Comment on "Penrose Tilings as Jammed Solids"

The discovery of isostaticity in sphere packings [1] and network glasses [2,3] has inspired a great deal of activity in the field of isostatic networks. Recent studies [4,5] suggest that all elastic moduli of geometrically disordered isostatic networks go to zero with increasing linear size L, if disorder is uncorrelated. Packings of hard frictionless spheres or discs, on the other hand, have nonzero compressive modulus B [6,7], despite being isostatic [1] and disordered, because their contact network is not random, but tuned to avoid negative forces. Contact disorder is correlated in these systems. Attempts to model sphere packings as randomly disordered isostatic networks have therefore failed. However, in a recent Letter [8], Stenull and Lubensky (SL) claim that randomly disordered Penrose networks have nonzero B for large L. The present numerical study, using a high-precision conjugate gradient to solve the elastic equations shows that the bulk modulus B actually goes to zero for large sizes. Figures. 1(a) and 1(b) show $B(\epsilon, L)$ for Penrose periodic approximants of orders 5 to 12 (up to 8×10^4 sites), whose sites are randomly displaced within a circle of radius ϵ.B behaves roughly as $1/L^2$ for large L [Fig. 1(b)]. However, because $B(\epsilon = 0, L) = 0 \forall L$ [8], B grows as $\epsilon^2 L^3$ [9] when $\epsilon^2 L^3 \ll 10^2$. The asymptotic regime $L \gg L_0 \approx (10/\epsilon)^{2/3}$ is hard to reach for small ϵ. This has been noted already [5] for other disordered isostatic networks. The data reported by SL [8] (derived from normal-mode calculations for a single, unspecified, value of ϵ) are similar to our results for $\epsilon = 10^{-2}$ in Fig. 1(a); i.e., B appears to saturate. Our scaling analysis in Fig. 1(b) shows that this is a finite-size effect: the true asymptotic behavior $B \sim L^{-2}$ would only be seen at much larger sizes for this value of ϵ. Further validation of our claim that $B \to 0$ for large L is provided by the following: fixing a line and a row of sites produces Penrose networks with fixed boundary conditions (FBCs). B^{FBC} is seen to go to zero with size when $\epsilon^2 L \gg 1$ [see Figs. 1(c) and 1(d)]. But B^{FBC} is a rigorous upper bound for B^{PBC}. Therefore, $B^{\mathrm{PBC}} \to 0$ for large L as well. We additionally mention that the effects of geometric disorder on elastic constants can be described analytically for small ϵ, giving rise to rational expressions for $B(\epsilon, L)$, that predict an asymptotic power-law behavior $B(\epsilon, L) \sim L^{-\mu}$ when $\epsilon \neq 0$. Details will be provided somewhere else [9]. We conclude that one of the points raised by SL [8] is not justified: generic Penrose networks with uncorrelated geometric disorder have zero bulk modulus for large sizes. Their asymptotic properties, therefore, seem comparable to those of previously studied isostatic networks with geometric disorder.

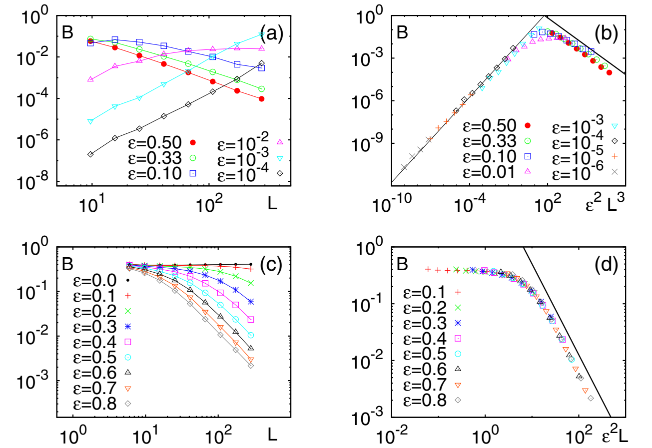

FIG. 1 (color online). Average bulk modulus $B(\epsilon, L)$ of randomly site-displaced Penrose approximants with period L and disorder strength ϵ, under periodic boundary conditions [PBCs, (a) and (b)] and fixed boundary conditions [FBCs, (c) and (d)]. Lines in (a) and (c) are guides to the eye. The thin line in (b) is $B \sim \epsilon^2 L^3$. Thick lines in (b) and (d) are, respectively, $B \sim (\epsilon^2 L^3)^{-2/3}$ and $B \sim (\epsilon^2 L)^{-1.6}$.

The authors thank CGSTIC of CINVESTAV for computer time on cluster Xiuhcoatl.

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Received 8 July 2015; published 11 November 2015
DOI: 10.1103/PhysRevLett.115.209801
PACS numbers: 83.80.Fg, 61.43.-j, 63.50.Lm, 64.70.Q-

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