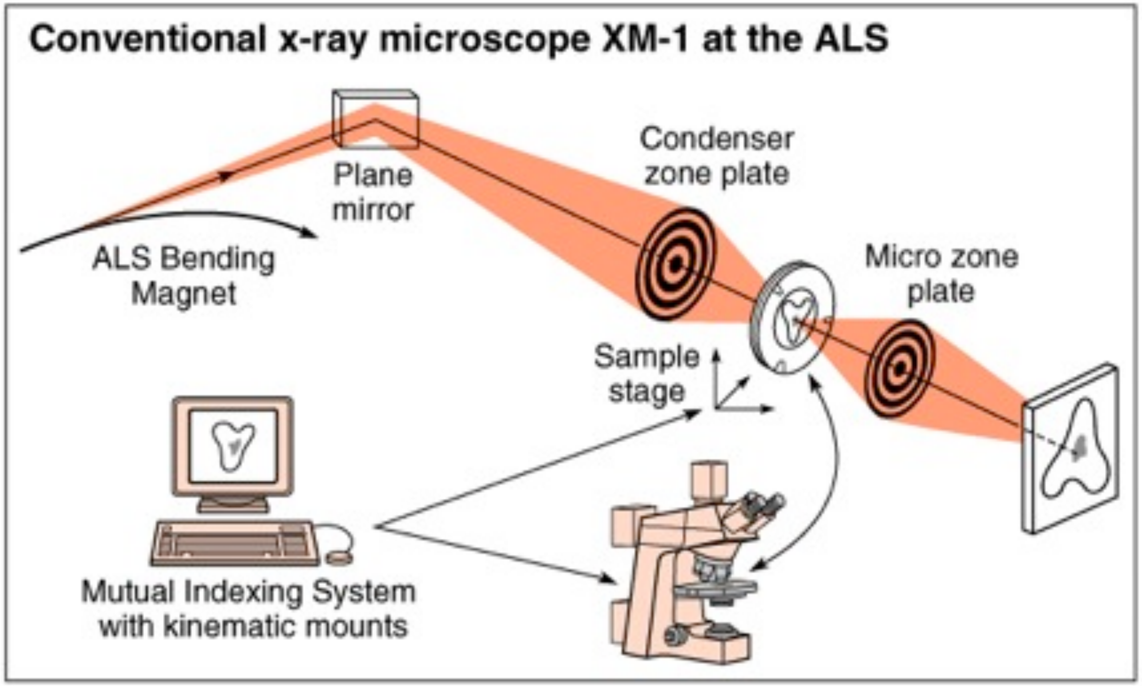
SR requirements:

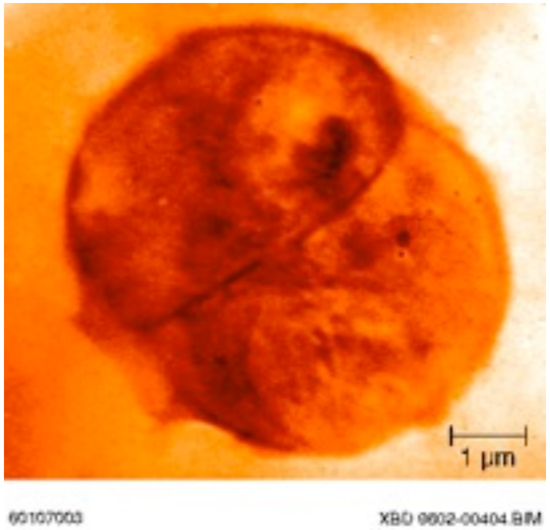
intensity stability: **10^{-3}**

energy resolution: **10^{-4}**

X-ray Microscopy and Micro-diffraction

Focus spot size to micron level to examine single micron-sized structures





white or monochromatic light, 100-1000 eV

SR requirements:

intensity stability: **10^{-3}**

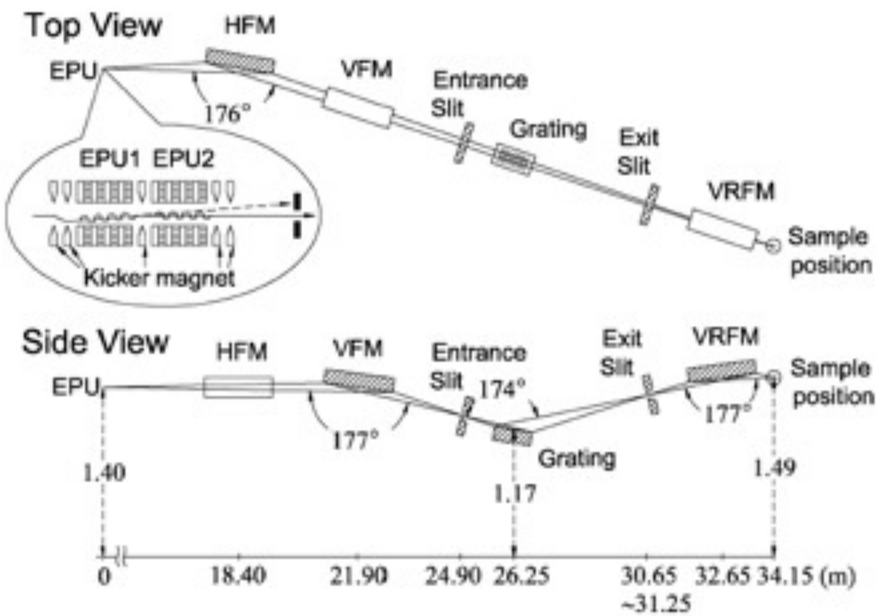
position stability: **~1 μm**

Circular Dichroism Beam Lines

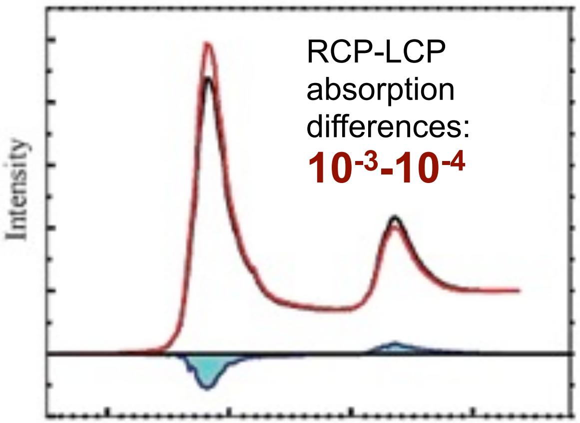
SR requirements:

intensity stability: <10^{-4}

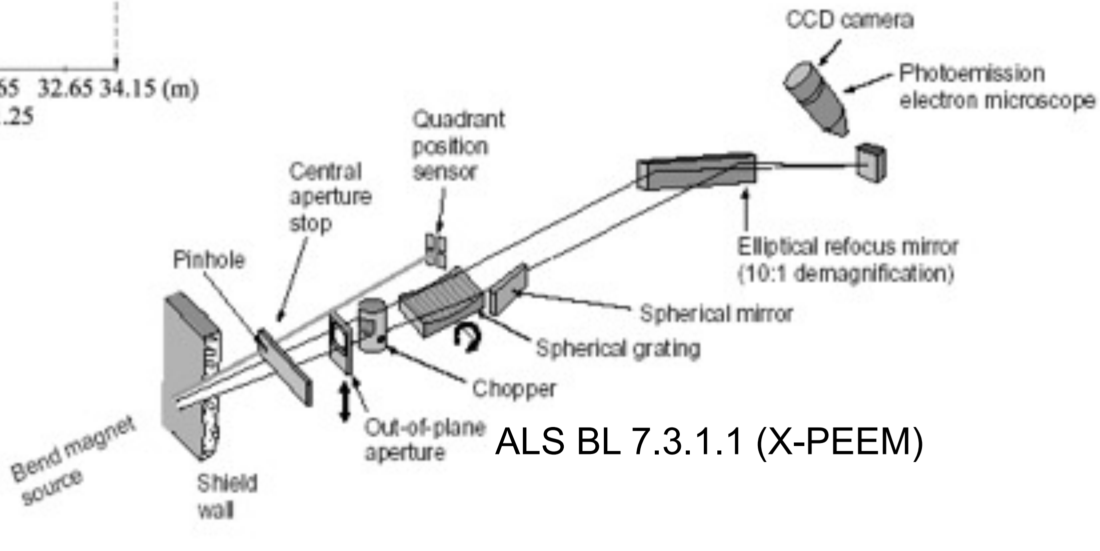
position stability: ~1 μm



PLS EPU6



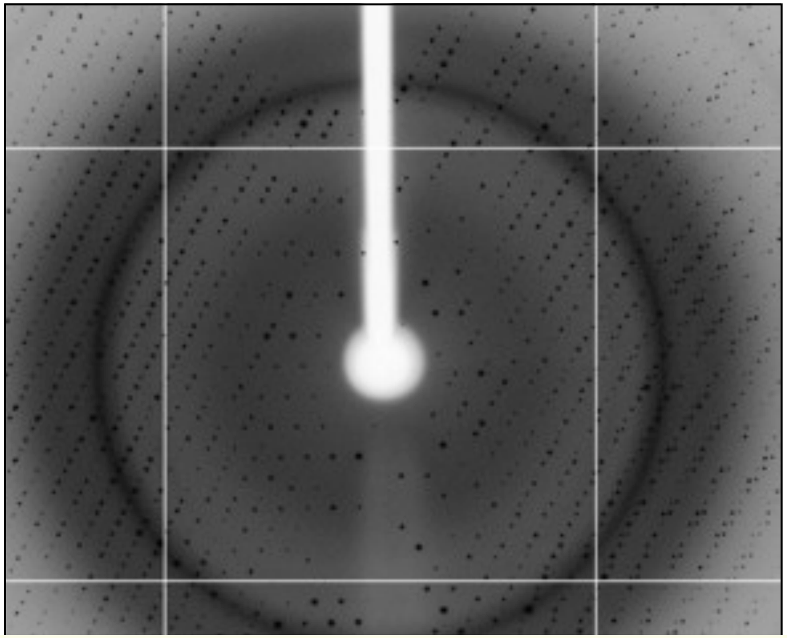
RCP-LCP absorption differences: 10^{-3}-10^{-4}



Schematic layout of Branchline 7.3.1.1.

ALS BL 7.3.1.1 (X-PEEM)

Macromolecular Crystal Diffraction Patterns



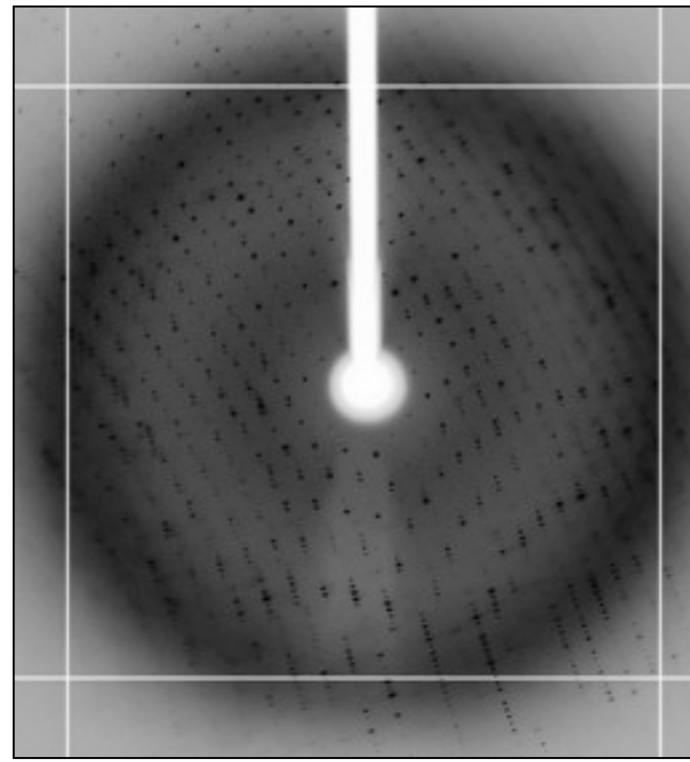
low mosaicity crystal

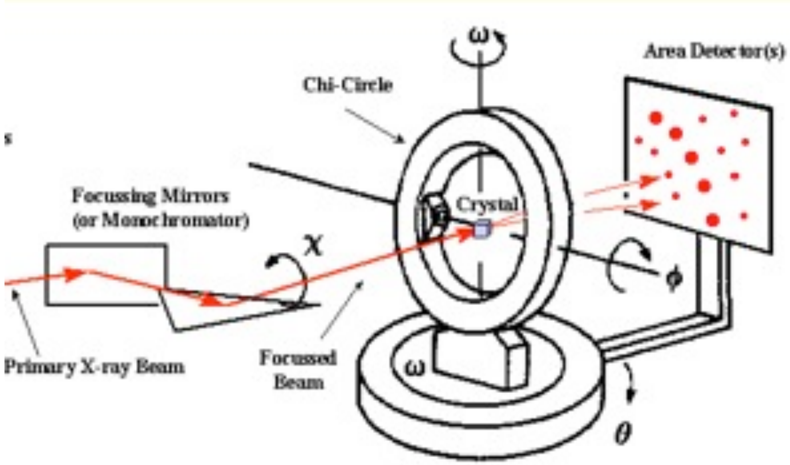
SR requirements:

intensity stability: **10^{-3}**

energy resolution: **10^{-4}**

high mosaicity crystal





4-Circle Goniometer (Eulerian or Kappa Geometry)

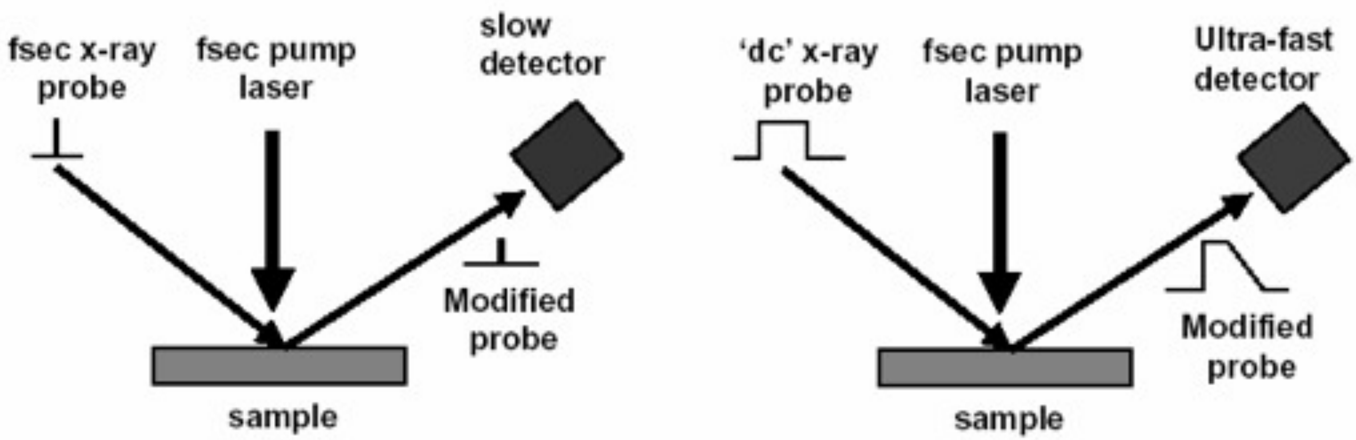
Femtosecond X-ray Spectroscopy and Diffraction

H. Padmore, ALS

Goal: measurement of structure on the fundamental time scale of a vibrational period ~100 fs

Research areas: ultrafast phase transitions, chemical reactions, biological processes

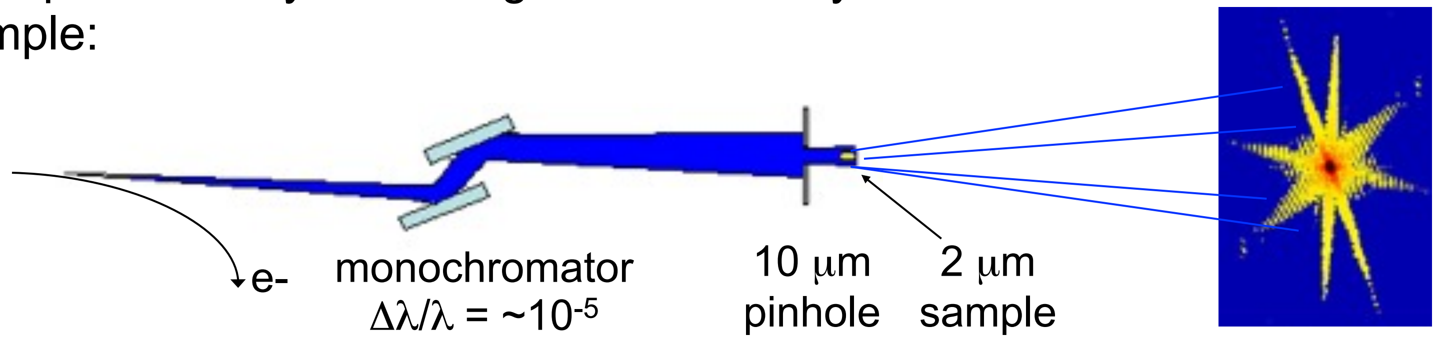
Probes: x-ray diffraction; ordered systems, structural phase transitions
spectroscopy; disordered/complex materials, chemical reactions



Timing stability requirement: pump-probe timing synchronization **< ~50 fs**, or else be able to measure actual shot-shot synchronization to that level

Coherence Experiments

Speckle pattern produced by scattering of transversely coherent photons in sample:



Longitudinal coherence length > sample thickness to obtain coherent speckle pattern

Longitudinal coherence length increased using narrow bandwidth monochromator:

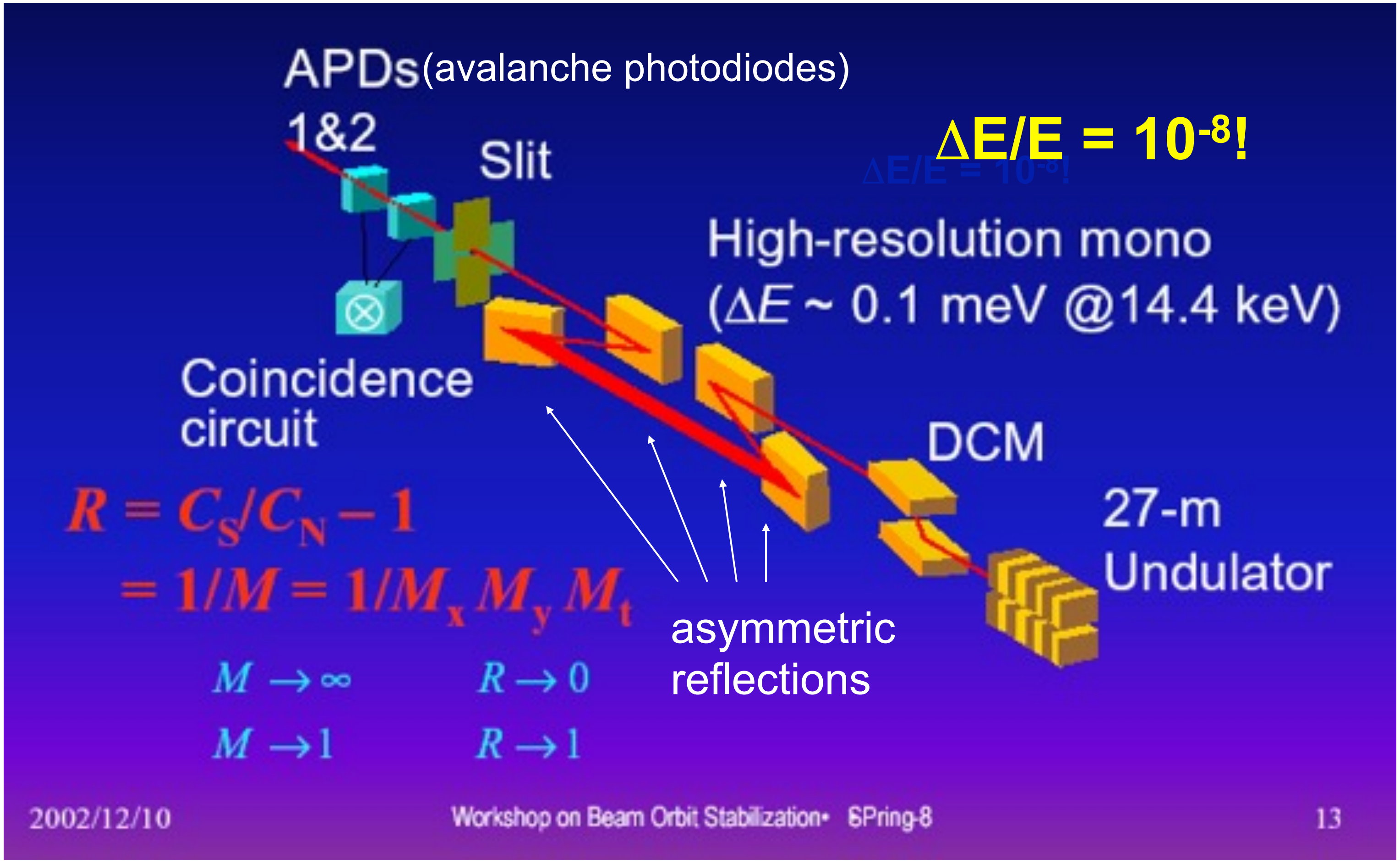
$$l_{coh} = \lambda_{ph}(\lambda/\Delta\lambda)_{mono} = \sim 20\ \mu\text{m for } 2\ \text{Å photons}$$

SR requirements:

intensity stability at sample: **~$10^{-3} - 10^{-4}$**

X-ray Intensity Interferometry

T. Ishikawa



Hambury Brown-Twiss Interferometer at SPring-8

Stability Relationships

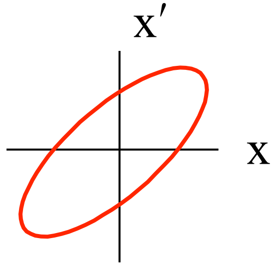
Can derive basic some basic relationships experimental observables and beam properties based simple (1st-order) dependencies (-- = 2nd order):

experiment parameters	e- orbit	e- size/ rotation	e- energy/ energy spread
intensity	x	x	x
energy resolution	x	--	x
timing, bunch length	--	x	x

Electron Beam Properties

Electron beam characterized by conjugate variable pairs in 6-D phase space:

x, x′	**y, y′**	**E, t (or ϕ)**
-------- transverse ----------		**longitudinal**



Electron Beam Properties

Electron beam characterized by conjugate variable pairs in 6-D phase space:

x, x' **y, y'** **E, t (or ϕ)**

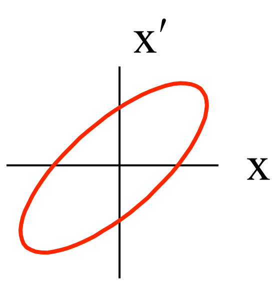
-------- transverse ---------- **longitudinal**

For each conjugate pair, beam occupies phase space ellipse of constant area - or emittance (A = $\pi\varepsilon$)

transverse:
$$\varepsilon = \gamma(s)x^2 + 2\alpha(s)xx' + \beta(s)x'^2 = \text{constant} \qquad \left(\alpha = -\beta'/2 \qquad \gamma = \frac{1+\alpha^2}{\beta}\right)$$
$$\varepsilon_y \cong k\varepsilon \qquad (k = \text{coupling, } k <\sim 0.1)$$

e- beam size:
$$\sigma_x(s) = \sqrt{\varepsilon_x\beta_x(s) + (\eta(s)\delta E/E)^2} \qquad \sigma_y(s) = \sqrt{\varepsilon_y\beta_y(s)}$$

e- divergence:
$$\sigma_{x'}(s) = \sqrt{\varepsilon_x\gamma_x(s) + (\eta'(s)\delta E/E)^2} \qquad \sigma_{y'}(s) = \sqrt{\varepsilon_y\gamma_y(s)}$$



longitudinal:
$$\Delta\phi\ (\text{rad}) = \frac{h\alpha_c}{\nu_s}\frac{dE}{E} \qquad \left(= \sim 40\frac{dE}{E} \text{ for SPEAR3}\right)$$

Electron Beam Properties – cont.

Have coupling between phase space planes:

- H-V by skew quads, orbit in sextupoles, resonances
- longitudinal-transverse (energy-orbit, $\Delta x = \eta \Delta E/E$)
- photon energy dependent on orbit through IDs
- photon polarization dependent on vertical orbit through dipole
- etc.

Experiment Sensitivity to Electron Beam Parameters

Response of experiment observable parameters to source point electron beam parameters: sensitivity matrix M(i,j)

$$[\Delta P_{exp}(i)] = \mathbf{[M(i,j)]}\ [\Delta P_{e-}(j)]$$

where

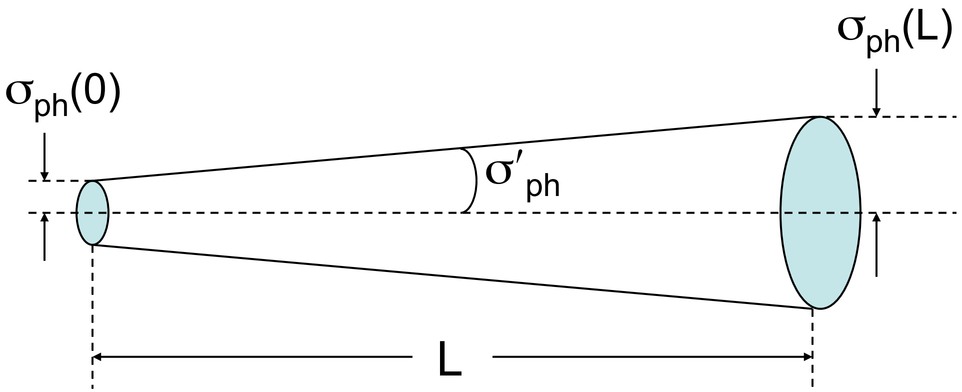
$$[\Delta P_{exp}(i)] = \begin{pmatrix} \Delta I_{ph} \\ \Delta E_{ph} \\ \Delta E_{ph}/E_{ph}\ (\mathrm{rms}) \\ \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\sigma'_x \\ \Delta\sigma'_y \\ \Delta\sigma_z \\ \Delta x \\ \Delta y \\ \Delta x' \\ \Delta y' \\ \Delta t_{bunch} \\ \text{polarization} \\ \text{coherence} \end{pmatrix}
\qquad
[\Delta P_{e-}(i)] = \begin{pmatrix} \Delta I_{e-} \\ \Delta E_{e-} \\ \Delta E_{e-}/E_{e-}\ (\mathrm{rms}) \\ \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\sigma_z \\ \Delta\sigma'_y \\ \Delta\sigma_z \\ \Delta\theta_{rot} \\ \Delta x \\ \Delta y \\ \Delta x' \\ \Delta y' \\ \Delta t\ _{bunch} \end{pmatrix}$$

Photon-Electron Relationships

Photon beam size:

- unfocused, vertical plane:

(assume depth of field = 0)



$$\sigma_{ph}(L) = [\sigma_{ph}(0)^2 + L^2\sigma_{ph}'^2]^{1/2}$$

$$\sigma_{ph}(0) = [\sigma_{e-}^2 + \sigma_{diff}^2(\lambda)]^{1/2}$$

$$\sigma_{e-} = [\varepsilon\beta(s) + (\eta(s)\delta)^2]^{1/2}$$

$$\sigma_{diff} = \frac{\lambda}{4\pi\sigma'_{\psi}(\lambda)} \left(= \frac{\sqrt{2L_u\lambda}}{4\pi} \text{ for undulator length } L_u \right)$$

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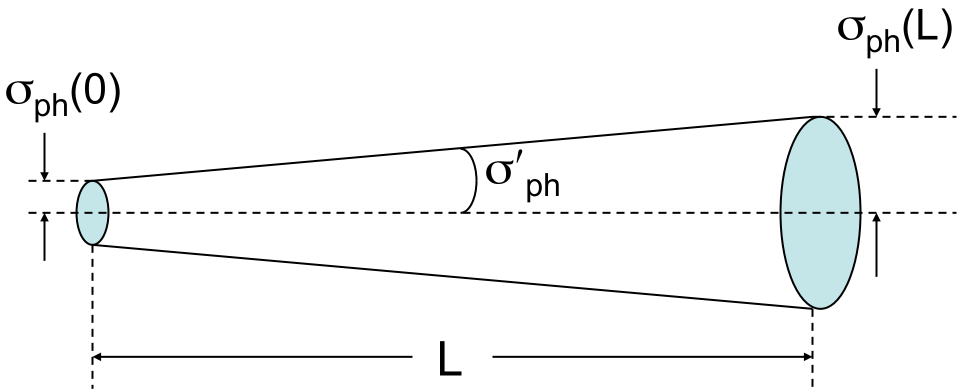

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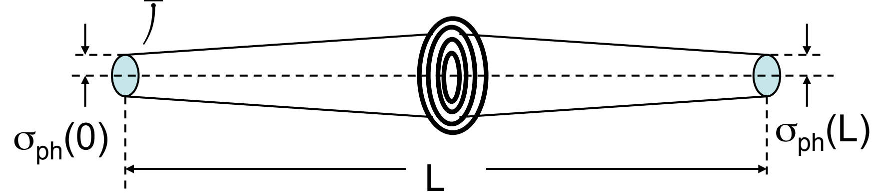
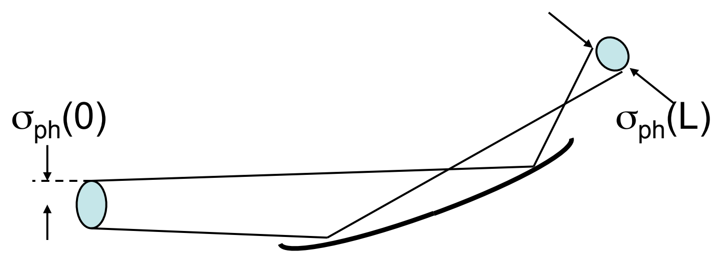


- focused (1:m, m = magnification):

$\sigma_{ph}(L) = m\sigma_{ph}(0)$

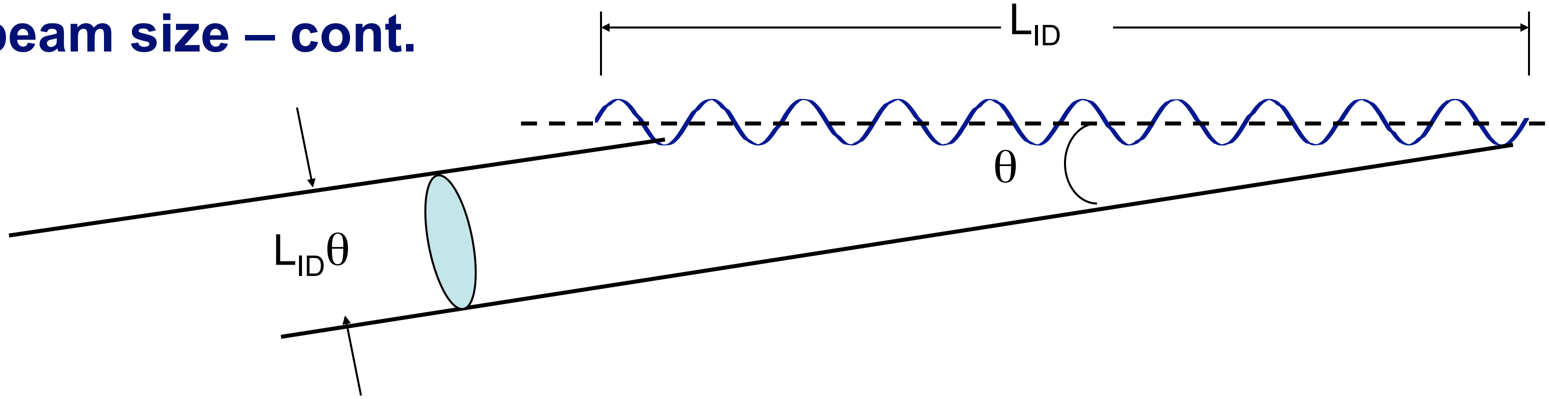
(~insensitive to angle)



$\sigma'_{ph}(L) = -\sigma'_{ph}(0)/m$

Photon-Electron Relationships – cont.

Photon beam size – cont.



- Off-axis view of ID radiation adds to focused beam size due to extended source
- On-axis beam size has additional terms arising from wiggle amplitude and ID length:

$$\sigma_{Tx}^2 = \sigma_r^2 + \sigma_x^2 + a^2 + \tfrac{1}{12}\sigma_{x'}^2 L^2 + \tfrac{1}{36}\varphi^2 L^2 \qquad \sigma_{Tx'}^2 = \sigma_{r'}^2 + \sigma_{x'}^2$$

$$\sigma_{Ty}^2 = \sigma_r^2 + \sigma_y^2 + \tfrac{1}{12}\sigma_{y'}^2 L^2 + \tfrac{1}{36}\psi^2 L^2 \qquad \sigma_{Ty'}^2 = \sigma_{r'}^2 + \sigma_{y'}^2$$

from I.V.Bazarov

- DIpole source size is slightly increased from finite depth of field and orbit arc

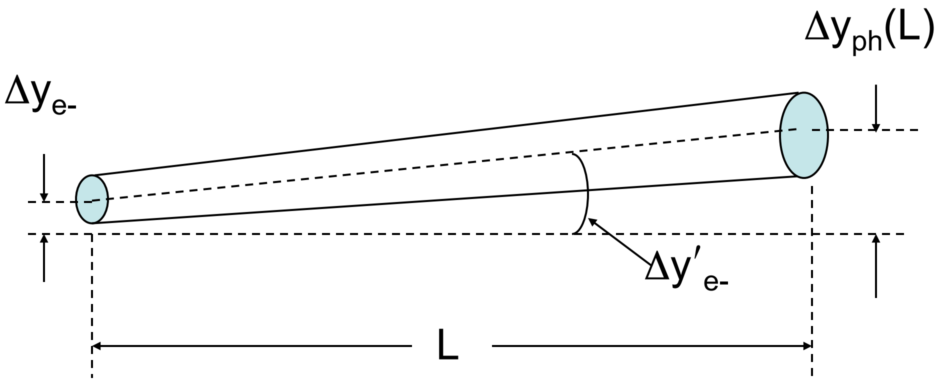
Photon-Electron Relationships – cont.

Beam line steering:

- pointing parameters (1st order) for **unfocused** photon centroid:

$$\Delta y_{ph}(L) = \Delta y_{e-} + L\Delta y'_{e-}$$

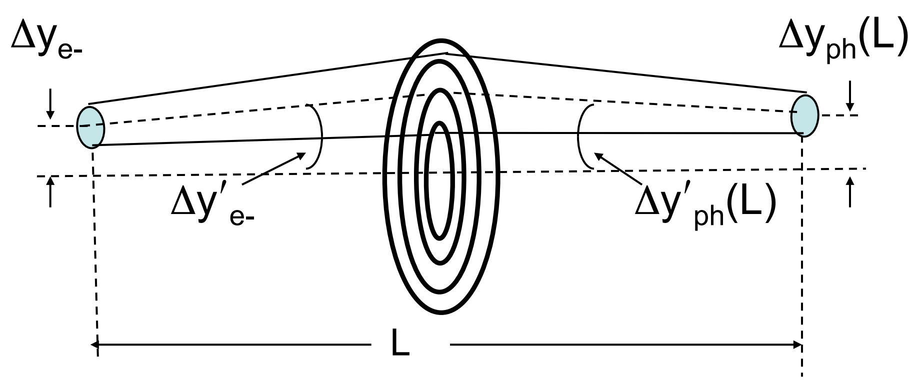
$$\Delta y'_{ph}(L) = \Delta y'_{e-}$$



focused (1:m) photon centroid:

$$\Delta y_{ph}(L) = m\Delta y_{e-}$$

$$\Delta y'_{ph}(L) = -\Delta y'_{e-}/m$$



Photon-Electron Relationships – cont.

Photon beam divergence:

$\sigma'_{ph}(L) = \sigma'_{ph}(0) = [\sigma'_{e-}{}^2 + \sigma'_{\Psi}{}^2]^{1/2}$ $\qquad$ $\sigma'_{e-} = [\varepsilon\gamma(s) + (\eta'\delta)^2]^{1/2}$

Photon-Electron Relationships – cont.

Photon beam divergence:

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for dipoles and wigglers:

$$\lambda_c = \frac{2\pi c}{\omega_c} = \frac{hc}{E_c} \qquad \text{h = Planck's const.}$$

= 4.14 x 10^{-18} keV-s

$$E_c(\text{keV}) = \frac{3hc\gamma^3}{2\rho} = 0.665\,B(T)\,E^2(\text{GeV})$$

$$\sigma'_{\psi}(\lambda) \cong \begin{cases} \dfrac{1.07}{\gamma}\left(\dfrac{\lambda}{\lambda_c}\right)^{1/3} & \lambda >> \lambda_c \\[2ex] \dfrac{0.64}{\gamma} & \lambda = \lambda_c. \\[2ex] \dfrac{0.58}{\gamma}\left(\dfrac{\lambda}{\lambda_c}\right)^{1/2} & \lambda << \lambda_c \end{cases}$$

Photon-Electron Relationships – cont.

Photon beam divergence:

$$\sigma'_{ph}(L) = \sigma'_{ph}(0) = [\sigma'^{\,2}_{e-} + \sigma'^{\,2}_{\psi}]^{1/2} \qquad \sigma'_{e-} = [\varepsilon\gamma(s) + (\eta'\delta)^2]^{1/2}$$

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for planar undulators:
(on-axis, central cone)

$$\sigma'_{\psi}(n) = \sqrt{\frac{\lambda_n}{2L_u}} = \frac{1}{\gamma}\left[\frac{\lambda_u(1+K^2/2)}{4nL_u}\right]^{1/2} = \frac{1}{\gamma}\left[\frac{1+K^2/2}{4nN_u}\right]^{1/2}$$

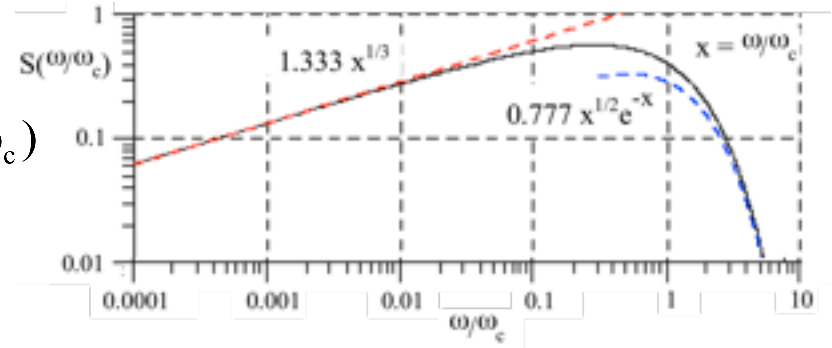
n = harmonic # Lu = undulator length λu = undulator period Nu = # periods K = ~1

Photon-Electron Relationships – Photon Emission

K-J Kim

Dipole spectral flux density (per horizontal mrad, integrated over vertical angle):

$$\frac{dF_{dip}(\omega)}{d\theta} = 2.457 \times 10^{16} \frac{\Delta\omega}{\omega} E_{e-}^{\ 2}(\text{GeV})\, I(\text{A})\, S(\omega/\omega_c)$$



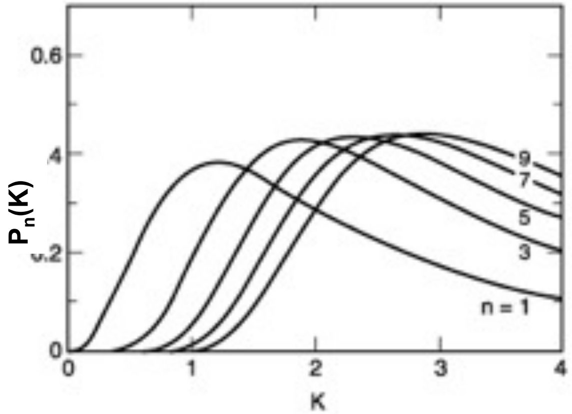
Wiggler spectral flux density:

$$\frac{dF_{wigg}(\omega)}{d\theta} =\sim N_{wigg} \frac{dF_{dip}(\omega)}{d\theta} \qquad N_{wigg} = \#\,\text{wiggler poles}$$

Undulator spectral flux density:

$$\left.\frac{d^2F_{und}(\omega_n)}{d\theta\, d\varphi}\right|_{\varphi,\theta=0} = 1.744 \times 10^{14} \frac{\Delta\omega}{\omega} N_u^{\ 2} E_{e-}^{\ 2}(\text{GeV})\, I(\text{A})\, P_n(K)$$

N_u = # undulator periods



Undulator Radiation

Angular distribution of 1st harmonic:

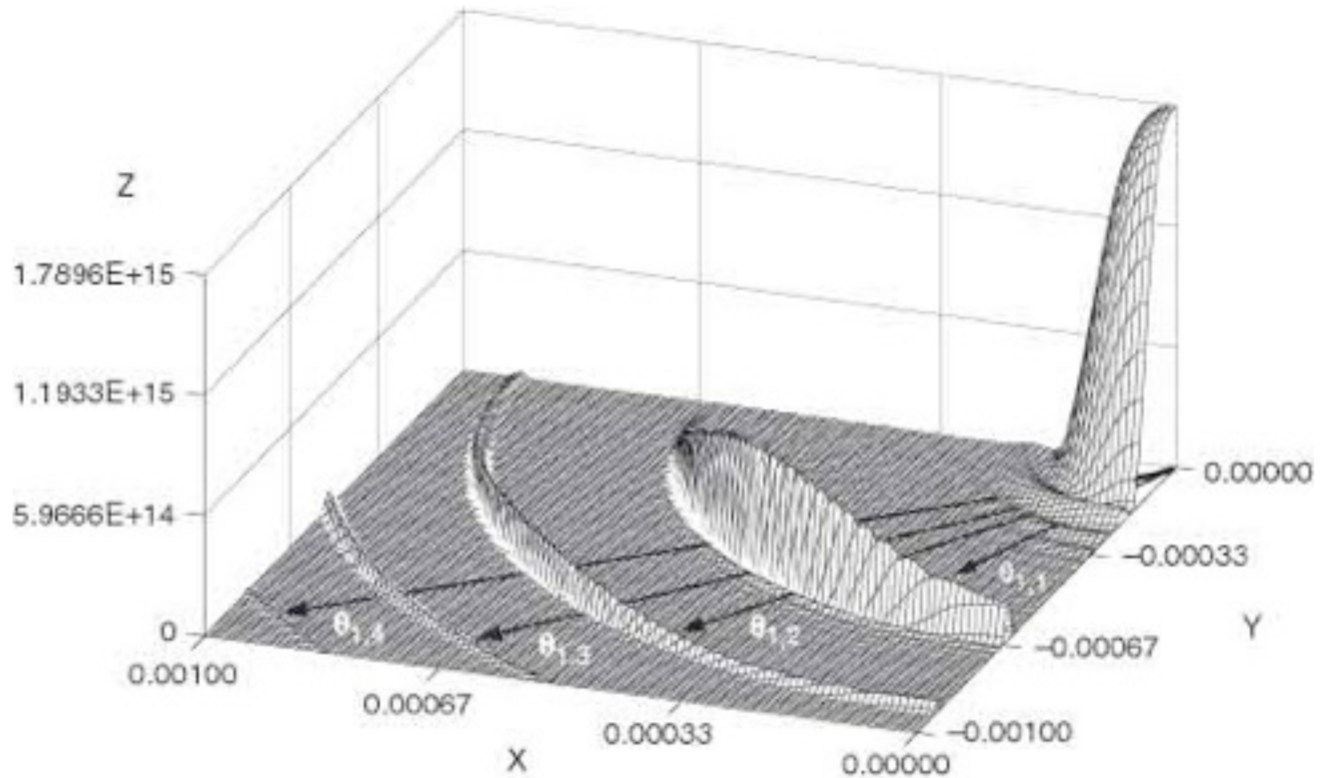


Fig. 2-5. *The angular distribution of fundamental ($n = 1$) undulator radiation for the limiting case of zero beam emittance. The x and y axes correspond to the observation angles θ and ψ (in radians), respectively, and the z axis is the intensity in photons·s^{-1}·A^{-1}·$(0.1\ mr)^{-2}$·$(1\%\ bandwidth)^{-1}$. The undulator parameters for this theoretical calculation were $N = 14$, $K = 1.87$, $\lambda_u = 3.5$ cm, and $E = 1.3$ GeV. (Figure courtesy of R. Tatchyn, Stanford University.)*

K-J Kim, from X-ray Data Booklet, LBNL

Photon-Electron Relationships – cont.

Typical photon beam dimensions

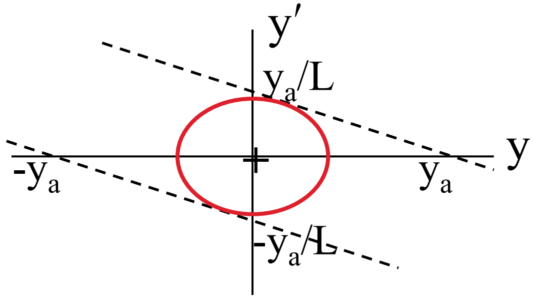
3 GeV 3rd generation source with ε = ~3 nm-rad, 0.1% coupling, E_c = 7.5 keV:

	dipole/wiggler		undulator (N=100, n=1, E = 2 keV)	
	hor	vert	hor	vert
σ_{e-} (μm)	75-300	7-20	75-300	7-20
σ'_e (μrad)	10-50	1-3	10-50	1-3
σ_{diff} (E_c) (μm)	0.12	0.12	3.6	3.6
σ'_ψ (E_c) (μrad)	107	107	14	14
σ_{ph} (E_c) (μm)	**75-300**	**7-20**	**75-300**	**8-21**
σ'_{ph} (E_c) (μrad)	**mrads**	**107**	**17-52**	**14**

For 100-period undulator, n = 7 (~12 keV), **σ'_{ph} (n = 7) = 5-6 μrad**

Experiment in Phase Space

Can represent experiment configuration in phase space

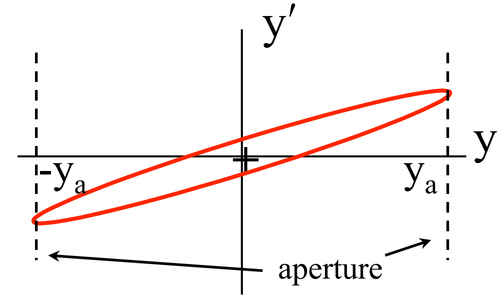


source point

$$\xleftarrow{\quad -L \quad}$$

$$y_{exp} = y_{sp} + Ly'_{sp}$$
$$y'_{exp} = y'_{sp}$$

$$\xrightarrow{\quad L \quad}$$

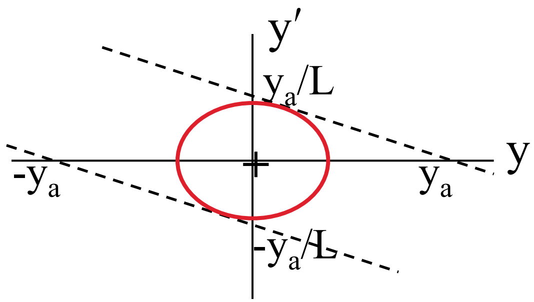


experiment

$$\begin{pmatrix} y \\ y' \end{pmatrix}_{s_1+L} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}_{s_1}$$

Experiment in Phase Space

Can represent experiment configuration in phase space

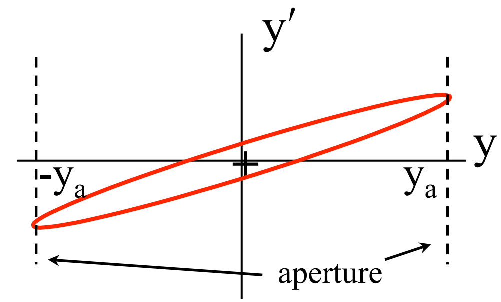


source point

-L ←

$$y_{exp} = y_{sp} + Ly'_{sp}$$
$$y'_{exp} = y'_{sp}$$

L →

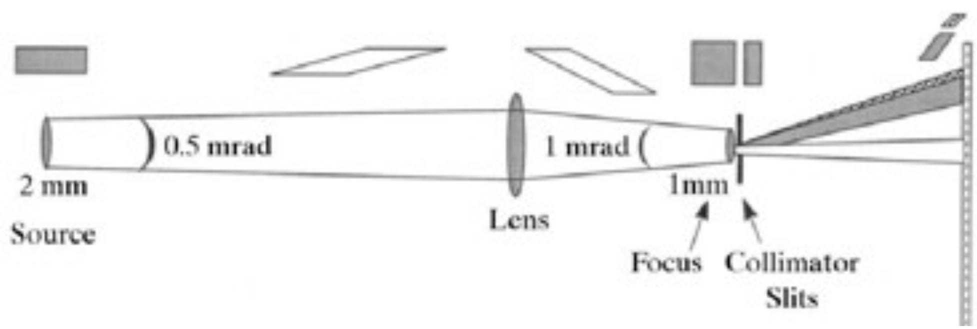


experiment

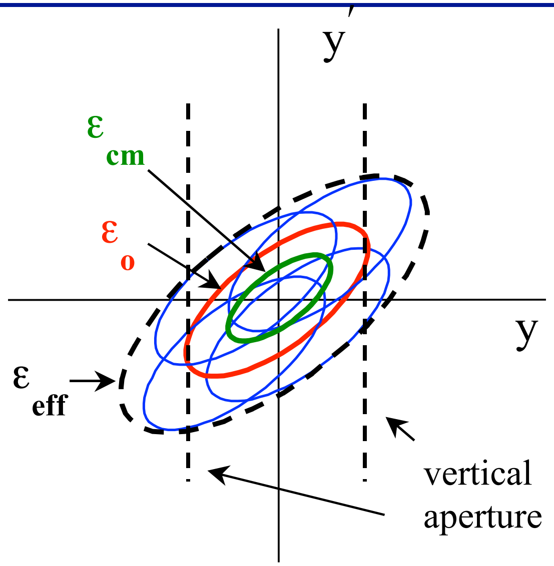
$$\begin{pmatrix} y \\ y' \end{pmatrix}_{s_1+L} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}_{s_1}$$

Can propagate beam phase space through beam line with transport matrices representing drifts, reflections, focusing, etc. **– ray tracing programs**

C. Nave



Beam Position Instability and Emittance Growth from Orbit Motion



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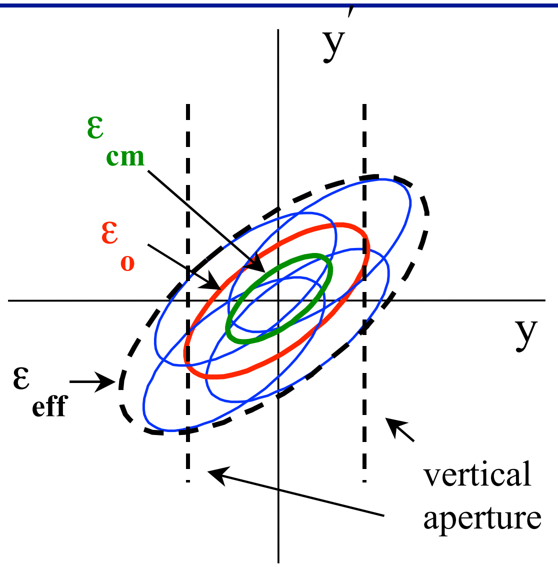

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(effective "blow-up" of emittance ellipse)

$\varepsilon = \varepsilon_o + \varepsilon_{cm}$ $\Delta\varepsilon/\varepsilon = \varepsilon_{cm}/\varepsilon_o$

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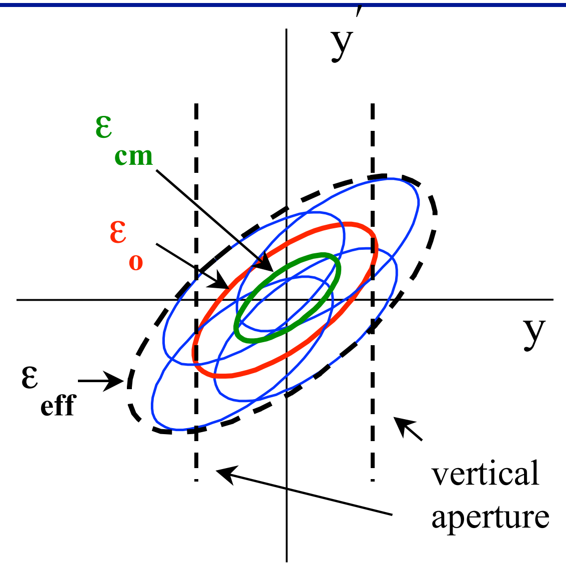

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("coherent displacement" of nominal emittance ellipse)

$\varepsilon \text{ (envelope)} = \varepsilon_o + 2\sqrt{\varepsilon_o \varepsilon_{cm}} + \varepsilon_{cm}$ $\Delta\varepsilon/\varepsilon \cong 2\sqrt{\varepsilon_{cm}/\varepsilon_o}$

($\varepsilon_{cm} << \varepsilon_o$; L. Farvacque, ESRF)

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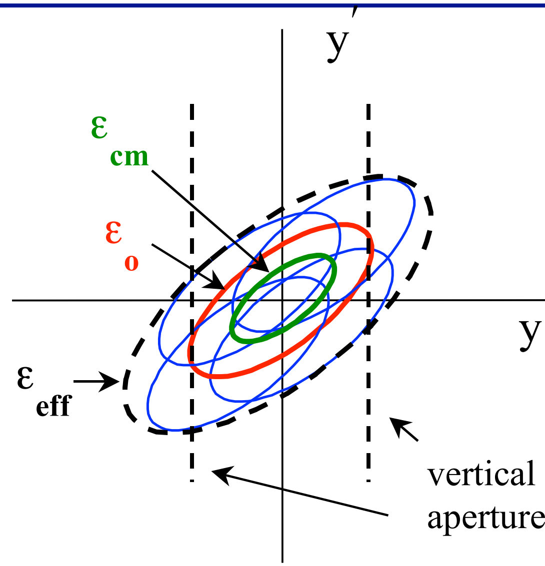

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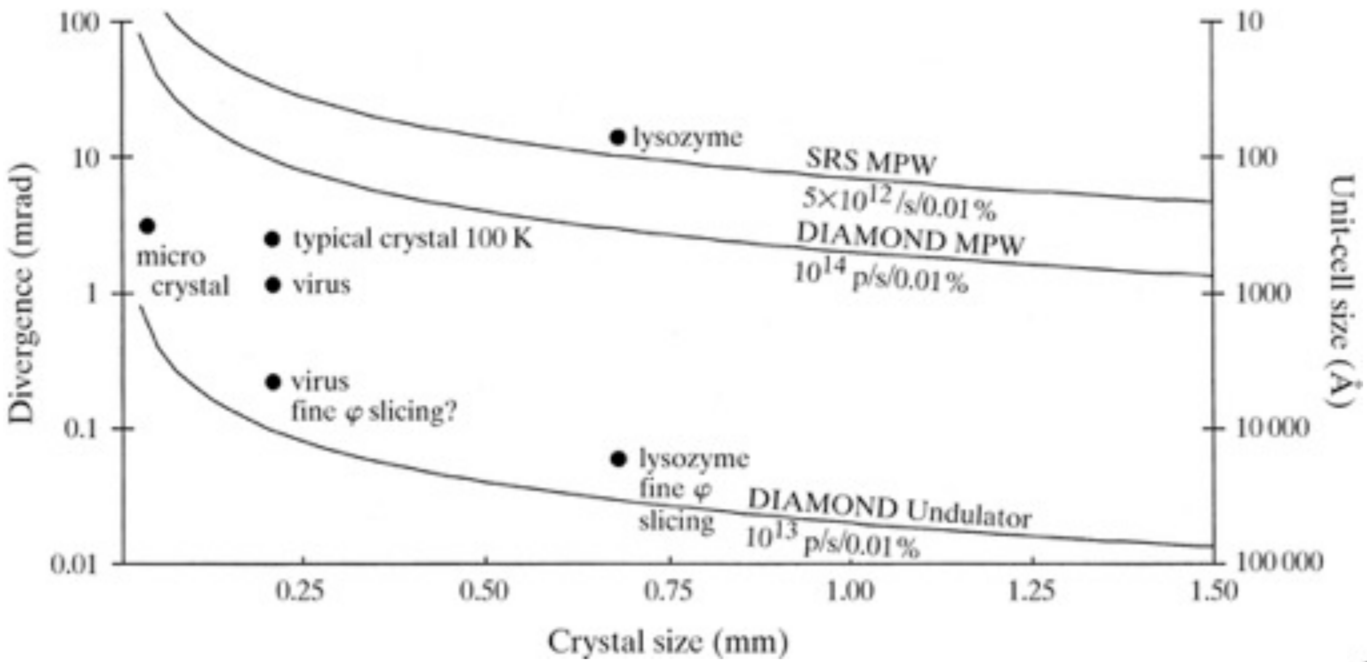
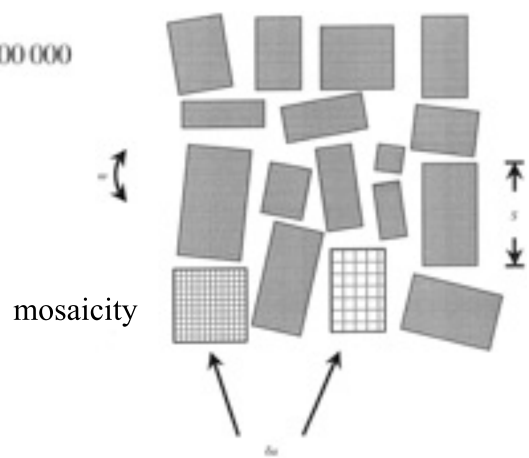
Note: can apply similar analysis to other phase space dimensions

Crystal Acceptance in Phase Space



C. Nave



Beam Stability Time Scales

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- **Disturbance time scale << experiment integration time:**

  Disturbances blow up effective beam σ and σ', reduce intensity at experiment, but do not add noise

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  Disturbances add noise to experiment

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- **Sudden beam jumps or spikes can be bad even if rms remains low**

  Peak amplitudes can be **> x5** rms level

Beam Stability Time Scales – cont.

Most demanding stability requirements:

- Orbit disturbance frequencies approximately bounded at high end by data sampling rate but and a low end by data integration and scan times

$\Rightarrow$ **noise not filtered out**

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- Some experiments average over much shorter times (e.g. 100 kHz)

$\Rightarrow$ **sensitive to synchrotron oscillations (~10 kHz)**

- Acquisition rates are increasing, averaging times decreasing

MHz for turn-turn measurements

single-shot acquisition for pulsed sources (e.g. pump-probe)

Sources of Beam Instability

Long term (weeks-years)

- ground settlement (mm)
- seasonal ground motion (< mm)

Medium term (minutes-days)

- diurnal temperature (1-100 μm)
- crane motion (1-100 μm)
- filling patterns (heating, BPM processing: (1-100 μm)
- gravitational earth tides (sun and moon, ΔC = 10-30 μm)
- river, dam activity (1-100 μm)
- machine fills (heating, BPM intensity dependence)
- RF drift (microns)
- coupling changes

Short term (milliseconds-seconds)

- ground vibration, traffic, trains, etc. (< microns, <50 Hz typ)

ground motion amplified by girder + magnet resonances (x~20 if not damped) and by lattice (x10-x40)

⇒ nm level ground motion can be amplified close to μm level

- cooling water vibration (microns)
- booster operation (microns)
- power supplies (microns)
- rotating machinery (air conditioners, pumps: microns)
- insertion device motion (1-100 μm)
- vac chamber vibration (microns)

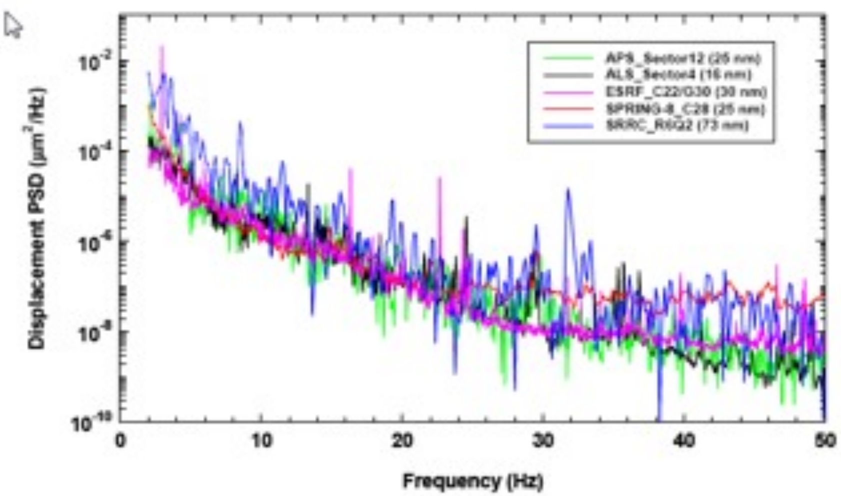
High frequency (sub-millisecond)

- high frequency PWM and pulsed power sources (microns)
- synchrotron oscillations (1-100 μm)
- single-/multibunch instabilities (1-100 μm)

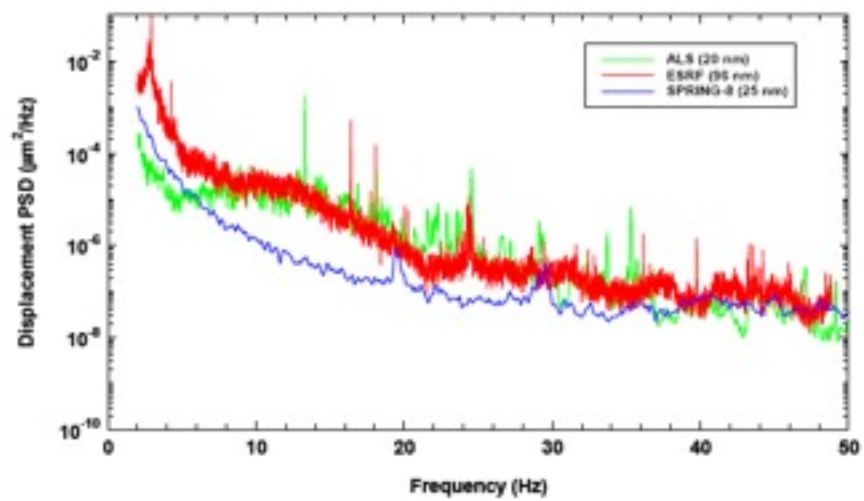
Note: relative component motion more critical than common mode motion

Ground Motion

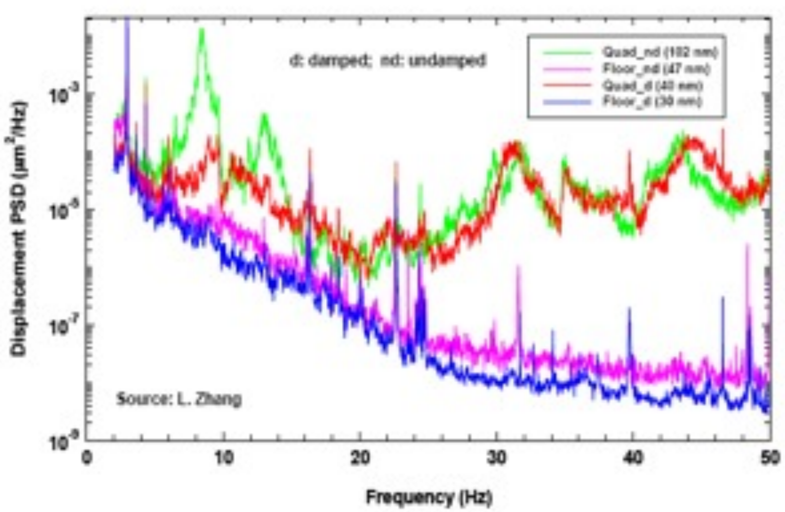
PSD - Horizontal Ground Motion



PSD - Vertical Ground Motion



ESRF
Horizontal Motion Spectra - Cell22/G30



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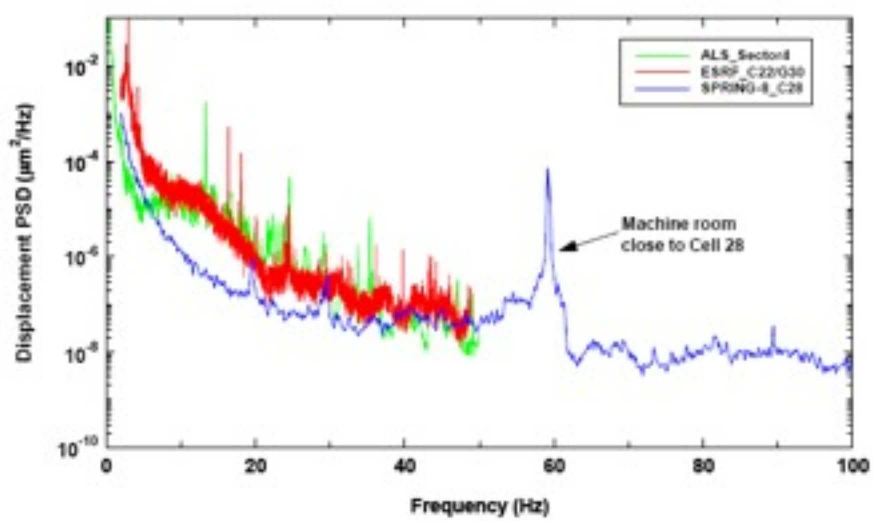

Intensity Stability

Want high level of flux (F) constancy through aperture or steering accuracy to hit small sample **(sample size on order of beam size σ)**

$\Delta F/F < 10^{-3}$ (typical)

Note: some experiments require **$< 10^{-4}$** flux constancy

e.g. **photoemission electron spectroscopy combined with dichroism spectroscopy (subtractive processing of switched polarized beam signals)**

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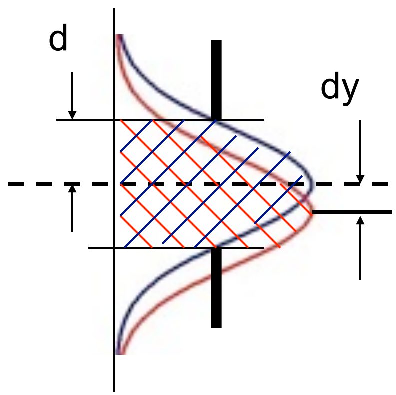
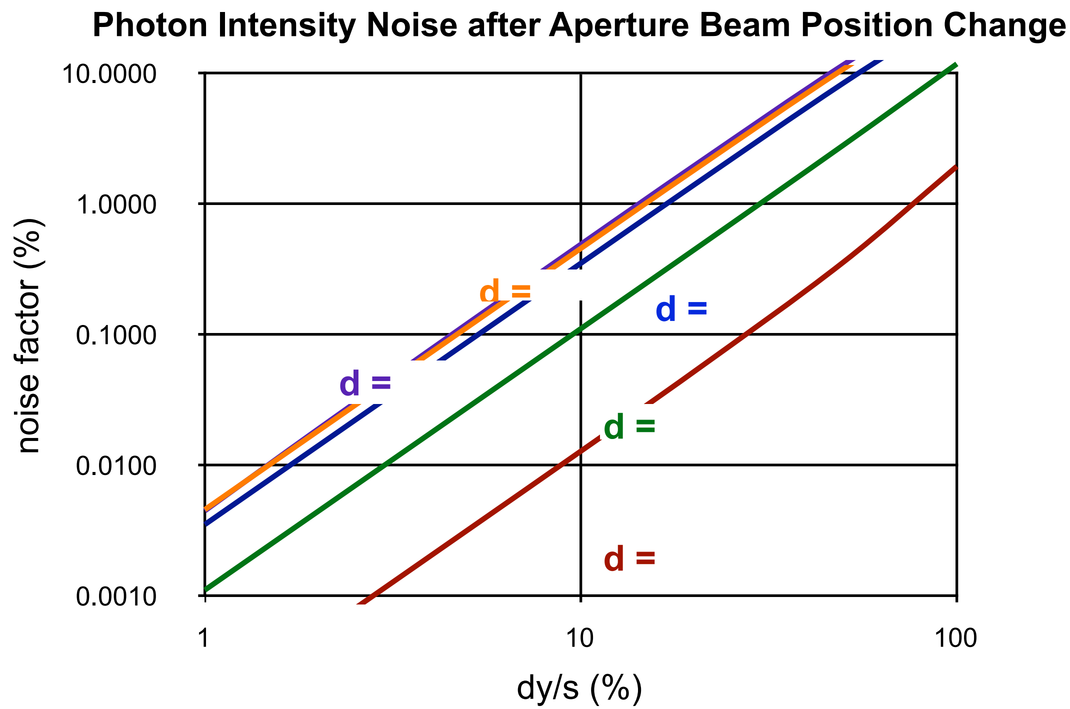
Flux variations caused by

- orbit instability
- beam size instability
- energy instability

Intensity Stability after Apertures – Beam Position

Sensitivity of intensity (flux) to beam position change:





Noise factor (position):

$|F_0 - F_{dy}|/F_0$ **~ dy^2**

d =half

dy =displacement from

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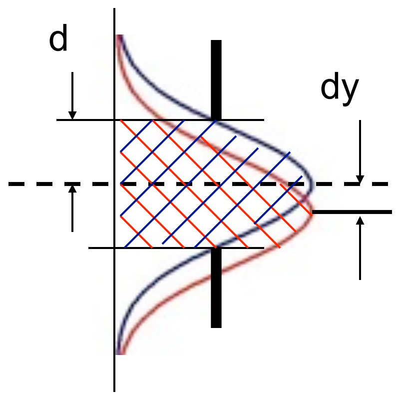

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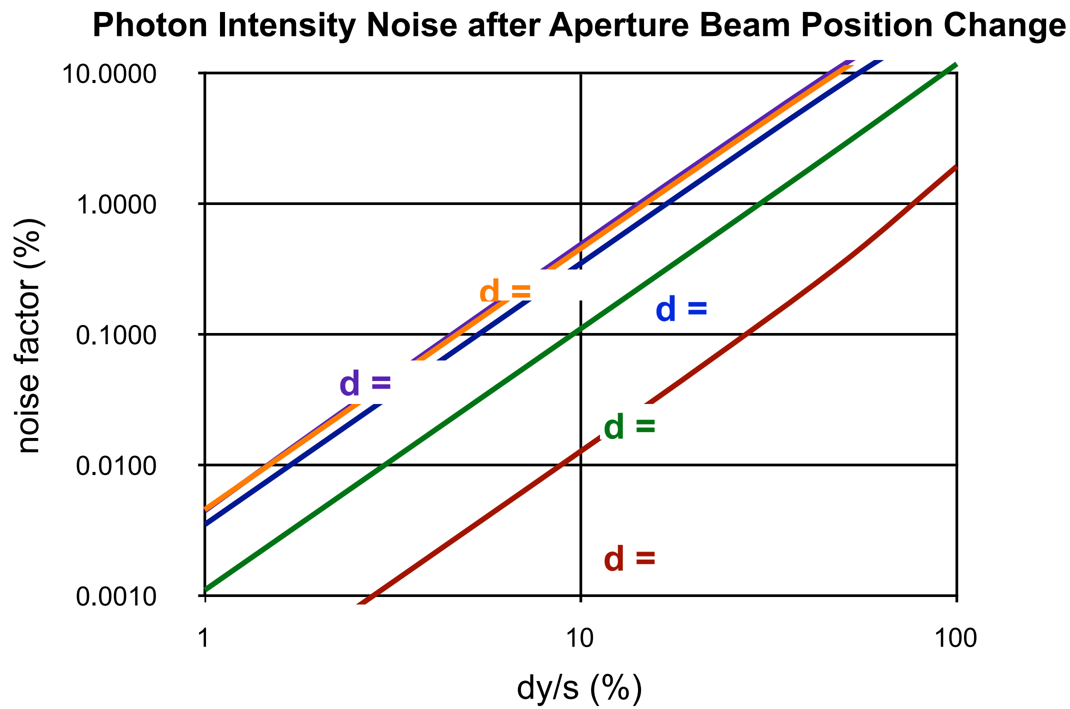

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For noise intensity stability <0.1%: **dy < 5% σ_y** **(<1.5% σ_y for 0.01% stability)**

Intensity Stability Sensitivity - Orbit

For 0.1% intensity stability, orbit stability should be

$\Delta x_{cm}, y_{cm} < .05\ \sigma_{x,y}$ at source point for ***focused*** beams

$\Delta x'_{cm}, y'_{cm} < .05\ \sigma_{x',y'}$ at source point for ***unfocused*** beams

Vertical (0.1% coupling): **~107 μrad** @ 3 GeV, N = 1

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$\sigma_y \sim$ **7-21 μm**

$\sigma_{y'}$ = **~107 μrad** @ 3 GeV, N = 1
~14 μrad @ 3 GeV, N = 100, n = 1
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$\Rightarrow$ $\Delta y_{cm} < \sim 0.4 - 1\ \mu m$, $\Delta y'_{cm} < \sim 0.25 - 5\ \mu rad$ for 3rd gen sources

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$\sigma_x \sim$ **75-300 μm** $\sigma_{x'}$ = **~17-52 μrad** @ 3 GeV, N = 100, n = 1

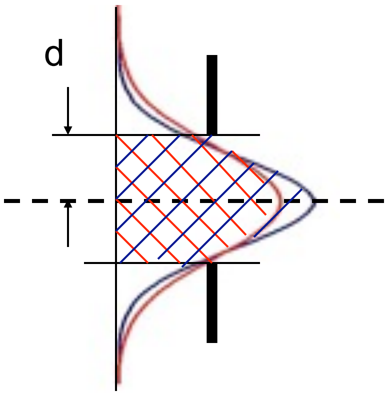
~6-19 μrad @ 3 GeV, N = 100, n =

7

But for wigglers: $dF(\omega)/d\theta \propto E_{e-}\ I_{e-} S(\omega/\omega_c)$ $\omega_c \propto E^2[1 - (\theta\gamma/K)^2]$ θ = **horiz view angle**

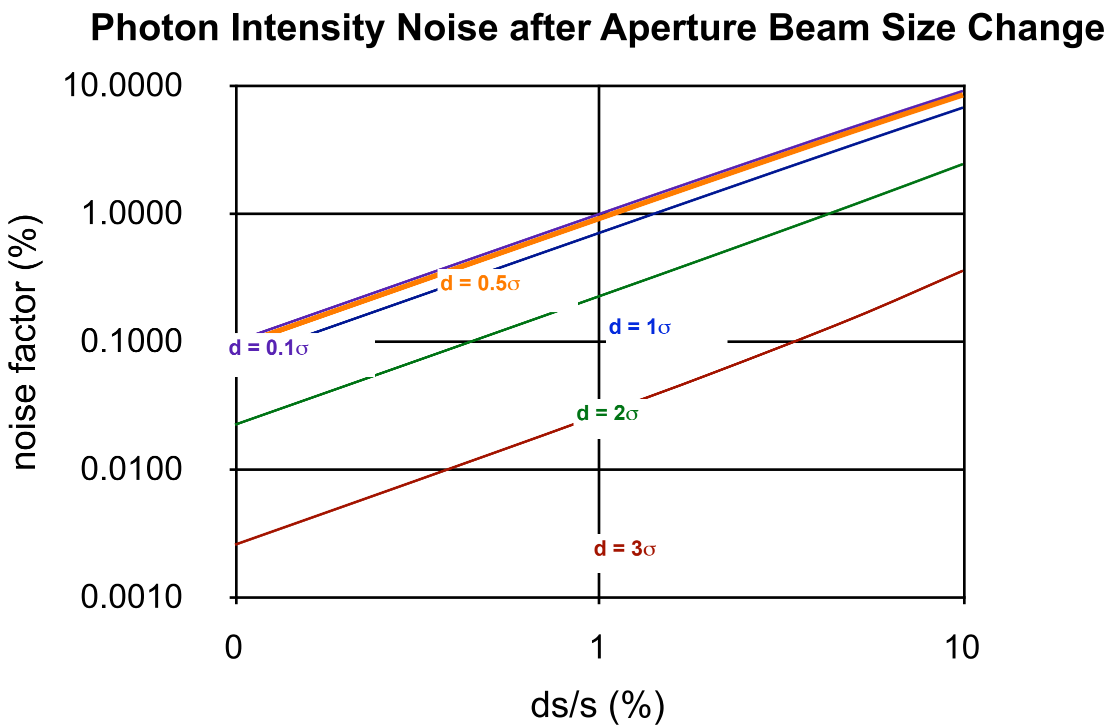
Intensity Stability after Apertures – Beam Size

Sensitivity of intensity (flux) to beam size change:



Noise factor (size):

$|F_{\sigma 0}-F_{\sigma 0+d\sigma}|/F_{\sigma 0}$ **~ dσ**



d =half aperture

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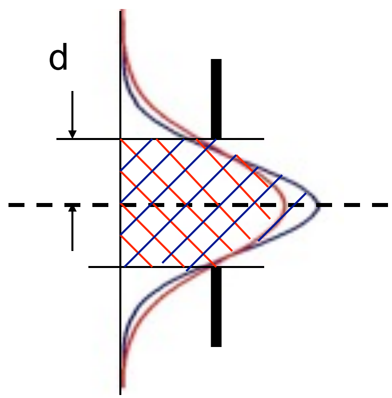

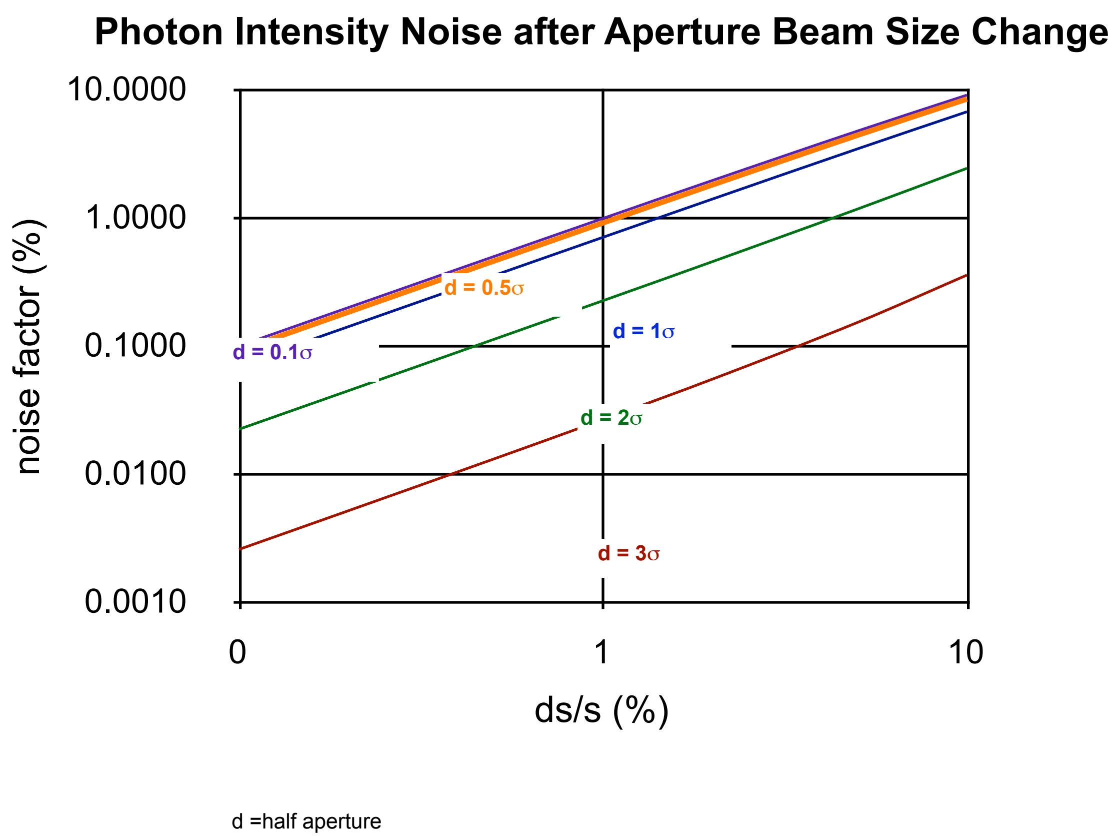

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Beam size-perturbing mechanisms:

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Beam size-perturbing mechanisms:

- changes in horizontal-vertical electron beam coupling

  ID gap change, orbit in sextupoles, energy ramp without coupling correction

- collective effects

  coupling resonances, single- and multibunch instabilities in transverse and longitudinal planes, intrabeam scattering

- gas bursts, ions, dust particles
- electron energy variations in lattice dispersion sections in times < data integration time
- synchrotron oscillations, Landau damping mechanisms, etc.

Intensity Stability Sensitivity – Energy

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Energy-dependent orbit perturbation:

Coherent electron energy variations in lattice dispersion sections at times > data integration time (synchrotron oscillations, RF)

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$\Rightarrow$ **ΔE/E (coherent) < ~10^{-4} - 10^{-5}** (η_x= ~0.1 m at dipole; η_y= ~0.02 m)

($\Delta\phi$ **< ~0.1° - 0.01°**)

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($\Delta\phi$ **< ~0.1° - 0.01°**)

$\Rightarrow$ **$\Delta f_{RF}/f_{RF} = \alpha_c\, \Delta E/E$ < ~10^{-7} - 10^{-8}** (α_c = ~10^{-3})

(Δf_{RF} **< 5- 50 Hz** for f_{RF} = 500 MHz); imposes limit on phase noise for RF source ~10 kHz BW)

Energy-dependent photon emission:

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Energy-dependent photon emission:

$dF(\omega)/d\theta \propto E_{e-}\, I_{e-}\, S(\omega/\omega_c)$ $\qquad \omega_c \propto E^2$ $\qquad$ (dipole, wiggler)

$dF(\omega_n)/d\theta\, d\psi \propto I_{e-}/\sigma'^{2}_{ph} \propto E_{e-}^2\, I_{e-}\, n\, N_u$ $\qquad$ (undulator)

$\Rightarrow$ **$\Delta E/E$ (coherent) $< \sim 10^{-3} - 10^{-4}$**

Intensity Stability Sensitivity – Energy (cont.)

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Energy-dependent beam size:

For electron energy variations in lattice dispersion sections at times < data integration time (i.e. **synchrotron oscillations**):

$$\sigma^2 = \varepsilon\beta + (\sigma_\delta \eta)^2 + (\eta \Delta E/E)^2 = \sigma_0^2 + (\Delta\sigma)^2$$

where σ_δ is natural energy spread of electron beam = ~0.1%

Also $\varepsilon \propto E^2$, but emittance change only happens for energy changes slower than damping times (~ms); synchrotron oscillations are too fast (0.1 ms)

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e.g. $(\varepsilon\beta)^{1/2}$ = ~350 μm, $\eta\,\sigma_\delta$ = ~100 μm for η = 0.1 m $\Rightarrow$ σ_0 = ~360 μm

$\Delta\sigma/\Delta\sigma_0 < 0.1\%$ $\Rightarrow$ **$\Delta E/E$ (rms) < ~ 10^{-4}-10^{-5}**

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Energy-dependent beam divergence:

$\sigma'_{ph} = [\sigma'^2_{e-} + \sigma'^2_{\Psi}]^{1/2}$ $\quad$ $\sigma'_{e-} = [\varepsilon\gamma(s) + (\eta'\delta)^2]^{1/2}$ $\quad$ $\sigma'_{\Psi} \propto 1/E$

Unfocused beam size: $\sigma_{ph}(L) = [\sigma_{e-}^2 + \sigma_{diff}^2(\lambda) + L\,\sigma_{ph}'^2]^{1/2}$

unfocused beam intensity affected by both horizontal and vertical size change

Photon Energy Stability and Resolution

Photon energy resolution <10^{-4} after monochromator:

Bragg: $$\frac{\Delta E_{ph}}{E_{ph}} = \frac{\Delta\theta}{\theta_B}$$ where θ_B = ~5°-45° (~90-800 mrad)

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$\Rightarrow$ $\Delta y'_{cm} <$ ~10 μrad

Note: some monochromators reaching 10^{-5} resolution $\Rightarrow$ $\Delta y'_{cm} <$ **~1 μrad**

Photon Energy Stability and Resolution

Photon energy resolution $<10^{-4}$ after monochromator:

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$$\Rightarrow \quad \Delta y'_{cm} < \sim 10\ \mu\text{rad}$$

Note: some monochromators reaching 10^{-5} resolution $\Rightarrow$ $\Delta y'_{cm} < \sim 1\ \mu$rad

Undulator line energy and width not degraded:

line wavelength = $\lambda_n = n\ \lambda_u(1+k^2/2)/2\gamma^2$

$$\Rightarrow \mathbf{d}\ \lambda_n / \lambda_n = -2\ \Delta E/E$$

natural width = $\delta\ \lambda_n/\lambda_n = 1/Nn$ (N = # periods, n = harmonic)

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Note: for 10^{-5} resolution $\Rightarrow$ $\Delta E/E$ (coherent) < ~7 x 10^{-6}

($\Rightarrow$ $\Delta\phi$ **< 0.01°** for SPEAR 3 Δf_{RF} **< 2.5 Hz** for f_{RF} = 500 MHz)

Timing and Bunch Length Stability

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$\Delta E/E$ (rms) < 10^{-3} $\Rightarrow$ $\Delta\sigma_{bunch} <$ 5% σ_{bunch}

Stability Tolerances

- Tolerance budget for electron beam parameters contributing to instability of a specific photon beam parameter can be derived from stability sensitivities, assuming random uncorrelated effects:

$$\sqrt{\sum_{i=1}^{n}\left(\frac{p_{tol}}{p_{sen}}\right)_i^2} < 1$$

p_{tol} = tolerance for parameter p, p_{sen} = sensitivity to parameter p

- e.g., to obtain <0.1% intensity stability, must reduce tolerances for orbit, beam size and energy stability below their sensitivity levels by ~1/√3 (0.57)

- Can increase tolerance for difficult parameters by reducing tolerance for easy parameters

Stability Requirements for Storage Rings - Summary

experiment parameters	beam orbit	beam size	beam energy/ energy spread
< 0.1% intensity steering to small samples	$\Delta x,y < 5\%\ \sigma_{x,y}$ $\Delta x',y' < 5\%\ \sigma'_{x,y}$	$\Delta\sigma_{x,y} < 0.1\%\ \sigma_{x,y}$ $\Delta\sigma'_{x,y} < 0.1\%\ \sigma'_{x,y}$	$\Delta E/E$(coher) $< 10^{-4}$ $\Delta E/E$(rms) $< 10^{-4}$
$< 10^{-4}$ photon energy resolution	$\Delta x' <$ ~5 μrad $\Delta y' <$ ~1 μrad (undulator)		$\Delta E/E$(coher) $< 5 \times 10^{-5}$ $\Delta E/E$(rms) $< 10^{-4}$ (und n = 7)
timing, bunch length		$\Delta\sigma_t < 0.1\%\ \sigma_t$	$\Delta E/E$(coher) $< 10^{-4}$

Conclusion

3rd generation stability requirements are stringent:

- intensity stability < 0.1%
- pointing accuracy < 5% beam dimensions
- photon energy resolution < 10^{-4}
- timing stability < 10% bunch length

⇒ vertical orbit < ~0.4-1 μm, <0.25-5 μrad

beam size < 0.1 %

e- energy < 5 x 10^{-5}

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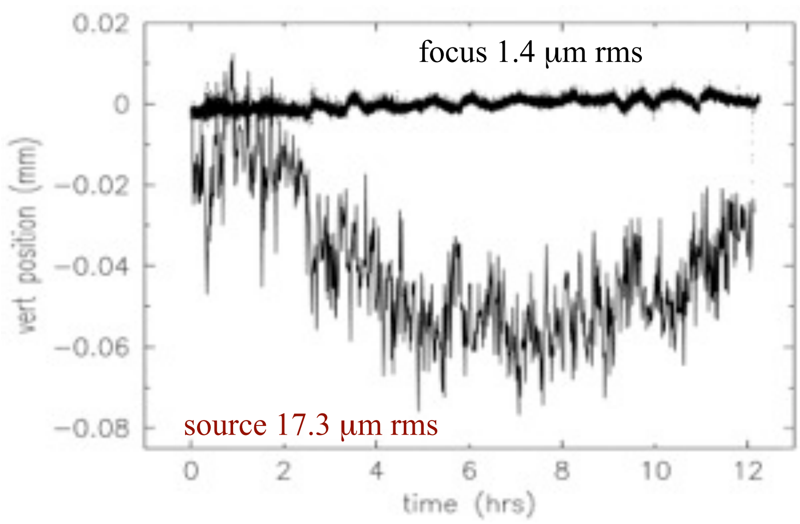
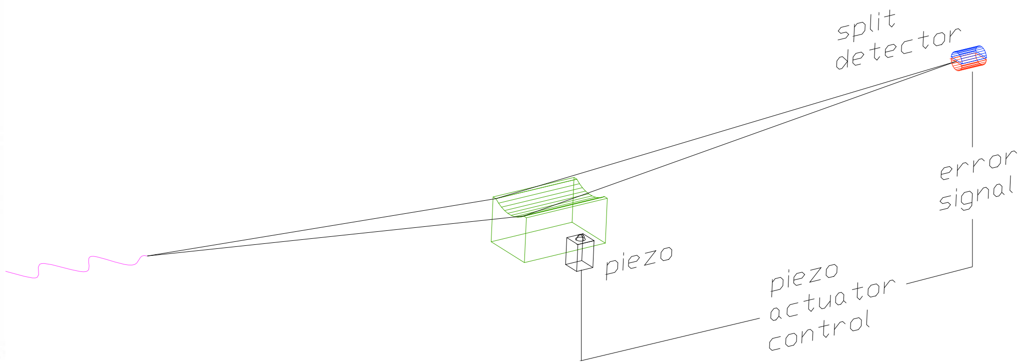
- absolute stability at sub-micron levels may be impossible
- achieving maximal performance may involve more complex ("smart") signal processing that accounts for residual instabilities (sample-by-sample intensity normalization, position detection/ binning, digital filtering, etc)

Mirror Pitch Feedback (SSRL, T. Rabedeau)

- error signal obtained from position sensitive (split) detector located near beam focus
- error signal used to control piezo high voltage via PI algorithm
- piezo provides mirror fine pointing control with typical full range of motion +/-~30 μrad





back-up slides

Orbit Noise

Orbit shift $\Delta y(s)$ caused by angular kick $\theta(s_0)$:

$$\Delta y(s) = \theta_y(s_0)\frac{\sqrt{\beta_y(s_0)\beta_y(s)}}{2\sin\pi\upsilon_y}\cos(|\varphi_y(s_0)-\varphi_y(s)|-\pi\upsilon_y)$$

Amplification factor A(s) for uncorrelated dipole kicks having RMS amplitude σ_θ:

$$\sigma_{x,y}(s) = A_{x,y}(s)\sigma_{\theta x,\theta y}(s)$$

$$A(s) = \frac{\sum_k \beta_k\beta(s)}{8\sin^2\pi\upsilon}$$

- A(s) ~ 10-20 for uncorrelated dipole kicks (e.g. quadrupole motion, corrector power supply ripple, etc).
- A(s) can be much less for correlated kicks – such as those caused by the common motion of quadrupoles on a girder.

Electron Beam Dimensions - Longitudinal

Longitudinal parameters:

emittance: $$\varepsilon_s = \int_S \frac{\Delta E}{\omega_{rf}} d\phi = \sigma_s \sigma_\delta \qquad \delta = \Delta E / E$$

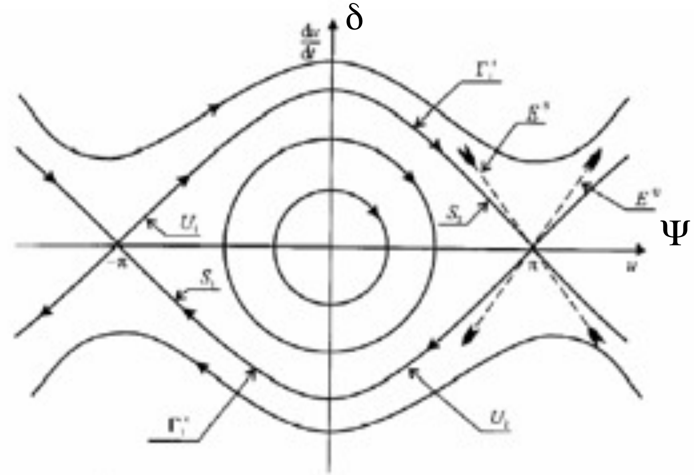
$$\delta = \delta_{max} \sin \Omega_s t$$

$$\delta_{max} = \frac{\Omega_s}{\alpha_c \omega_{rf}} \phi_{max} = \frac{\nu_s}{\alpha_c h} \phi_{max}$$

synchrotron frequency: $$\Omega_s = \sqrt{\frac{\alpha_c \omega_{rf} e V_{rf}^0 \cos \phi_s}{E_0 T_0}}$$

bunch length (m): $$\sigma_s = \frac{\alpha_c c}{\Omega_s} \sigma_\delta$$

bunch length (s): $$\sigma_t = \frac{\alpha_c}{\Omega_s} \sigma_\delta$$



Phase-space portrait of a pendulum

α_c = momentum compaction factor

ν_s = synchrotron tune

h = harmonic #

ϕ_s = synchronous phase

$T_0 = 2\pi h/\omega_{rf}$ = rev period

V_{rf}^0 = peak rf voltage

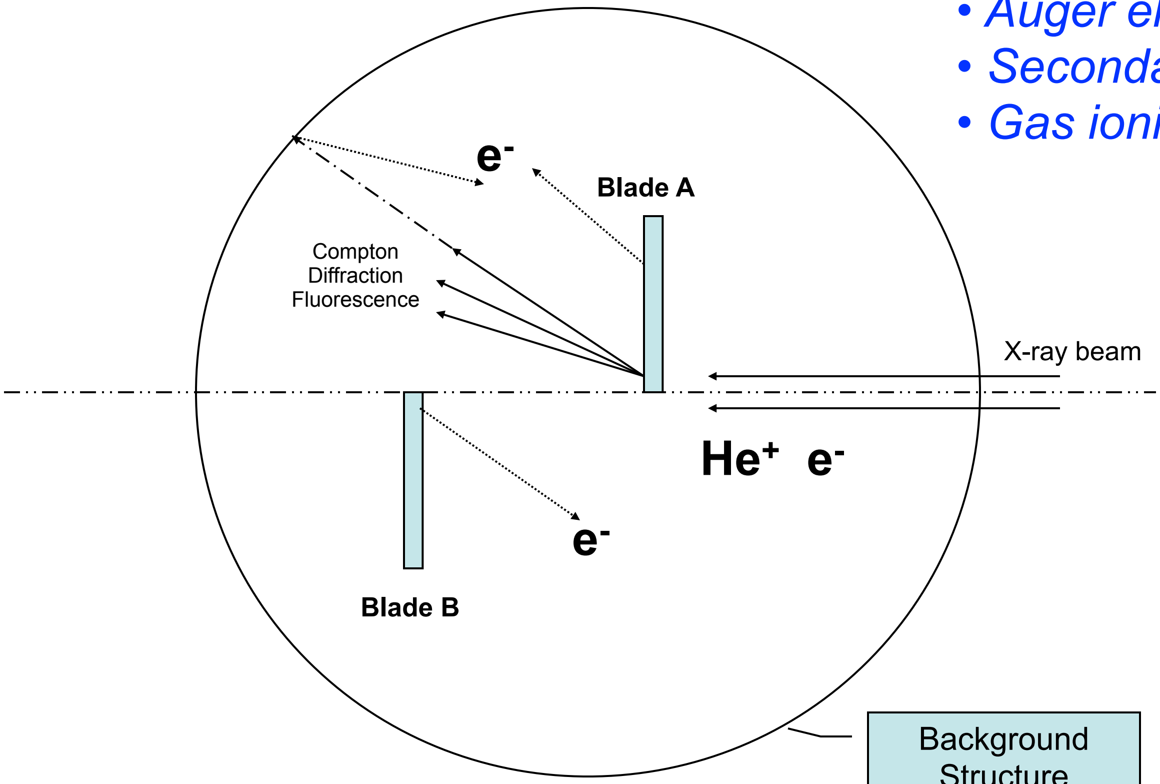
Mirror Pointing Feedback

Gas Amplified Electron Yield Detection

- Be blades with Ti/Al coatings
- He gas amplified electron yield

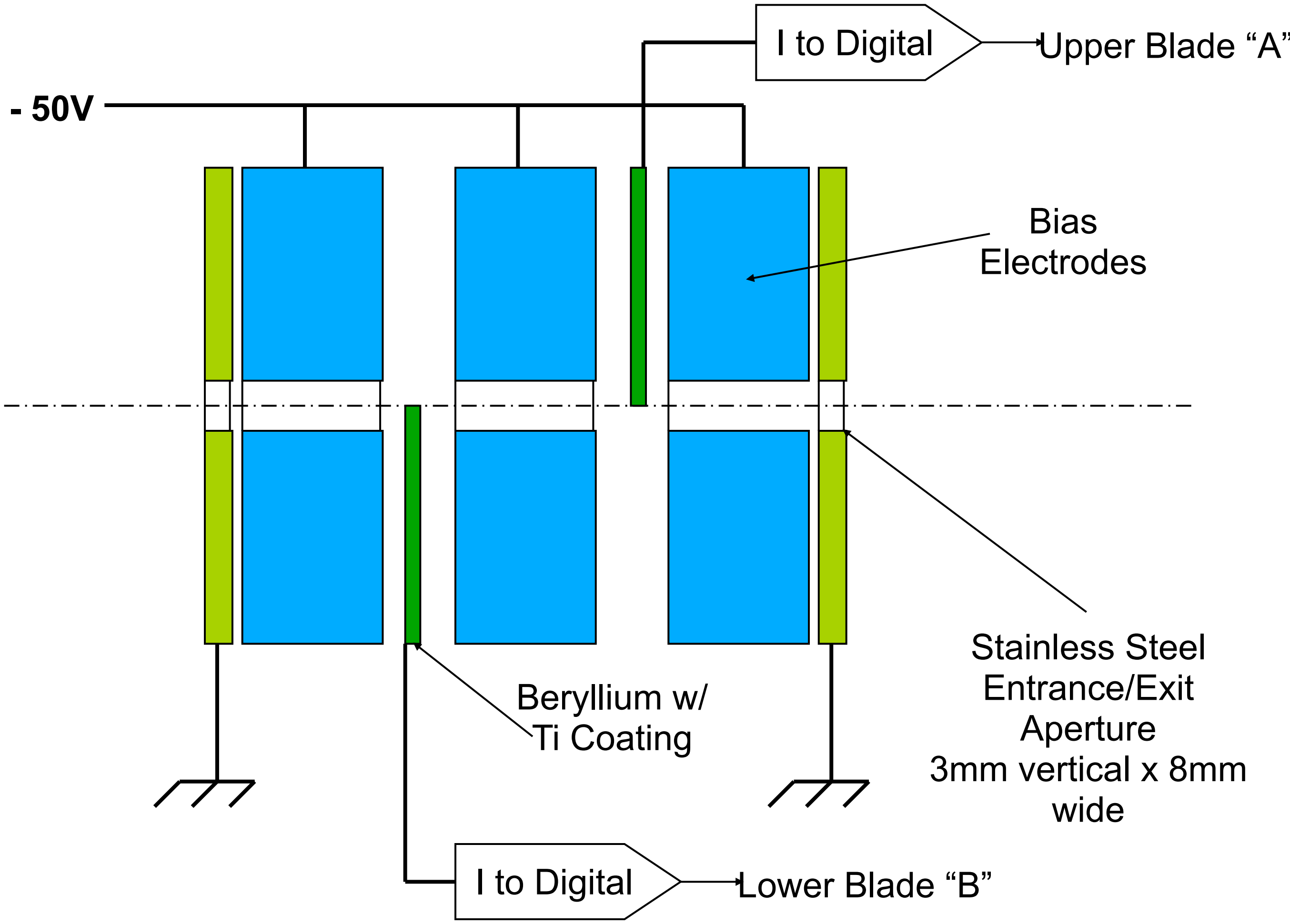
<u>Electron Processes</u>

- Photoelectrons
- Auger electrons
- Secondary electrons
- Gas ionization



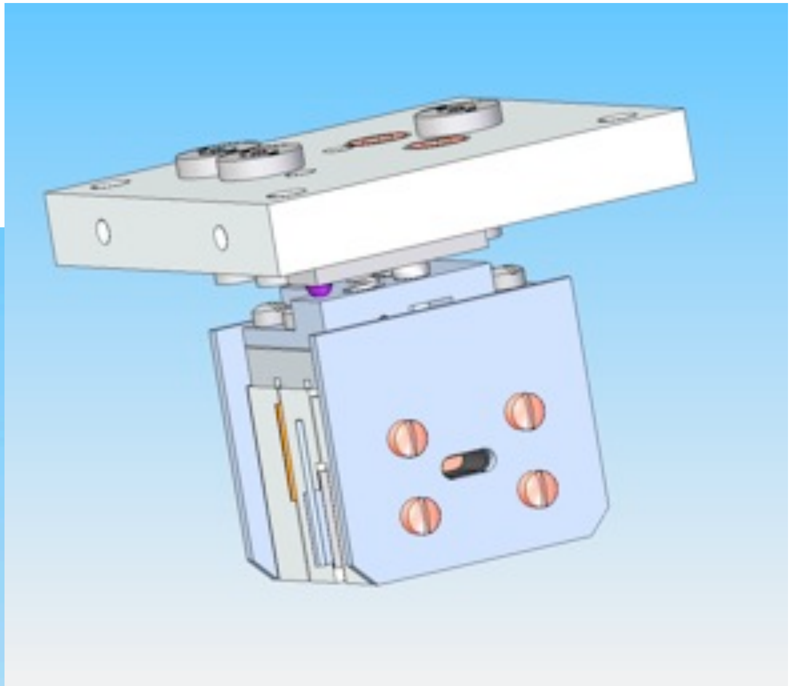
Mirror Pointing Feedback

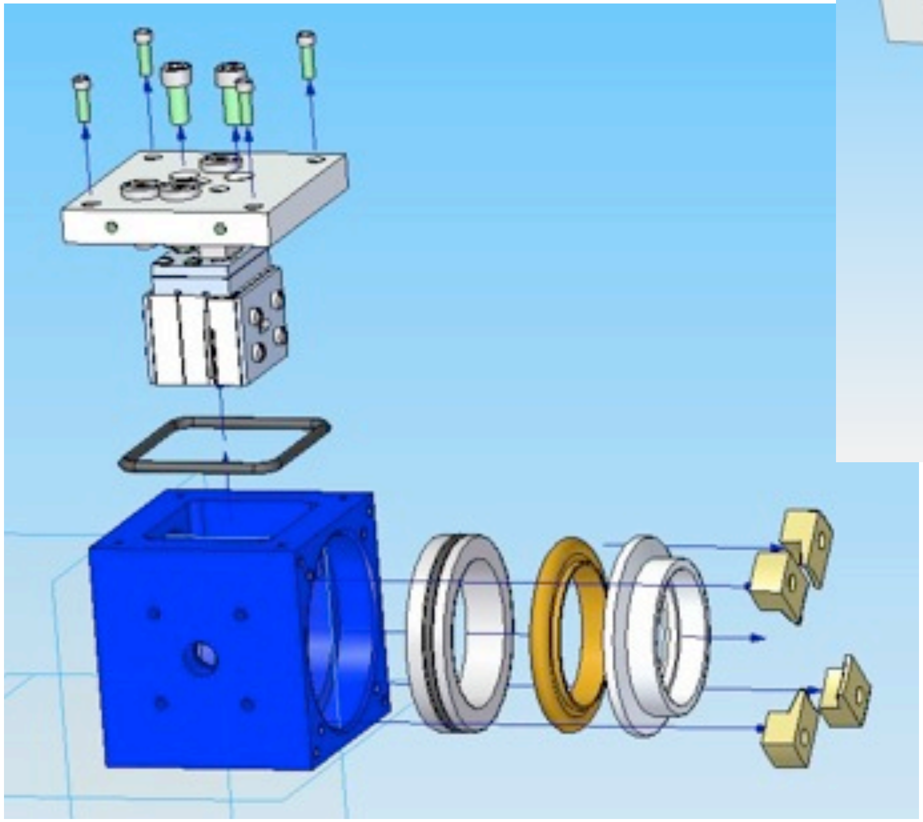
Detector Cross Section



Mirror Pointing Feedback

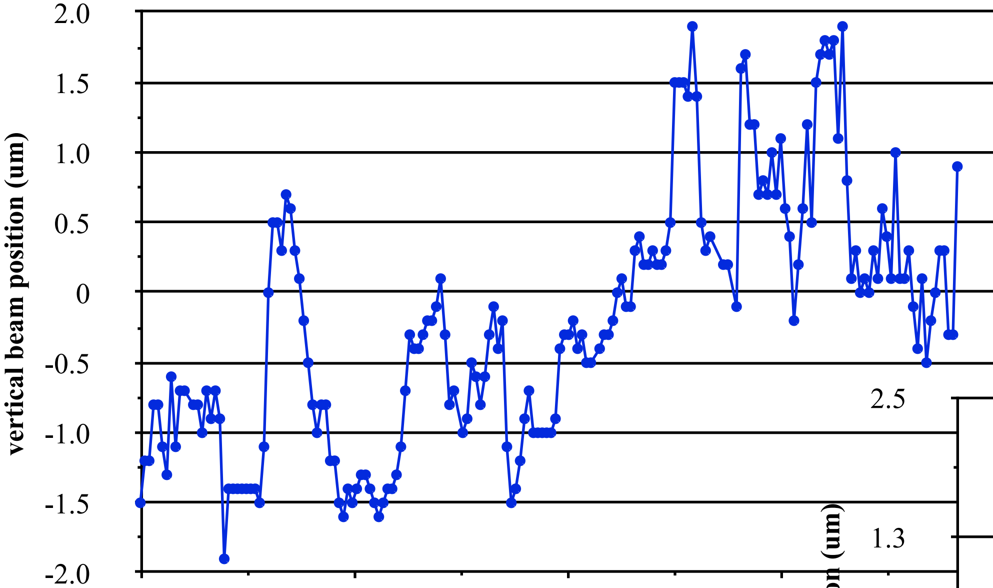
Detector Mechanical Model





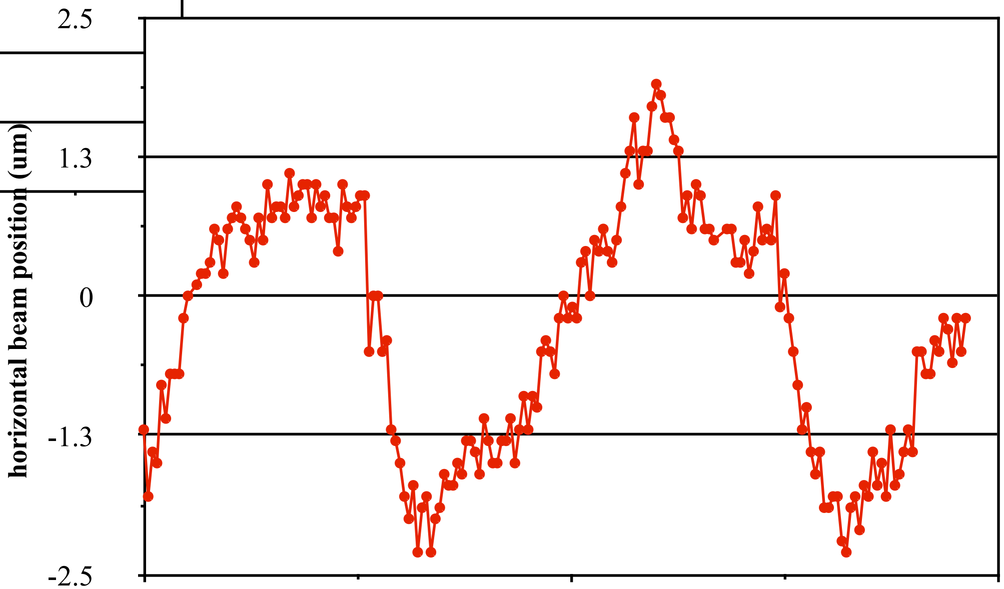
Mirror Pointing Feedback

Pointing Stability Results, 48 Hrs



0.9um rms variation over 48hrs
16% of beam 5.6um fwhm

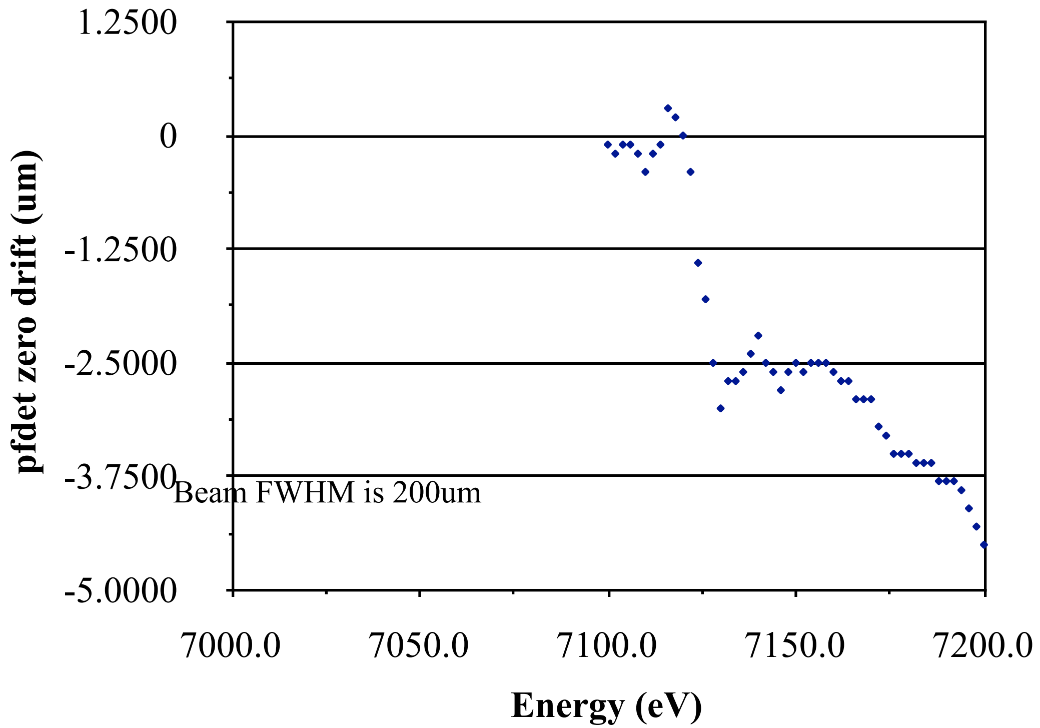
- *10Hz to diurnal bandpass*



1.1um rms variation over 48hrs
1.3% of beam 82.3um fwhm

Mirror Point Feedback

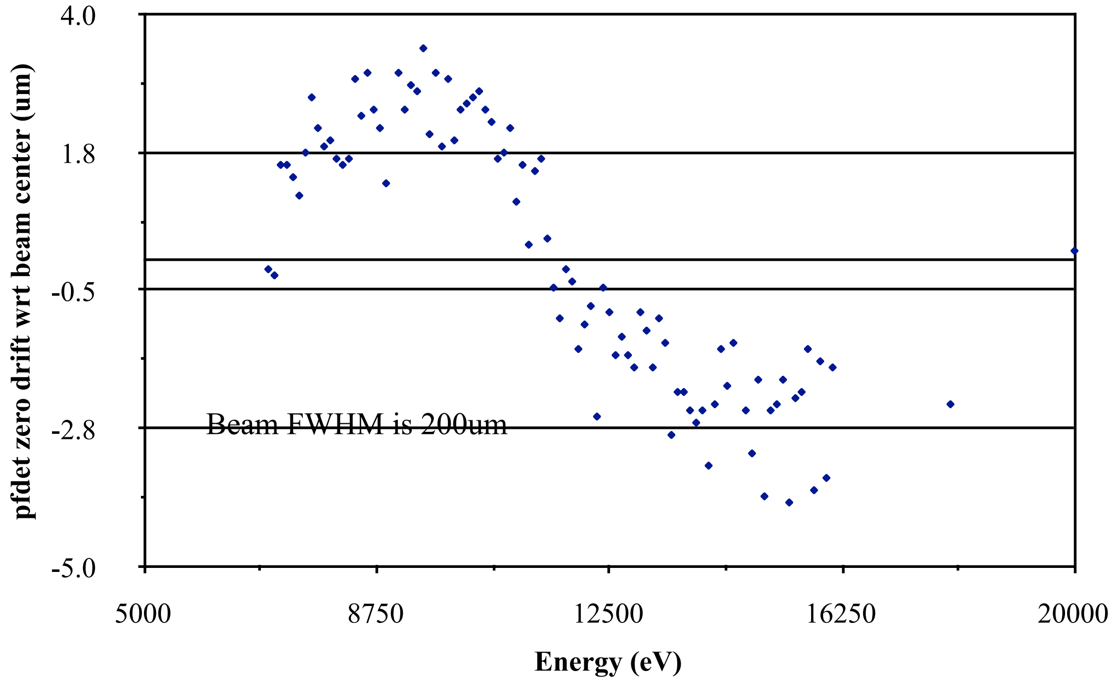
Photon Energy Dependence



- ~300ppm Fe in Beryllium (IF1 Grade)
- 1.5% effect in position center
- Bury the Fe signal under Ti coating (data at left collected with only one side of Be blade Ti coated)

Mirror Point Feedback

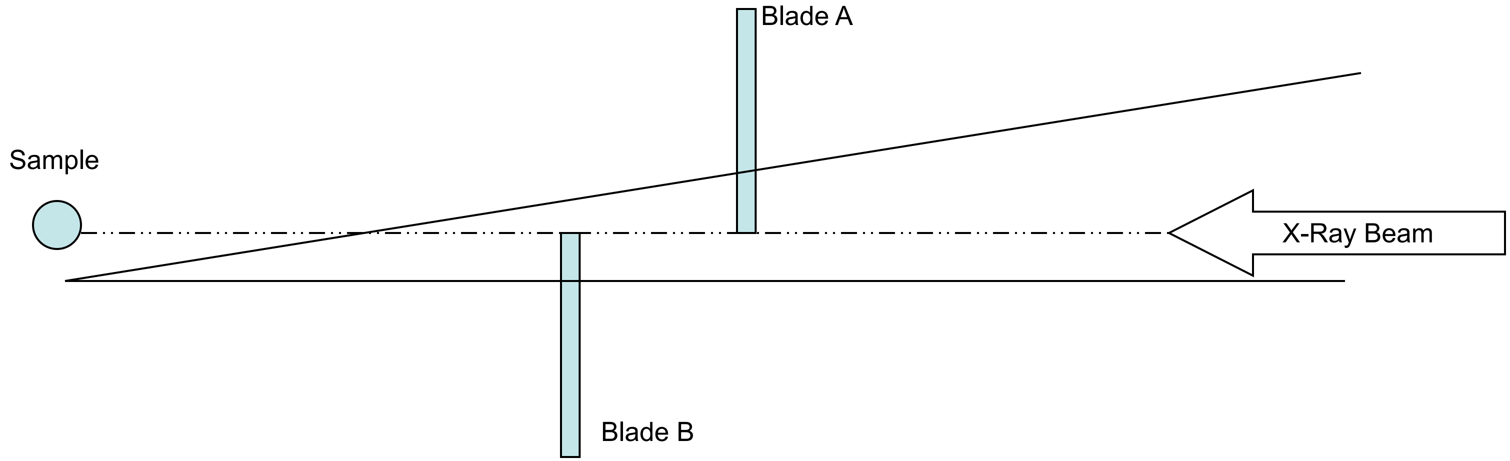
Photon Energy Dependence, cont.



- *photon flux and beam profile highly varied in scan*
- *no significant energy dependence since electron yield is non-resonant*

Mirror Pointing Feedback

Detector Non-Idealities



- detector center shift with upstream filter insertion
- detector center shift with beam shape changes or slit insertion
- Be blade edge irregularities result in x – y positioning cross talk
- Be blade affect on beam coherence
- lever arm effect – sample to detector parallax
- detector response coupling to beam angle through detector
- detector vs. sample support relative motion
- limited aperture size, 3x8mm

Mirror Point Feedback

Detector Specifications

- Max Current: 0.1nA – 100nA
- Sensitivity: ~1:1000 (ie: 0.25um/250um)
- Bandwidth: 10Hz to diurnal
- Sampling: 100Hz or equivalent
- Temperature Stability: +/- 2^{o}C affects only least significant bit
- Bias supply: -50VDC

Orbit-Perturbing Mechanisms

Long term (weeks-years):

- ground settlement (mm)
- seasonal ground motion (< mm, sometimes more)

Medium term (minutes-days):

- diurnal temperature (1-100 μm)
- crane motion (1-100 μm)
- fill patterns (1-100 μm)
- coupling changes
- river, dam activity (1-100 μm)
- machine fills (heating, BPM intensity dependence)
- RF drift (microns)
- gravitational earth tides (ΔC = 10-30 μm)

Short term (milliseconds-seconds):

- ground vibration, traffic, trains, etc. (< microns, <50 Hz typ)

  ground motion amplified by girder + magnet resonances (x~20 if not damped) and by lattice (x ~5+)

  ⇒ nm level ground motion can be amplified close to μm level
- cooling water vibration (microns)
- booster operation (microns)
- power supplies (microns)
- rotating machinery (air conditioners, pumps) (microns)
- insertion device motion (1-100 μm)
- vac chamber vibration from BL shutters, etc. (microns)

High frequency (sub-millisecond):

- high frequency PWM and pulsed power sources (microns)
- synchrotron oscillations (1-100 μm)
- single- and multibunch instabilities (1-100 μm)

Lifetime: Intensity Constancy

- **Lifetime contributors:**
  - quantum lifetime
  - gas scattering lifetime (Coulomb, bremsstrahlung)
  - Touschek lifetime
  - ions and dust particles
- **Touschek often dominant lifetime factor:**

$$\tau_{Touschek} \propto \frac{\sigma_{x'}\sigma_x\sigma_y\sigma_s\gamma^3\left(\delta p/p\right)^2}{N}$$

$\delta p/p$ = ring momentum acceptance N = number of particles in bunch

$\Rightarrow$ **control and stabilize bunch volume**

e.g. increase vertical coupling, lengthen bunch with harmonic cavity

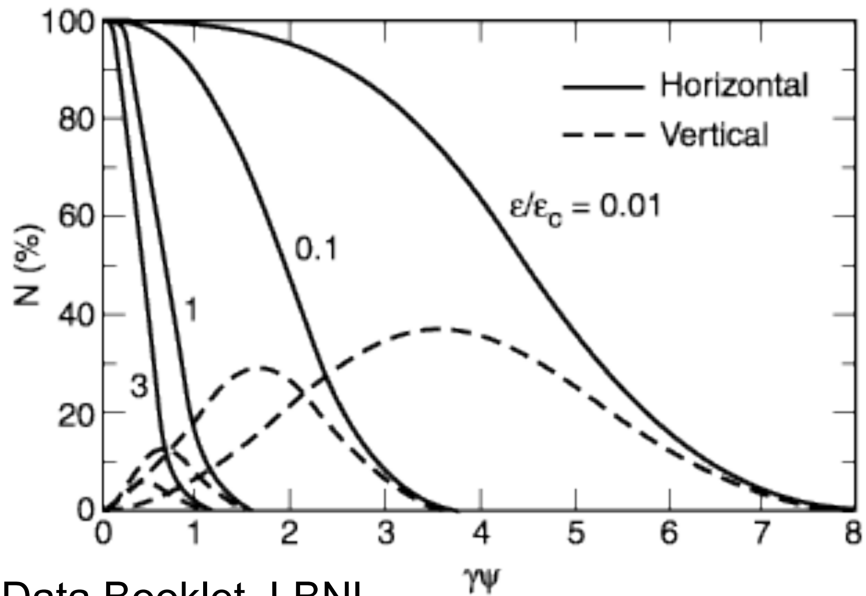
- **Ion trapping prevented by having gap in bunch fill pattern**
- **Top-off injection can solve lifetime woes**

Photon-Electron Relationships - Polarization

- SR from dipole is linearly polarized in horizontal plane when viewed in this plane
- Polarization is elliptical when viewed out of horizontal plane

  rotation sense reverses as vertical angle changes from positive to negative

- Elliptical polarization can be decomposed into horizontal and vertical components:



K-J Kim, from X-ray Data Booklet, LBNL

Relationship Between Photon and Electron Parameters

TABLE II. Relationship of photon and electron parameters (approximate, with constant lattice Twiss parameters; γ_s is Twiss parameter; N_u = number of undulator periods; n = undulator harmonic number).

Photon parameter	Relationship to electron parameters
1. Size at L	$\sigma_{ph}(L) = [\sigma_{e^-}{}^2 + \sigma_{diff}^2(\lambda) + (L\sigma'_{ph})^2]^{1/2}$ (unfoc) $\sigma_{ph}(L) = \sigma_{ph}(0)$ (1:1 foc) $\sigma_{diff}(\lambda) = \lambda/[4\pi\sigma'_{\psi}(\lambda)]$ $\quad \sigma_{e^-} = [\epsilon\beta(s) + (\eta(s)\delta E/E)^2]^{1/2}$
2. Divergence at L	$\sigma'_{ph}(L) = \sigma'_{ph}(0) = [\sigma'_{e^-}{}^2 + \sigma'_{\psi}{}^2]^{1/2}$ (unfoc) $\sigma'_{ph}(L) = -\sigma'_{ph}(0)$ (1:1 foc) $\sigma'_{e^-} = [\epsilon\gamma_s + (\eta'\delta E/E)^2]^{1/2}$ $\quad \epsilon \propto E^2$ $\sigma'_{\psi} \propto \gamma_{e^-}{}^{-1}$ (dip wigg) $\quad \sigma'_{\psi} \propto \gamma_{e^-}{}^{-1}(nN_u)^{-1/2}$ (und)
3. Position at L	$\Delta y_{ph}(L) = \Delta y_{e^-} + L\Delta y'_{e^-}$ (unfoc) $\quad \Delta y_{ph}(L) = \Delta y_{e^-}$ (1:1 foc) $\Delta y_{e^-}(\Delta E_{e^-}) = \eta\Delta E_{e^-}/E_{e^-}$
4. Angle at L	$\Delta y'_{ph}(L) = \Delta y'_{e^-}$ (unfoc) $\quad \Delta y'_{ph}(L) = -\Delta y'_{e^-}$ (1:1 foc) $\Delta y'_{e^-}(\Delta E_{e^-}) = \eta'\Delta E_{e^-}/E_{e^-}$
5. Critical freq/ undulator harm	$\omega_c \propto E_{e^-}{}^2$ (dip) $\omega_c \propto E_{e^-}{}^2[1-(\theta\gamma/K)^2]$ (wigg), θ = horiz view ang $\omega_n \propto nE_{e^-}{}^2/[1/2 + K^{-2} + (\theta\gamma/K)^2]$ (und) $\quad K/\gamma$ = ID deflect ang
6. Energy/freq resolution	$\Delta E_{ph}/E_{ph} = \Delta y'_{ph}/\theta_B$ (xtal mono; $\theta_B = \sim 90-900$ mrad) $\Delta\omega_n/\omega_n = 1/nN_u$ (undulator)
7. Spectral flux density	$dF(\omega)/d\theta \propto E_{e^-}I_{e^-}S(\omega/\omega_c)$ (dip, wigg on-axis; $S(\omega/\omega_c)$ in Fig. 3) $dF(\omega_n)/d\psi d\theta \propto I_{e^-}/\sigma'_{ph}{}^2(\omega_n)$ (und, on-axis) $F(\omega)$ = photons//s/unit freq BW; $I_{e^-} = e^-$ curr
8. Bunch length	$\sigma_t(\alpha/\omega_s)\delta E_{e^-}/E_{e^-}$ $\quad \alpha$ = moment. compact, ω_s = synchrotron freq
9. Bunch time osc	$\Delta t_b = \Delta\phi/\omega_{rf} = (\alpha/\omega_s)\Delta E_{e^-}/E_{e^-}$ $\quad \omega_{rf}$ = rf frequency

$$\alpha = -\beta'/2$$

$$\gamma_s = \frac{1+\alpha^2}{\beta}$$

$$\gamma_{e-} = \frac{E_{e-}}{m_{e-}c^2}$$