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Collective behaviour of random-activated mobile cellular automata

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Dynamical properties of 2D cellular automata with mobile elements are examined qualitatively. Results show that a system containing elements with local interactions but with no fixed connections, due to movement and connection breaking, are able to display periodic oscillations by collective synchronization of non-periodical randomly activated elements. The system studied is found to be robust. Spatial dynamics is shown to generate interesting spatial structures suggesting the presence of self-organization. Moreover, maximum Lyapunov exponents and fractal dimension of attractors have been calculated in order to show that the dynamics of interaction among elements is chaotic.

1. Introduction

The list of phenomena that involves oscillatory processes arising from the coupling of interacting elements in biology, is almost endless [1–3]. Recently, this list has been further enriched with the discovery of short-period oscillations in the activity behaviour of confined *Leptothorax* ant colonies [4–7]. It has been established, through careful experimental procedures, that individual ants are not periodic oscillators and that they can activate or deactivate through direct physical contact and, when isolated, they become active spontaneously without external stimulus. Moreover, it has been found that this process of spontaneous activation involves the presence of low-dimensional chaos [8].

Those findings seem to support the claim that social behaviour, or at least an important part of it, can be regarded as a global complex self-organized process originating in the non-trivial interactions among the social units and where chaos is very likely to occur as an essential constituent of the system dynamics.

In order to gain more insight and predictive capacity from these exciting ideas of social behaviour it seems necessary to develop mathematical tools that can capture the nature of such complex systems and be able to display the maximum range of behaviour with the simplest assumptions. It is our belief that recent developments in the field of parallel distributed models and discrete dynamical systems offer valuable tools for accomplishing this task. In particular it appears that cellular automata and neural networks are appropriate choices and, in fact, models of ant behaviour based on them have been presented elsewhere [9, 10].

While it is true that the original motivation that led us to the present study is the experimental work with confined ants cited above, we want to explore here, in greater generality, the properties of a model that is essentially a cellular automaton but where the elements are endowed with the capability of movement and can activate spontaneously if isolated or through mutual excitatory local interactions. The interacting units may be cells, insects, robots or any kind of mobile excitatory objects.

We will show the emergence of collective periodic oscillations in the temporal evolution of the model and demonstrate that mobility and change of connections makes the system more robust than the non-mobile counterpart. On the other hand, the space domain will be analyzed and self-organized structures that suggest the spatial organizations present in the nests of some social insects will be shown to exist. Finally the presence of chaos in the dynamics of interactions will be evinced.

2. Mobile cellular automata

Cellular automata in two dimensions are discrete dynamical systems that consist of a regular lattice of sites. Each site takes on a set of possible values, and is updated in discrete time steps according to fixed rules [11]. We will name here *mobile cellular automaton* (MCA) a system that is essentially a 2D cellular automaton but with the property that only a subset A of lattice cells will be updated in the time evolution. The elements that belong to A will be able to move over the lattice randomly.

A mobile cellular automaton is defined over a two-dimensional rectilinear lattice $\Lambda(L)$:

$$\Lambda(L) = \{(x, y) | 1 \le x \le L_n, 1 \le y \le L_m\} ,\quad L_n, L_m \in \mathbb{N} . \tag{1}$$

A nine-cells-square is considered as the neighborhood where interactions among elements will occur (this is the frequently used Moore neighborhood). The set of cells that belong to this neighbor will be labeled M. A collection of n objects, labeled a_i, will be considered as the elements of the set A:

$$\mathrm{A} = \{a_1, \ldots, a_n\} ,\quad n \in \mathbb{R} . \tag{2}$$

Each object a_i in this 2D lattice is further characterized by four quantities:

$$a_i = \{x_i, y_i, m_i, S_i\} , \tag{3}$$

x_i and y_i are the two integer space coordinates. New values for x_i and y_i are calculated at random. The movement pattern is thus essentially a random walk subject to the following two constraints: (i) no two objects will be placed at the same position at the same time, and (ii) new position will be selected randomly among the lattice cells that belong to M. If the cell selected as new position is engaged, then the object will look for another one, this procedure being repeated until a free cell is found or after six attempts are made. If no free cell is found the object stops until the next time step.

The local variable m_i is a Heaviside function of a threshold variable S_i:

$$m_i = \begin{cases} 1 , & \text{if } S_i > 0 , \\ 0 , & \text{otherwise} . \end{cases} \tag{4}$$

If $m_i = 1$ then the object a_i is regarded as active and may move, otherwise it will be considered as inactive and will remain motionless (both x_i and y_i are updated only if $m_i = 1$).

If the number of objects equals the number of lattice cells, then the system is said to be fully saturated (density = 1) and the objects will not move due to the lack of free lattice cells. If this limit condition holds, then a 2D MCA is equivalent to a "classical" 2D cell automaton, where all sites in the lattice are to be updated without motion and where the number and type of connections (the wiring diagram of the network) will be invariant through the entire evolution of the system.

Variable S_i represents the object *activity value* and will range continuously from -1 to 1 ($S_i \in \mathbb{R}$). It is calculated by assigning the local field around a_i that is generated by the presence of other objects $a_j \in \mathrm{M}$ and by a_i itself, if self-interaction is considered. The expression for this variable at time t is given by

$$S_i^t = \tanh\left\{ g\left[\left(\sum_{j=1}^{k} J_{ij} S_j^{t-1} \right) + J_{ii} S_i^{t-1} \right] \right\}, \qquad (5)$$

where J_{ij} are coupling coefficients taken from an interaction matrix $\mathbf{C}$ and k is the number of neighbors of a_i. Note that term $J_{ii}S_i^{t-1}$ represents the contribution of the self-interaction. $\mathbf{C}$ is the square interaction matrix defined as follows:

$$\mathbf{C} = \begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix}$$

whose real-valued entries are selected according to table I.

The parameter g has been introduced in order to allow the control of the slope of the hyperbolic tangent function. In neural network theory, the parameter g is commonly used for fine control of the firing rate or firing threshold of a neural element (e.g. ref. [12]). An equal role will be played here: the value of g will determine the rate of activation and deactivation of the mobile objects. We will further refer to g as the *gain* parameter.

Table1
Assignment of coupling coefficients in eq. (5). c_1 represents a active–active interactions, c_2 and c_3 represent active–inactive and inactive–active interactions, while c_4 represents inactive–inactive interactions. The sign of the i and j elements are considered together. For instance, if $a_i > 0$ and $a_j > 0$ then the interaction is of the active–active type and $J_{ij} = c_1$.

$a_i > 0$	$a_i \leq 0$	
c_1	c_2	$a_j > 0$
c_3	c_4	$a_j \leq 0$

3. Collective oscillations and system robustness

Let us consider first the general case of a system where self-interaction is allowed and suppose that there is only one single object on a lattice. For this isolated object, all terms S_j^{t-1} in (5) are zero and the state variable S_i is reduced to:

$$S_i^t = \tanh(gJ_{ii}S_i^{t-1}) . \qquad (6)$$

In the infinite limit, S_i^t will be zero, due to the fact that the tanh function goes exponentially to zero if self-iterated. When modeled in a digital computer the activity state of an isolated object will be zero after a finite number of time steps if $S_i^t \leq \epsilon$, where ϵ is an arbitrary real number representing a zero value threshold. In this work ϵ was taken as the internal computer IEEE double precision real type or 1×10^{-16}.

Without any external input (no interactions) an isolated inactive object will remain immobile unless it is randomly activated by assigning a non-zero value to S_i, this value will be labeled s_a and we will refer to it as the *spontaneous activity level*. This process of random activation takes place only if $r > p_a$, where $r \in [0, 1]$ is a random variable and $p_a \geq 0$ represents a probability value threshold, the *probably of spontaneous activation*.

Consider fig. 1, where the graph of the sum $S_1^t + S_2^t$ (total activity) of two interacting objects in a small lattice is shown. Because of the exist-

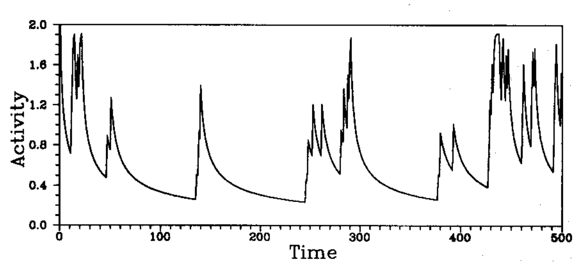

Fig. 1. Time evolution of the sum of individual activity of a pair of self-interacting objects. Increments in activity signals the time of an interaction and the interchange of activity. If no interactions occurs, the activity decays monotonously. Lattice size 10×10, $g = 1$, coupling coefficients all one.

ence of interactions total activity is not globally monotonously decreasing to zero, rather, we can see the existence of peaks where interactions occur that signals an increase in the total activity and the interchange of activity among the pair of objects. In the intervals where no interactions occur the curve is monotonously decreasing as expected.

When mobile elements do not interact they tend to freeze because the impossibility of activity interchange among them. When random activation is not permitted, it is possible to provoke the freezing of any arbitrary number of mobile objects, if the lattice size is large enough so as to make the average time for an interaction to occur larger than the time needed for the individual activity to decrease to zero. If these conditions hold the objects are, in the average, isolated. Other way to provoke the freezing of the system is to make the time needed for decreasing to zero shorter than the average interaction time, specially if the lattice size is not very large. This is accomplished by setting the value of the gain parameter to an adequate value (usually a value $\leqslant 1$). Quite the opposite, if one desires to prevent the system to attain the zero state value, the gain could be adjusted for this purpose (usually a value ≈ 1).

Now consider the case when no self-interaction is allowed and suppose the existence of only one single object in the system. In this case, the internal state will be zero at time t independently of the state value at time $t-1$ as follows from eq. (5). Under these circumstances, an isolated object will freeze immediately. It is possible then to contrast these two examples and to conclude here that self-interaction is acting as a sort of internal memory serving the purpose of delaying object deactivation. From the point of view of the number of connections, self-interaction may be regarded as an additional connection added to the external set of external connections so when isolation occurs, at least one connection is preserved. This of course does not mean that self-interaction is indispensable in order to prevent a collective of objects attaining a zero state value. A system of non-self-interacting objects could be guaranteed not to collectively converge to zero state value as long as the density of the system is high enough as to make the number of connections per object not zero.

In this article, we are concerned not only with the study of the dynamics of the activity states of individual objects but with the macroscopic behaviour of the system as described by the activity of the whole lattice. In order to accomplish this, it is possible to study the number of active objects rather than the individual activity summed, that is, to study the dynamics of the m_i variable and this will now be done. Later we will return to the analysis of the activity variable in order to characterize the dynamics of the interactions.

Consider the series of graphs that appear in fig. 2 showing the time evolution of a number of active self-interacting objects in a given system (lattice size 10×10, $g=0.05$, $s_{\mathrm{a}}=0.01$, $p_a=0.01$, coupling coefficients all 1, initial values of S_i were assigned randomly in the interval $[-1, 1]$ and initial positions were chosen randomly as well). In fig. 2a, the temporal evolution of the system containing just one object is represented. As expected, the occurrence of the peaks showing when the object is active are distributed randomly. In fig. 2b the same situation is represented but with ten objects. Subsequent graphs are for 20 objects (2c), 40 objects (2d), 60 objects (2e), 80 objects (2f) and 100 objects (2g).

Analysis of these graphs permits us to identify a very interesting phenomenon in relation to the system density. In particular, it is possible to observe the emergence of well defined oscillations in the range of medium densities (20–40 objects). In the range of higher densities (80–100 objects) the periodic nature of the oscillations is evident. In contrast, in the range of lower densities (1–10 objects), no periodic behaviour in the temporal evolution is present at all. In order to compare the behaviour of the

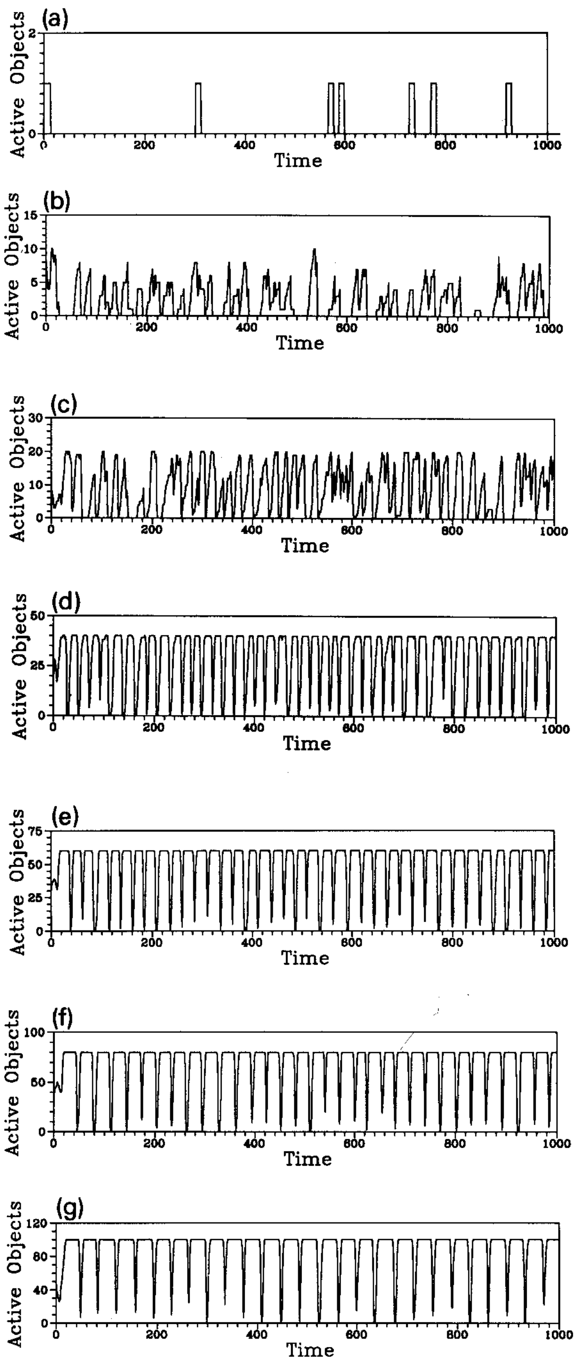

Fig. 2. Temporal evolution of a MCA with random activation of self-interacting objects showing the emergence of periodic oscillations in the number of active objects as the system density is increased. Density is (a) 0.01, (b) 0.1, (c) 0.2, (d) 0.4, (e) 0.6, (f) 0.8 and (g) 1.0. See text for system parameters.

system previously described with a motionless cellular automaton, we present the results shown in the graphs in the fig. 3. In this case, the objects were forced to stand still. When the motionless system has a population totaling 100

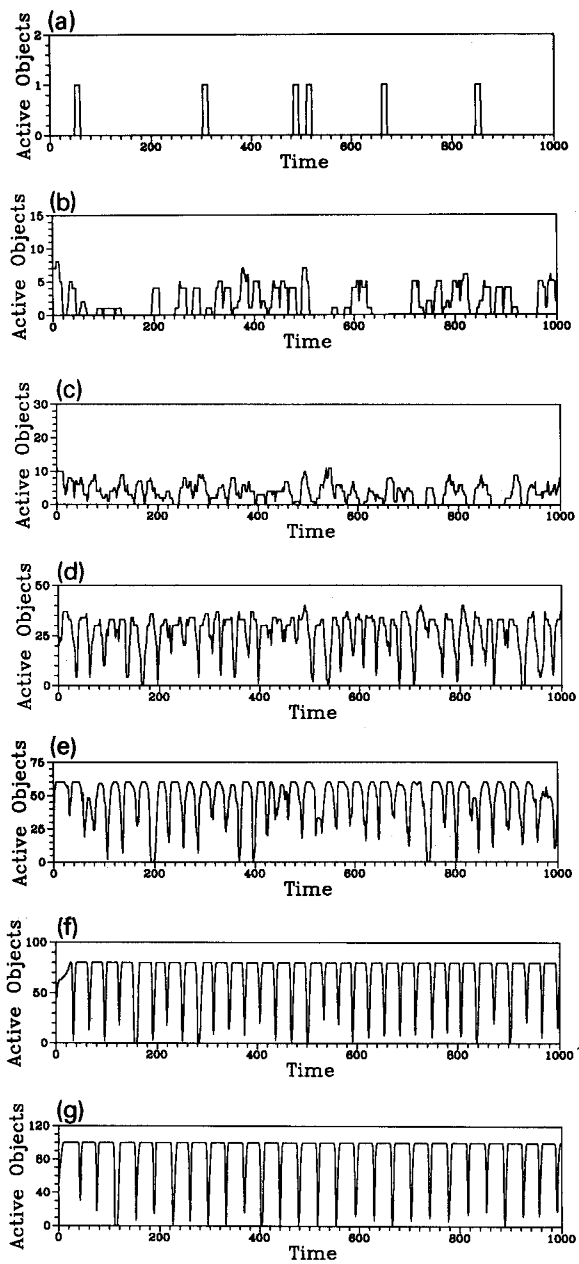

Fig. 3. Same system parameters as in fig. 2 but the objects are forced to remain motionless.

objects, the behaviour is identical with that of the mobile system because both are saturated and their parameters equal.

Qualitative comparison of both systems gives the following results:

(i) Collective periodic oscillations are present in both systems.

(ii) There is a clear density-dependent phenomenon regarding the emergence of collective periodic oscillations.

(iii) Both systems are robust: periodic oscillations are preserved even if a relatively large number of objects are suppressed from the lattice.

(iv) Collective periodic oscillations appear at lower densities in the mobile system, making mobility an important condition for greater robustness.

In order to clarify further these ideas the relationships among robustness, period of oscillations and system density were examined in more detail. A set of time series of both systems was prepared for different density values and a Fourier transform was applied to them in order to obtain the period value through the fundamental harmonic in the power spectrum. Results obtained from this procedure are represented on the graph in fig. 4a. When density is 1, both systems behave identical giving the same period value of 36.5. As density is lowered the period value difference among the two systems begins to increment. When density is around 0.35, divergence between both systems is obvious. The motionless system departs from the linear trend at density value of about 0.35 while the system with mobile objects does it at density value between 0.20 and 0.25. The point at which the period value departs from the linear trend marks the value of the density at which collective periodic oscillations begins to disappear.

What can be learned from this experiment is that the relationship between density and period is linear and tends to decrease as density decreased. When density is lower, the rapid increase in the period value is only apparent: in

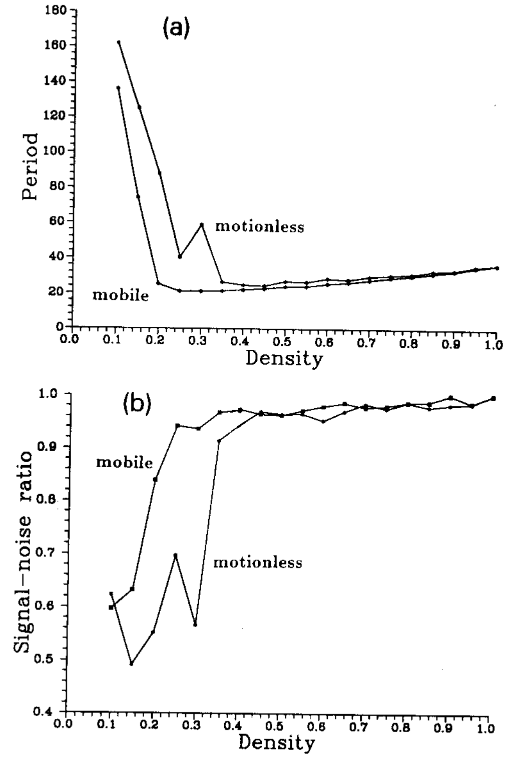

Fig. 4. (a) Relationship between system density and period. The period has been calculated with a Fourier transform on 1024 time steps long data series. For each density value an average of five samples period was made. Same system parameters as in fig. 2. (b) Signal to noise ratio as function of system density. Signal to noise ratio was defined as SNR = $(\bar{p}/\bar{p} + \sigma)$ where $\bar{p}$ is the average period and σ is the standard deviation of the five data samples. Signal to noise ratio as defined gives a good measure of system robustness. In this case it is easy to conclude that a system with mobile objects is more robust than the motionless counterpart.

this situation the power spectrum gives a wide range of period variance and the error in determining the period is large because the value of the period is the one associated with the highest peak in the power spectrum but other peaks as high as this are present. This fact is more evident in the graph shown in fig. 4b, where the relationship among density and the "signal to noise ratio" of the spectrum is presented graphically. Signal to noise ratio was defined here as SNR = $(\bar{p}/\bar{p} + \sigma)$ where $\bar{p}$ is the average period and σ is the standard deviation of the sample (each point is the average of five different replica). When the signal is pure ($\sigma = 0$) the SNR = 1. When the error in the signal is equal to the value of the signal the SNR = 0.5. Signal to noise ratio as defined above gives a good measurement of the degree of system robustness and serves the purpose of comparing a mobile cellular automaton with its motionless counterpart. In fact, from fig. 4b it is clear that the mobile system is more robust.

Up to here, the discussion has been centered on considering self-interacting objects, but what happens if self-interaction is omitted in a mobile system? In order to explore this crucial question the term $J_{ii}S_i^{t-1}$ in eq. (5) was omitted. Periodic oscillations were still present but they appeared at higher density values than in the case of the system with self-interaction (typically around density value of 0.4). Another remarkable difference among these two systems was that, at lower densities, non-self-interacting objects tended to remain more inactive than their self-interacting counterpart. Analysis of this behaviour permit us to conclude that robustness in this model is closely correlated with the average number of connections available per object in the lattice.

For self-interacting objects at least one connection is guaranteed to exist and, as proposed before, it serves as a deactivation delay. This in turn implies that self-interacting objects will on average, be more active than non-self-interacting objects. On the other hand, no self-interacting objects need a higher system density in order to preserve at least one connection.

Mobility increments robustness because it guarantees that interactions will occur more frequently. In fact, an encounter between any pair of mobile objects in a closed space is non zero probability event while two non mobile objects, not in their immediate neighborhood, will never interact. This fact is also reflected in an increase of the average number of connections in mobile systems. In a motionless system, isolation of individual objects or small domains of objects may occur while mobility assures that isolation will, on average, not occur. Isolation in domains implies that collective oscillatory behaviour may be desynchronized due the absence of links for activity interchange. Isolated domains may well be pulsating out of phase of the rest of the lattice objects.

In a motionless system, activity spreads over the lattice at speed equal to one lattice per time unit (speed of light = 1) while in mobile systems the speed of light is twice this value. A faster activity release over the lattice tends to decrease coupling delays making collective synchronization more easily attained. In this sense, mobile objects resemble mobile vectors for disease transmission: collective disease infection occurs faster in a space with mobile infected agents.

3.1. System parameters

As stated before, the gain controls the rate of deactivation of an isolated object and also the activation rate of an interacting object as is clear from (5). If g is made to have a large value (usually $g > 1$), S_i^t will flip-flop between values 1 and -1 or 1 and 0 if only positive values are involved. Under these circumstances the system approaches the discrete binary limit. Gain g also has a very important role in determining the period of the collective oscillations. As a general trend, the period is incremented with increments in g. Nevertheless this trend will be broken because, as stated, a large value for g will make the system to approach the discrete limit and if this happens periodic behaviour is lost.

Probability p_a has the overall effect of increasing the number of active objects but does not have an important role in determining the period of the oscillations, once oscillations are observed.

Coupling coefficients in matrix **C** play an important role. It is possible to regard eq. (5) as a weighted process since these coefficients "weight" the relative importance of the objects in the interaction. Coupling coefficients may be interpreted as weights for the three types of interactions (active–active, inactive–active and inactive–inactive).

Since the entries of the matrix are real-valued constants, complete examination of all combinations of matrix entry values is not possible, nevertheless an exhaustive research of their values and the qualitative system behaviour was carried out and the main result points toward considering the coefficient c_1 (active–active interaction) as the most important one (fig. 5). It was found that the period T decreases when the value of c_1 is decreased following an exponential relationship of the type $T = \alpha\, e^{c_1} + \beta$ where α and β are constants whose values depend on g, n and the values of the other coupling coefficients as well. If was found that when c_1 approaches zero ($c_1 \approx 10^{-4}$), there is a sharp transition in the dynamics of the system. A small value for c_1 implies that the coupling links between active self-interacting objects is weakened. In the limiting case when $c_1 = 0$, active objects are prevented from interacting and cannot access their internal memory. This gives rise to a new dynamic situation where collective periodic behaviour is lost and quasi-periodicity arises.

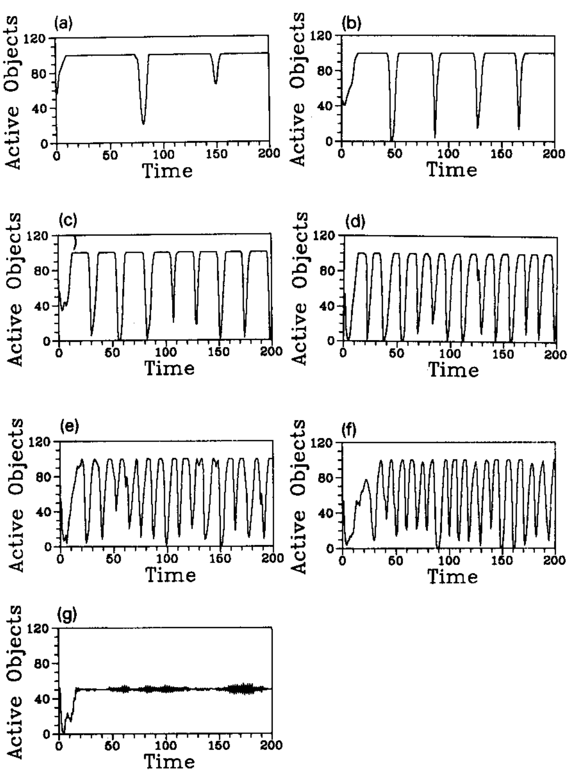

Fig. 5. Value of coupling coefficient c_1 and temporal evolution of a MCA lattice (see text). (a) $c_1 = 1.5$, (b) $c_1 = 1$, (c) $c_1 = 0.5$, (d) $c_1 = 0.1$, (e) $c_1 = 0.05$, (f) $c_1 = 0.01$, (g) $c_1 = 0$, (lattice size $= 10 \times 10$, $s_a = 0.01$, $p_a = 0.01$, $g = 0.05$).

4. Space structure and self-organization

In the case of a fully saturated lattice, the behaviour of the MCA is roughly equivalent to a model of an excitable medium. Excitable media are very well known for their property of supporting the propagation of pulses of excitation due to the ability to activate collectively following a perturbation of its resting state. This perturbation could be a single stimulus that goes beyond a given activation threshold value [13, and references therein]. This property of excitability and wave propagation is present in the MCA model.

Let us start the discussion of the spatial evolution in terms of a saturated lattice. Consider the series of snapshots shown in figs. 6a–6e. At time $t = 0$ an initial perturbation was introduced by assigning a value of 1 to the activity variable of the object located at the lattice center, the rest of objects being set to 0. Successive time steps show activity propagation as a wave of excitation. This wave has an initially square front ($t = 5$ and $t = 10$) but transforms quickly into a circular waveform because objects at the corners of an initial-squared wave deactivate first (they have more inactive neighbors than those objects in the edges or in the inner zone of the active domain). At $t = 20$ the wave touches the borders, at $t = 30$ it begins to enter the refractory state, in which the wave collapses towards the center to disappear completely at $t = 42$. The pictures show activity values in a black and white scale, white indicating no activity while black indicates

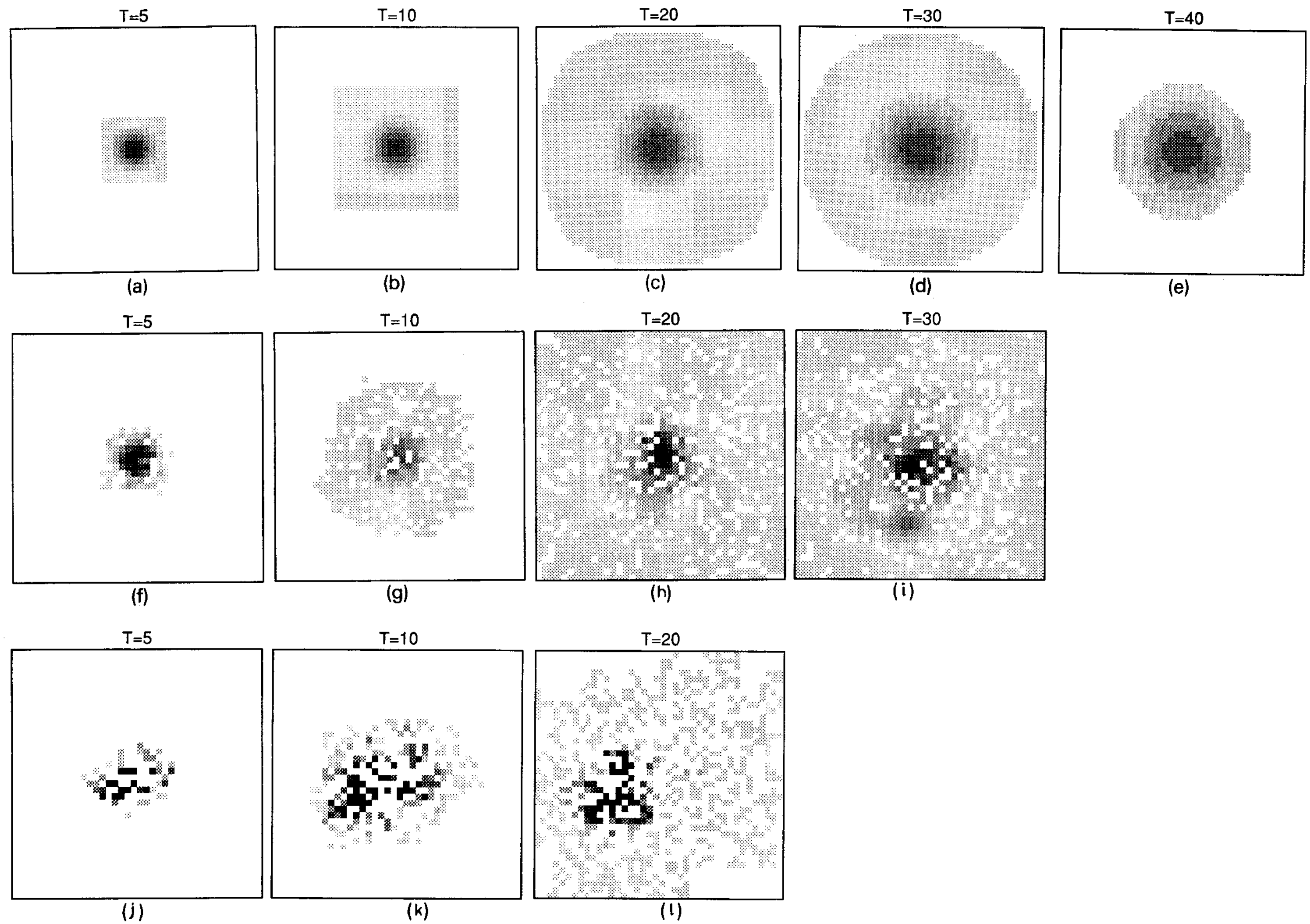

Fig. 6. The space dynamics of a MCA shows a behaviour similar to an excitable medium. An initial perturbation was introduced by setting the activity value of a center-located object to 1, the rest of the object were set to zero (lattice size = 41 × 41, $s_a = 0$, $p_a = 0$, $g = 0.05$, all matrix coefficients 1 except $c_4 = 0$). (a)–(e) System density = 1. A wave of excitation is seen expanding through the lattice, initially as a square wavefront and later as a circular wavefront. Activity is indicated by means of a black and white scale. White represents inactivity while black represents maximal activity. Note that scale is not absolute between frames. (f)–(i) System density = 0.8, white spaces indicate empty spaces and inactive objects as well. Activity propagates faster than in a saturated grid and the wavefront is roughly circular. Activity wave collapses faster than the wave in the saturated grid. (j)–(l) System density = 0.2. wave propagation is rather irregular, a cloud of maximal activity can be seen travelling over the lattice and in (l) it has moved from the center towards the left.

maximal activity, gray tones are intermediate values (the scale is not absolute between frames so only qualitative comparison can be made).

Fig. 6f–6i show the same lattice but with a density of 0.8 (20% of the space is available for movement). Objects were located randomly over the grid with zero activity value except for a single one located at the center with a value of activity $= 1$. White dots indicate both empty spaces and inactive objects. At $t = 10$ the activity has propagated between the objects and because of their movement they break the square waveform more quickly. The adoption of a circular wavefront is a very desirable property of a cellular automata model of excitable medium since it implies space isotropy, that is, activity will spread at the same velocity no matter in what direction [14]. In the case shown, the region of excitation is roughly circular and hence isotropy exists only on average. The roughly circular wave front in a mobile system comes from the combination of two phenomena. The spontaneous formation of the circular front as observed in a saturated grid and because territory coverage in a system of random walkers has a characteristic roughly circular geometry as long as the density is high [15]. On the other hand it is necessary to point out that the velocity of propagation has increased because of mobility as can be clearly seen from fig. 6h where activity has reached all regions of the lattice. At $t = 32$ the activity wave has completely collapsed and disappeared.

Finally, figs. 6j–l show a MCA with density of 0.2. Because of the low density, the roughly circular wave is not present, rather, the dispersion of activity is non-isotropic and depends strongly on the space distribution of the inactive objects. Clearly, if density is lower than a certain value the activity wave is not propagated. This is quite compatible with the dynamics in the time domain where periodic cycles are not present if density is above a certain value (and, possibly, can be a clue for the existence of percolation-like phenomena). When $t = 20$ another aspect of the behaviour of a MCA is evident: the region of greater activity has moved from the center towards the left side. If the life time of the activity wave is long enough it is possible to observe this region moving randomly over the lattice. Nevertheless the objects that form the region are not the same all the time except, perhaps, for a very few including the initial seed. Finally, total collapse of activity occurs at $t = 22$.

It appears adequate to regard the MCA model as a special kind of excitable medium in which space and time are discrete and excitable objects can move and interact. Now, one is tempted to question what kind of long-term space behaviours can exist in a system where excitable units move randomly and can activate randomly. At first, it appears reasonable to assume that, due to such great randomness, no-long term spatial organization can exist and, in fact, we never have come across with such structures as spirals waves or other spectacular patterns. While we cannot rule out *a priori* the possible existence in this model of such manifestations of spatial organization, specially in the case of saturated or high density lattices, we turned our attention to another kind of long-term spatial behaviour as will be explained below.

It is possible to perform an interesting experiment in which the number of times a given lattice cell has been active are counted. Results with eight different matrices are shown in fig. 7. The lattice is saturated and initial positions and activity values were chosen randomly. As can be seen, a number of them have developed concentric patterns in which activity has accumulated in the center of the lattice, minor activity being registered towards the periphery. The finding of concentric rings with greater activity at the center is consistent with another experiment in which the sites where random activations occurred were mapped using the same matrices that produce concentric rings. In this case random activations were found to be mostly confined to the periphery indicating regions of lower activity (random activations occur only if the

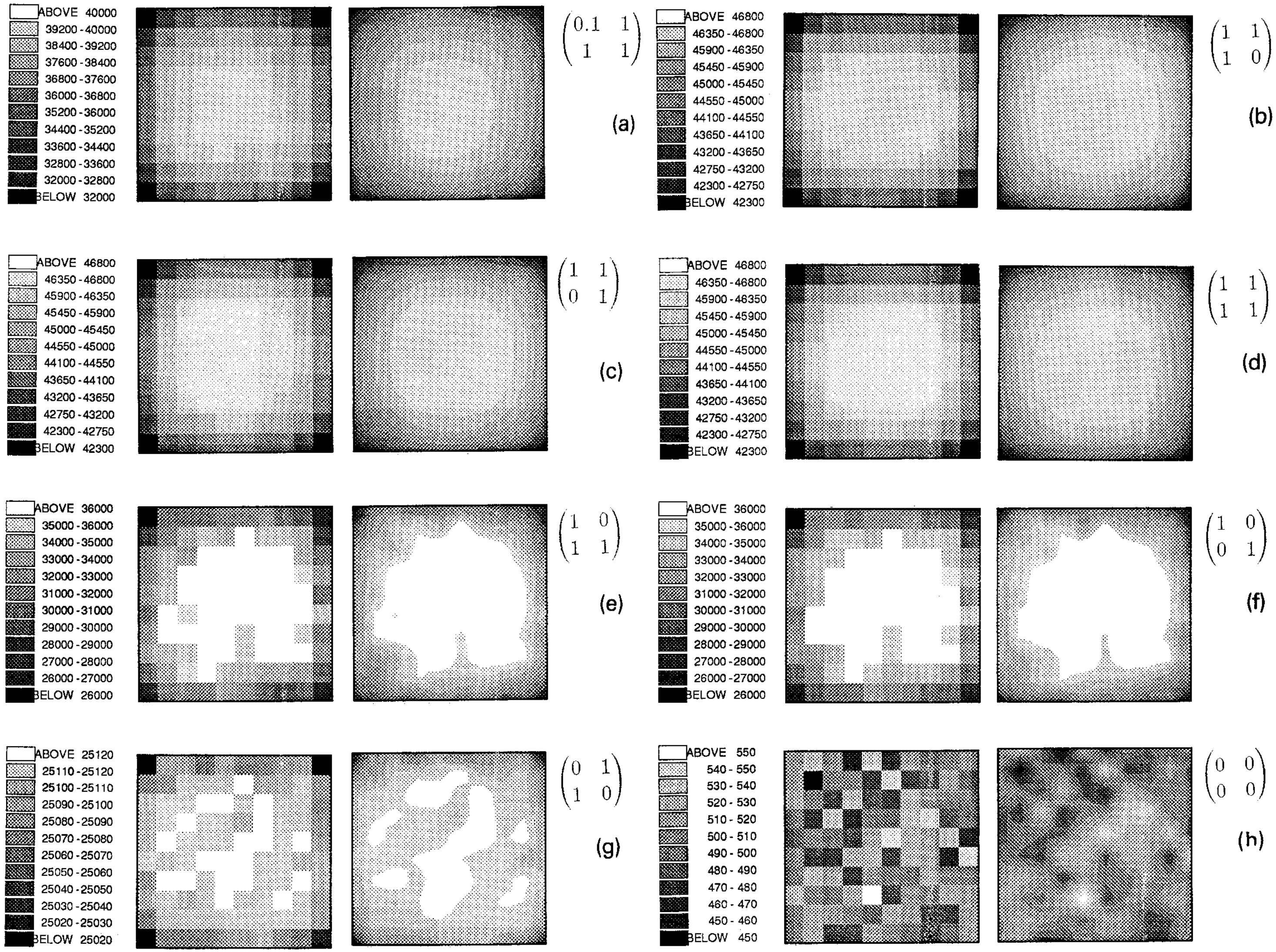

Fig. 7. Space patterns produced after 50 000 time steps are shown for four different matrices. Value at each lattice site represents the number of times the cell was active (density = 1, lattice size = 10×10, $s_a = 0.1$, $p_a = 0.01$, $g = 0.05$. Initial activity values were set randomly in the interval $[-1, 1]$. Objects with self-interaction were considered). Two representations for each matrix is shown, a discrete lattice that is the correct representation and a contour map that represents an interpolation and is presented only as a visual aid. (a)–(d) The pattern developed consist of a central region of maximum activity, lower activity was recorded at the periphery. (e)–(f) The pattern shows concentration of greater activity at the center of the lattice, but not in a clear pattern of concentric rings. (g) The pattern developed shows lower activity at the lattice borders but no clear pattern is seen at the center. (h) The pattern obtained was completely random. Notice that, since the matrix values for this case rule out any kind of interactions, the activations were only due to the random process of spontaneous activation, as can also be seen from the fact that an average of 500 activations occurred after 50 000 time steps giving a probability of 0.01. The value given to p_a was exactly this.

objects are inactive). Since no initial information was given on where greater activity should occur, it appears appropriate to consider the existence of such patterns to be the result of a self-organized phenomenon.

It is necessary to mention that only those matrices that produce oscillations in the time domain were found to produce concentric patterns in space. It is not clear whether this is a general rule in the model or not.

Existence of self-organized patterns may be relevant due to the fact that little or no encoding about space usage has to be previously stored in the individual interacting mobile objects. Again, as in the case of the global clock, a global map containing information about space distribution of activity can emerge from the process of local interactions only.

At this point one is tempted to question if this result is relevant. Fortunately, collectives of mobile interacting objects do exist apart from computer models: social insects provide us again with good examples of space organization in which concentric regions of greater activity (correlated with metabolic rates) are well defined, and we would like to mention here two examples.

The honey bee develops a 3D pattern of concentric patterns of spatial organization on their beehives [16, 17]. If the space activity pattern of a wax comb (a 2D section of the 3D nest) is observed, it is possible to identify concentric well

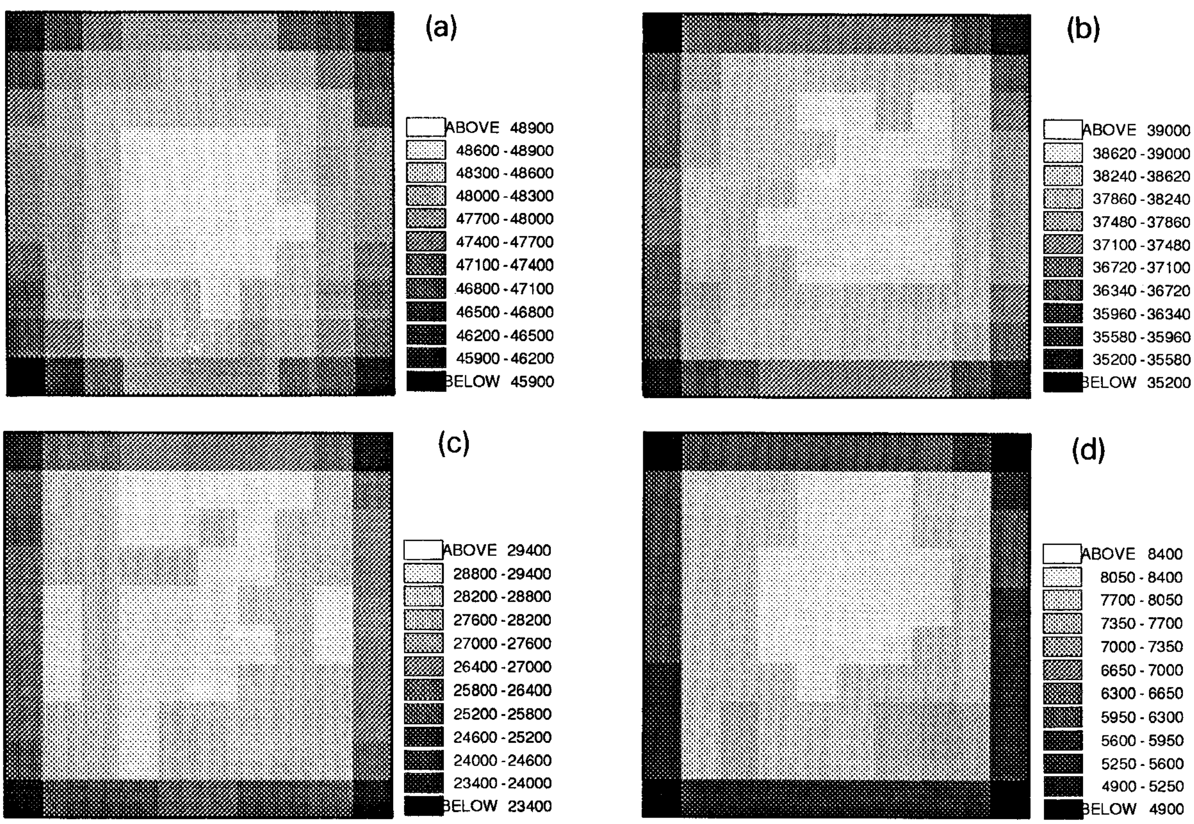

Fig. 8. Space patterns developed after 50 000 time steps are shown for four different density values. As in fig. 7, each lattice site value represents the number of times the cell was active (lattice size = 10 × 10, $s_a = 0.1$, $p_a = 0.01$, $g = 0.05$, matrix coefficients all 1 except $c_4 = 0$. Initial activity values were set randomly in the interval $[-1, 1]$. Objects with self-interaction were considered). Density values are (a) 1, (b) 0.8, (c) 6.0 and (d) 0.2.

differentiated ring-like regions, a central one where the major activity goes on (it includes the region of egg and brood care) and two peripheral rings where pollen and honey are stored. The other example is another *Leptothorax* species, in which concentric patterns of activity in their nests have been identified recently [18]. In this case, the brood is arranged in a cluster with a characteristic 2D pattern of concentric rings. Smaller items are placed at the centre while the largest are located towards the periphery.

Since the above examples of spatial organization involve mobile individuals it is necessary to check if the spatial organization in a saturated lattice is preserved when mobility is introduced. In order to answer this important question, the experiment shown in fig. 8 was performed. The pictures represent a lattice with different densities. As can be seen, concentric rings are lost as the density is lowered, but it is important to point out that clustering in a region of greater activity occurs despite low density and random mobility. No patterns of random distribution of activity (like the ones shown in figs. 7g and 7h) were detected.

Whether the social insects in the two examples just mentioned, show any kind of oscillatory behaviour in the temporal pattern of activity of their individuals is an open question, as is the problem of finding evidence of any pattern of spatial activity in the case of the ants with known oscillatory behaviour. Clearly further experimental work on this is very desirable.

Another case of interesting spatial organization in this model is present when $c_1 = 0$ and the system is fully saturated. Under these conditions coupling among active objects is lost as well as self-interaction. Interaction is limited to active–inactive and inactive–inactive pairs and the system becomes "diluted" since the number of connections per object is reduced. Because of no self-interaction (no deactivation delays) the objects tend to oscillate individually at higher rates between active and inactive phases but oscillations at lower rates are present at large temporal scales. A temporal evolution of such a system

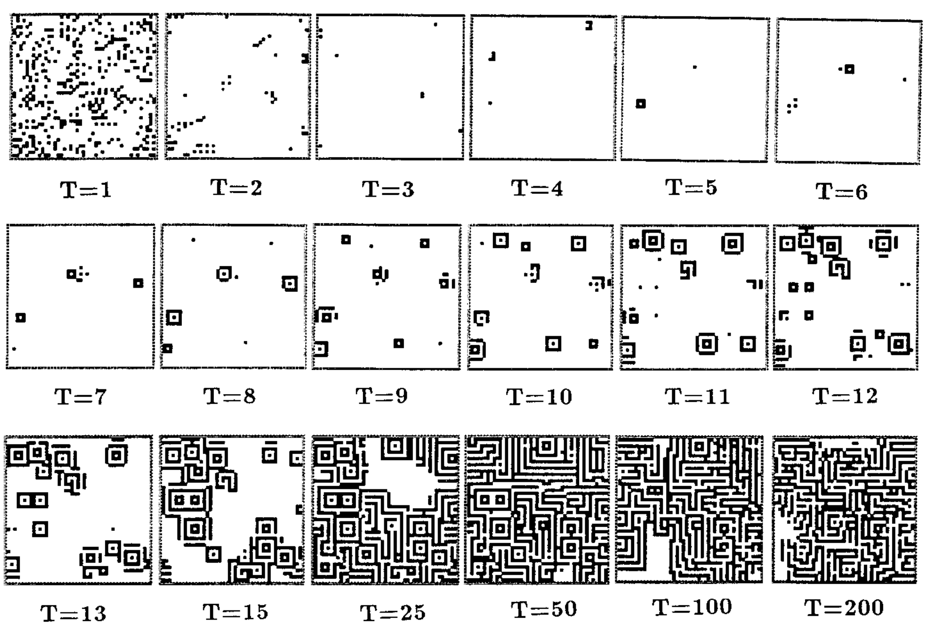

Fig. 9. Spatio-temporal evolution of a diluted and saturated MCA lattice showing emergence of long-term spatial organization. Black dots represent active objects while white dots represent inactive ones. Lattice size = 45×45, $n = 2025$, $g = 0.05$, $s_a = 0.01$, $p_a = 0.01$, $c_1 = 0$ and the rest of coupling coefficients are one. Time steps are shown.

proves to be quasi-periodic with largest Lyapunov exponent of −0.056, and it give rise to very interesting long term spatial structures such as those shown in fig. 9.

5. Dynamics of interactions: chaos is present

The origins of collective behaviour in any connected system have their ultimate explanation in the non-linear interactions among their elements. From this point of view, it is very important to describe the dynamics that are present not only on the global or macroscopic system scale as represented by counting the number of active objects in a given time step, but by studying the dynamics of interactions through the time evolution of the activity variable S_i^t.

Consider a system of n mobile self-interacting objects with no random activation. As stated before, the gain g may be adjusted to an adequate value so as to guarantee that no single object will lock in the stable attractor (zero value). Let us define total activity as $\Sigma_{i=1}^{n} S_i^t$ and explore its evolution through the example shown in fig. 10. The graph in fig. 10a corresponds to the time evolution of a lattice with six mobile self-interacting objects ($g = 0.95$, lattice size = 16×16, $p_a = 0$, $s_a = 0$, coupling coefficients all 1, all initial state variables are 1, first 500 time steps were discarded). The dynamics of this system was found to be chaotic and a Poincaré map is drawn in 10b showing clearly a well defined structured attractor. Further characterization of this attractor by means of an optimized box-assisted algorithm [19] showed a fractal dimension of 1.5. Moreover, the largest Lyapunov exponent calculated according to the well known Wolf et al. algorithm [20] showed a value of 0.474 indicating strong trajectory divergence typical of chaotic attractors.

It is worth mentioning that other lattices with different number of objects and system parameters also showed this chaotic behaviour remarking the non-linear nature of the model. Nevertheless no evident pattern was found to exist between the values of g, system density and the values of fractal dimension and largest Lyapunov exponent of the corresponding attractors.

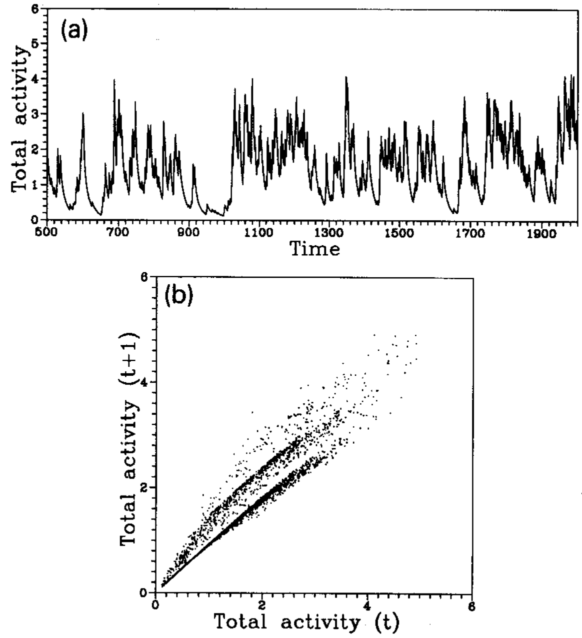

Fig. 10. (a) Time evolution of the total activity on a lattice with six mobile objects. This system was found to be chaotic. (b) The Poincaré map showing the attractor of the time series. Fractal dimension is 1.5 and maximum Lyapunov exponent is 0.474. See text for system parameters.

6. Summary and discussion

The main aim of this article was to demonstrate the existence of periodic oscillations in a system in which elements are not periodic oscillators. Oscillations are an emergent property of the system and arise from the interactions among the excitable lattice elements. Since the objects over the lattice can move, we named such a system *mobile cellular automaton* so as to distinguish it from classical cellular automata in which lattice elements are not mobile.

Inactive objects activate at random and once active they spread their activity through the lattice space by means of local interactions and because active objects can move, they spread activity through the space at a higher rate in comparison with non-mobile systems.

It was shown that a system with mobile self-interacting objects is more robust than the non-mobile counterpart and more robust than a system of non self-interacting objects. Robustness then seems to be associated with the average number of connections per object, a condition closely related to system density rather than to absolute number of objets.

System robustness is very important in relation to the internal complexity of individual objects. If periodic oscillations serve a purpose, complexity in system design is reduced because no internal clock has to be defined for each object. A very precise clock will be an emergent property of this connected collection of objects. In fact, if mobility and self-interaction are present, it will be a more efficient clocked system since mobile self-interacting objects could preserve the collective periodic behaviour with the smallest number of objets.

The presence of interesting self-organizing spatial structures was shown to exist, demonstrating the power of this model to account simultaneously for temporal and spatial collective behaviour. As in the case of the global clock, a global map develops from random disordered initial states, without any initial clue about the final spatial distribution of activity. It is also quite remarkable that spatio-temporal symmetry breaking in this model exhibits qualitative behaviour like those present in social insects.

A study was carried out to characterize the maximum Lyapunov exponents and fractal dimension of attractors in the time series representing the dynamics of the interactions and the existence of chaos was demonstrated to exist.

It is our opinion that a model like the one studied here offers a good approach to the phenomenon of the emergence of collective behaviour in systems with mobile elements where coupling by means of local interactions occurs. We hope it will be useful and stimulating for the current discussion of whether social behaviour is a complex form of self-organization.

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