COURSE
on
Effective optical properties of disordered systems

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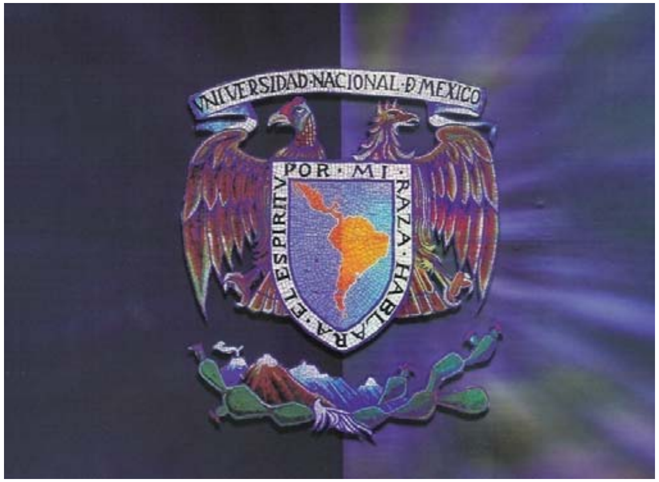
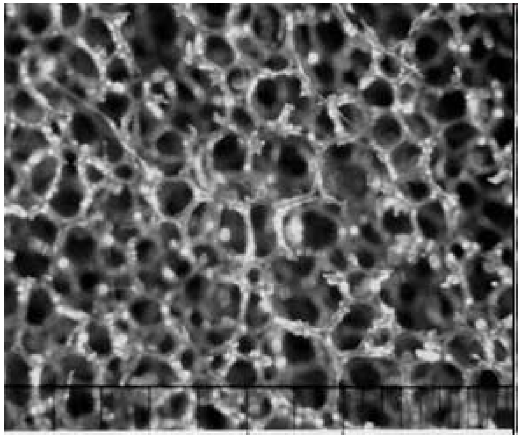
Mexico

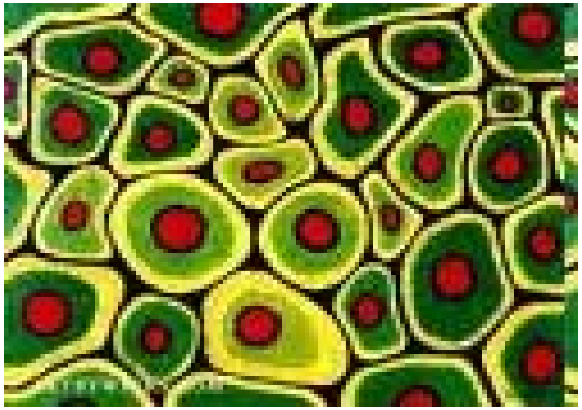


Programa

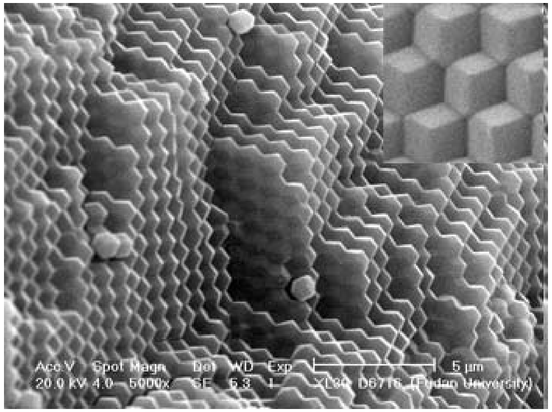
Propiedades ópticas de medios heterogéneos



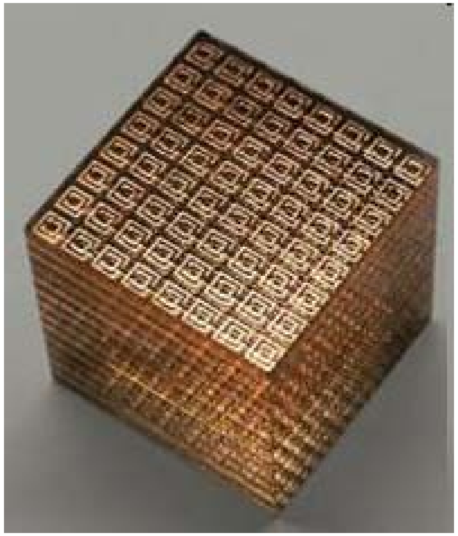
Materiales porosos



tejidos

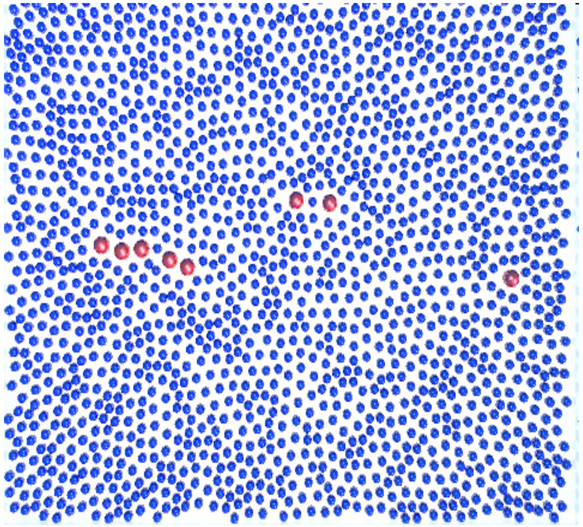


cristales fotónicos



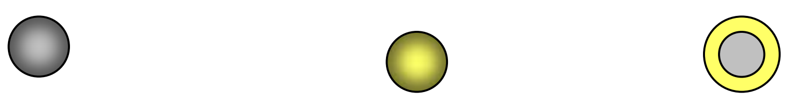
metamateriales

alas de mariposas



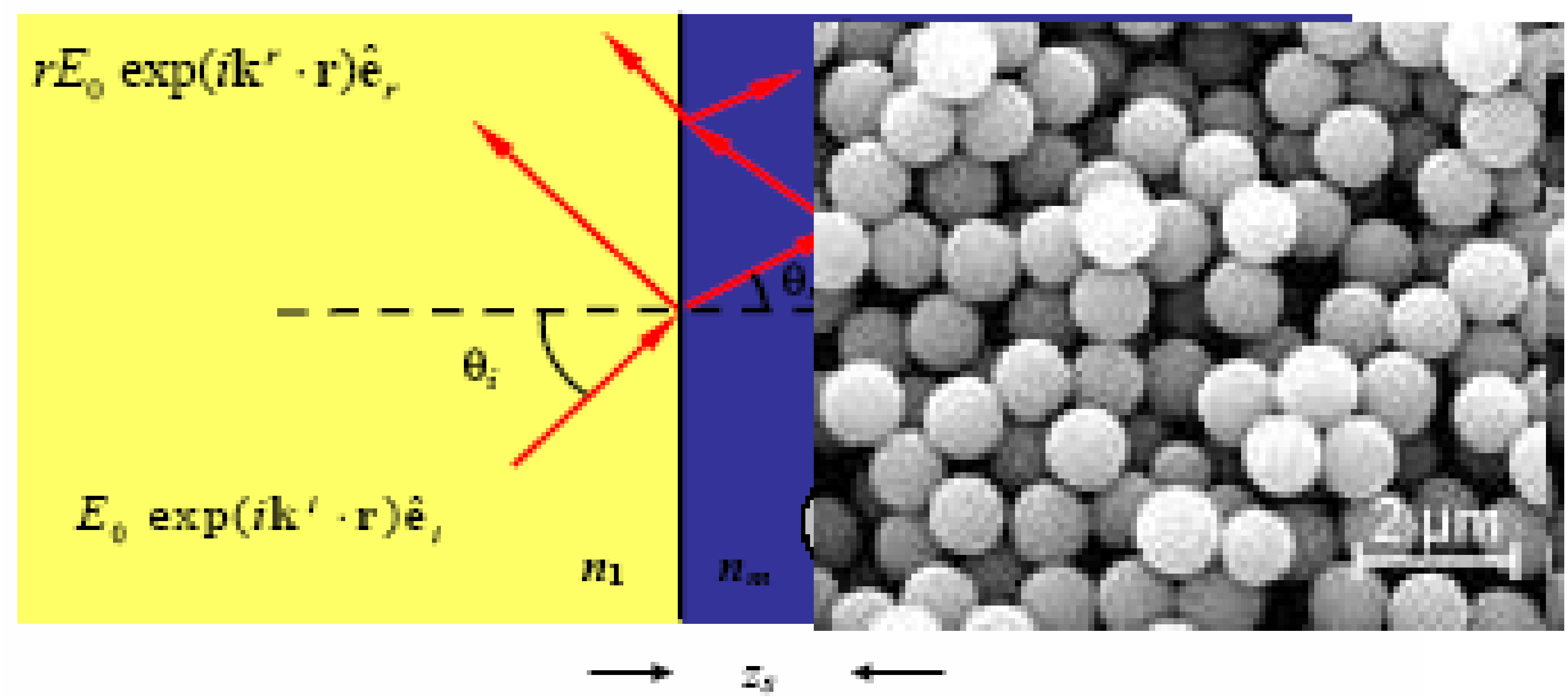
coloides

Entender…



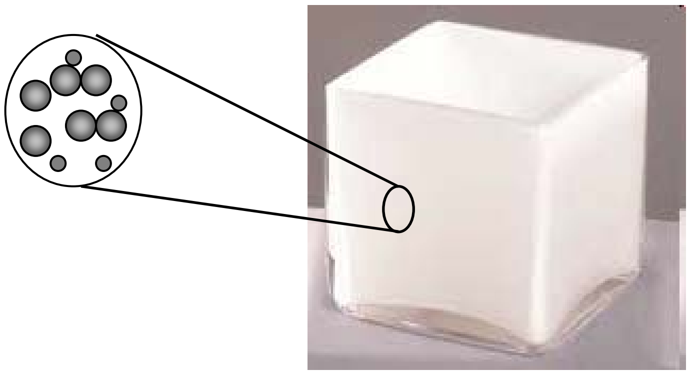
Ag Au Ag/Au

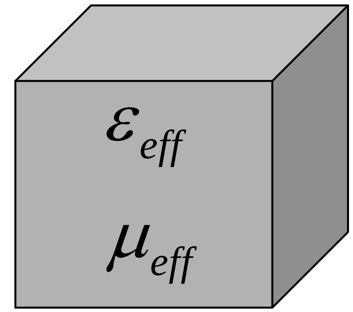
Transmisión y reflexión



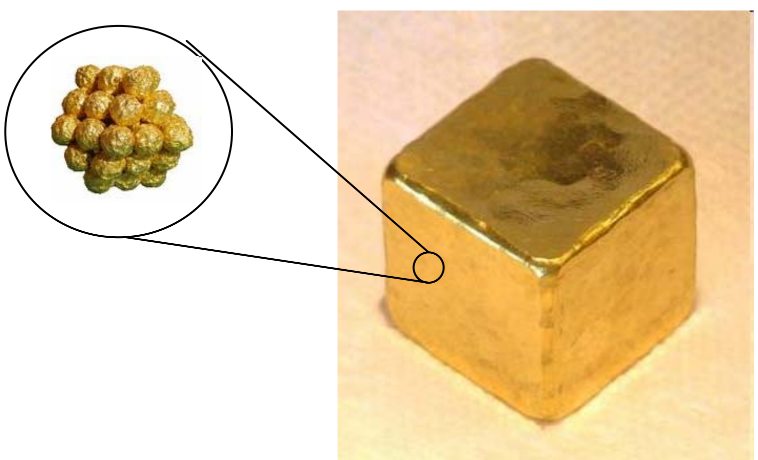
Teorías del medio efectivo

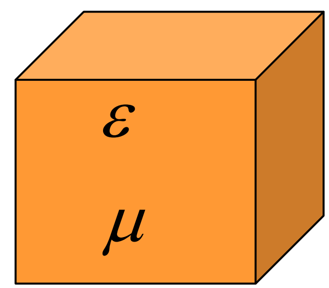
Promedio





Homogenización





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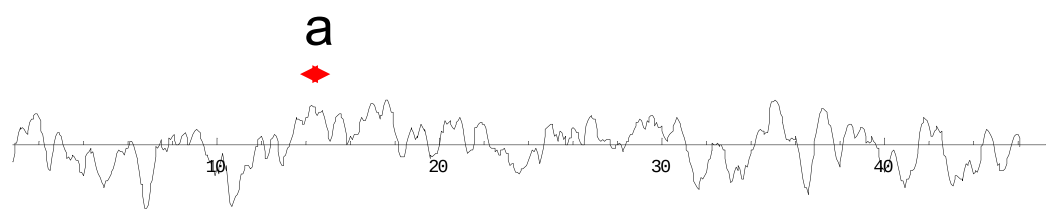
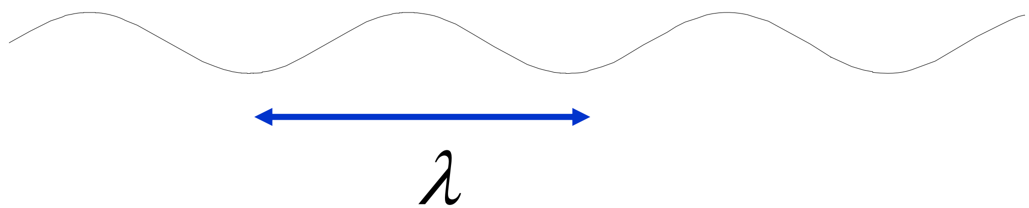
electrodinámica del medio contínuo

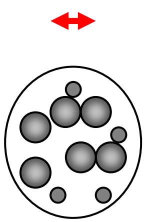
electrodinámica “macroscópica”

Promedio



$$\hat{P}_{av}\vec{E} = \left\langle \vec{E} \right\rangle$$





Promedio espacial

$$a \ll \lambda$$

mesoscópico

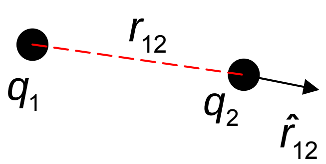
ESENCIA: ¿cómo promediar correctamente?

Electrodinámica continua

Electrodinámica en el vacío

cargas puntuales

acción "a distancia"



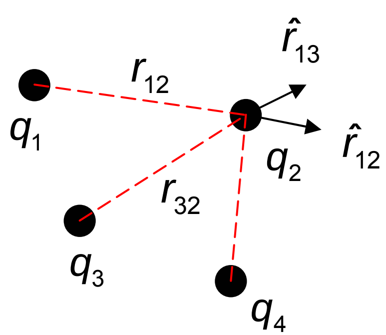
$$\vec{F} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

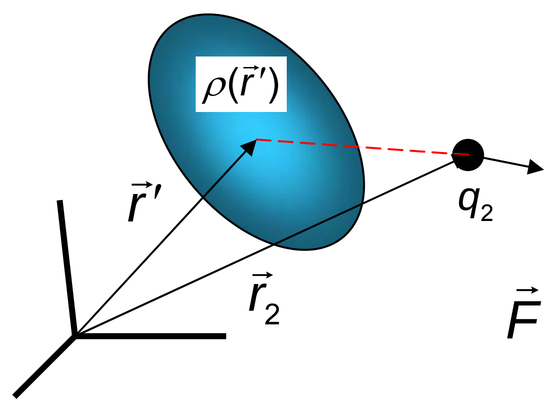
en reposo

superposición (propiedad intrínseca)

$$\vec{F} = kq_2\left[\frac{q_1}{r_{12}^2}\hat{r}_{12} + \frac{q_3}{r_{13}^2}\hat{r}_{13} + \dots\right] = kq_2\sum_i \frac{q_i}{r_{i2}^2}\hat{r}_{i2}$$

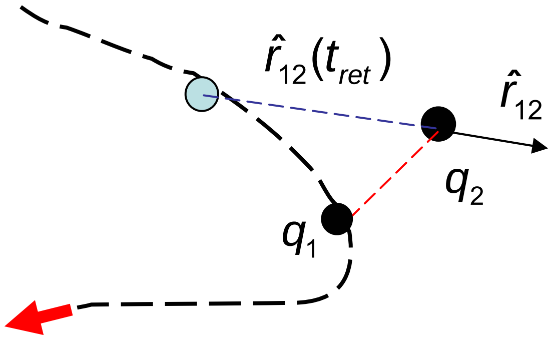
distribución de carga





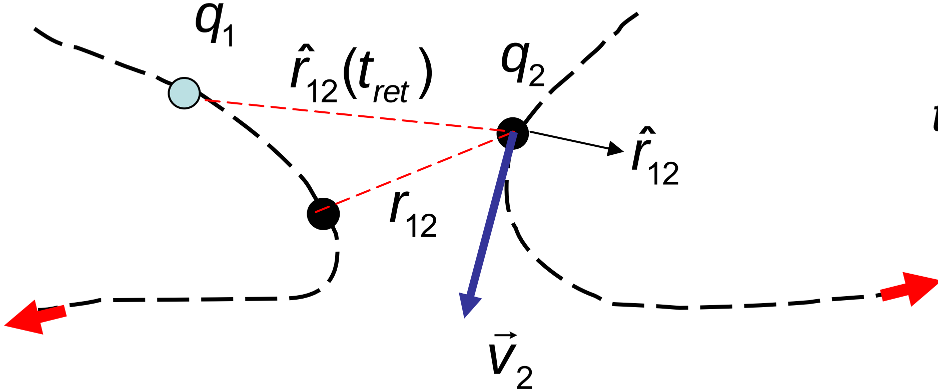
$$\vec{F} = kq_2\int_V \frac{\rho(\vec{r}\,')}{|\vec{r}\,' - \vec{r}_2|}\widehat{(\vec{r}\,' - \vec{r}\,)}$$

leyes de la electrodinámica



$$\vec{F} = kq_1q_2\left\{\left[\frac{\hat{r}_{12}}{\kappa r_{12}^2}\right]_{ret} + \frac{\partial}{c\,\partial t}\left[\frac{\hat{r}_{12}}{\kappa r_{12}}\right]_{ret} - \frac{\partial}{c^2\partial t}\left[\frac{\vec{v}_{12}}{\kappa r_{12}}\right]_{ret}\right\}$$

$$\kappa = 1 - \frac{\vec{v}_{12}\cdot\hat{r}_{12}}{c}$$



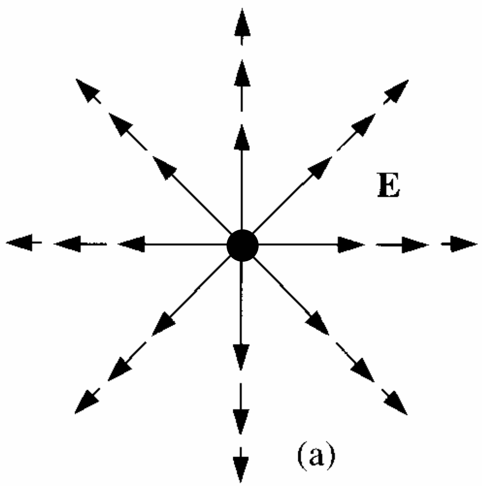
$$t_{ret} = t - \frac{\hat{r}_{12}(t_{ret})}{c}$$

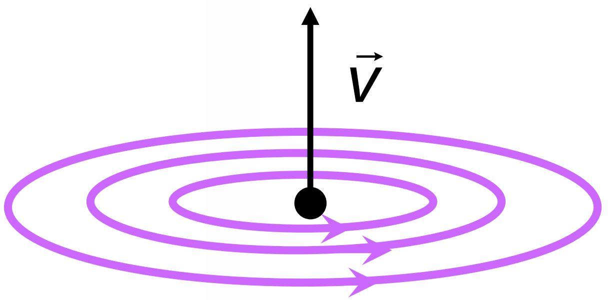
$$\vec{F} = kq_1q_2\left\{\left[\frac{\hat{r}_{12}}{\kappa r_{12}^2}\right]_{ret} + \frac{\partial}{c\,\partial t}\left[\frac{\hat{r}_{12}}{\kappa r_{12}}\right]_{ret} - \frac{\partial}{c^2\partial t}\left[\frac{\vec{v}_{12}}{\kappa r_{12}}\right]_{ret} + \frac{1}{c^2}\vec{v}_2\times\left(\left[\frac{\vec{v}_{12}\times\hat{r}_{12}}{\kappa r_{12}^2}\right]_{ret} + \frac{\partial}{c\,\partial t}\left[\frac{\vec{v}_{12}\times\hat{r}_{12}}{\kappa r_{12}}\right]\right)\right\}$$

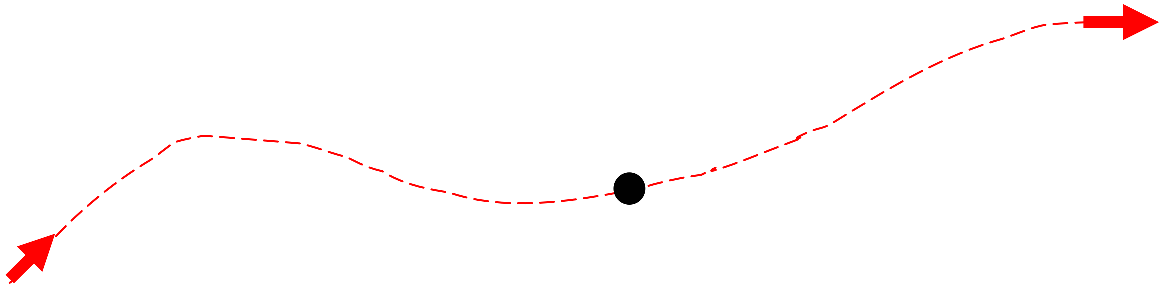
Teoría de campo

$$\vec{E}(\vec{r},t)$$

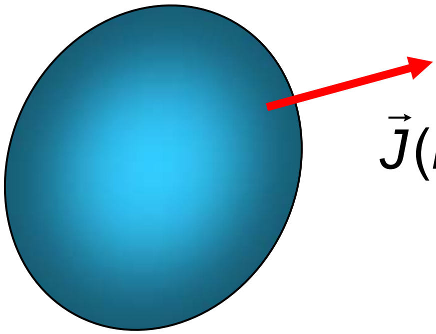
$$B(\vec{r},t)$$







Fuentes

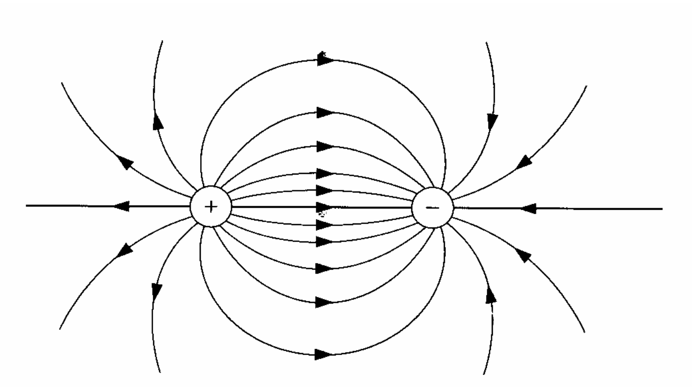


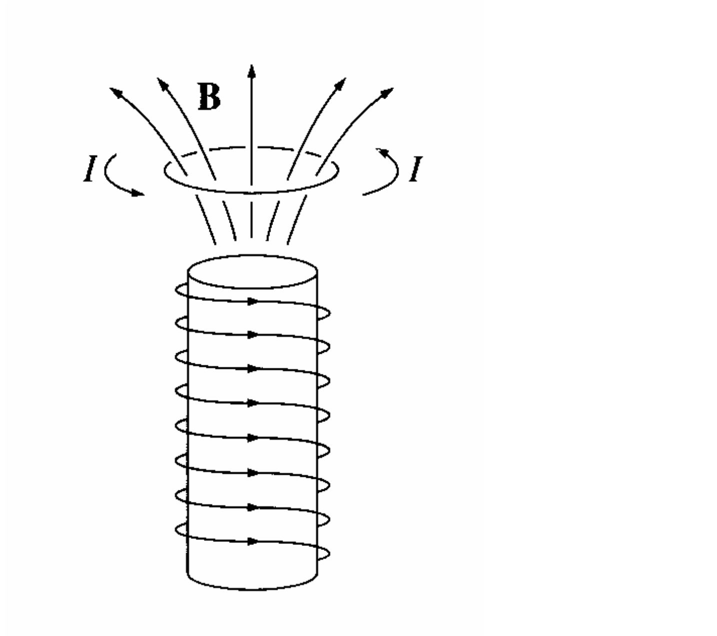
$\vec{J}(\vec{r},t)$

$\rho(\vec{r},t)$

conservación de la carga

$$\nabla \cdot \vec{J}(\vec{r},t) + \frac{\partial \rho(\vec{r},t)}{\partial t} = 0$$





$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

COULOMB

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

BIOT-SAVART
MAXWELL

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

FARADAY

$$\nabla \cdot \vec{B} = 0$$

NO HAY MONOPOLOS

$$\vec{F} = q\left(\vec{E} + v \times \vec{B}\right)$$

LORENTZ

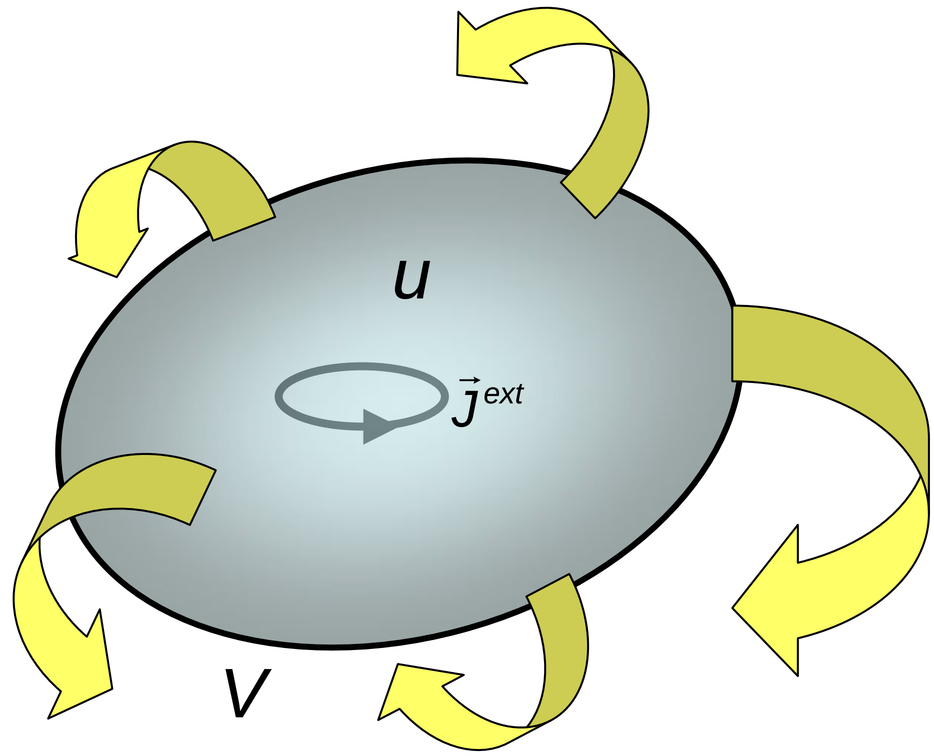
UNIDADES: SI

La energía

$$\nabla\cdot\underbrace{\left(\vec{E}\times\frac{\vec{B}}{\mu_0}\right)}_{\vec{S}}+\frac{\partial}{\partial t}\frac{1}{2}\underbrace{\left(\varepsilon_0 E^2+\frac{B^2}{\mu_0}\right)}_{u}=-\vec{E}\cdot\vec{J}^{ext}$$

$$\nabla\cdot\vec{S}+\frac{\partial u}{\partial t}=-\vec{E}\cdot\vec{J}^{ext}$$

$$\oint\vec{S}\cdot d\vec{a}=-\frac{dU}{dt}-W^{ext}$$

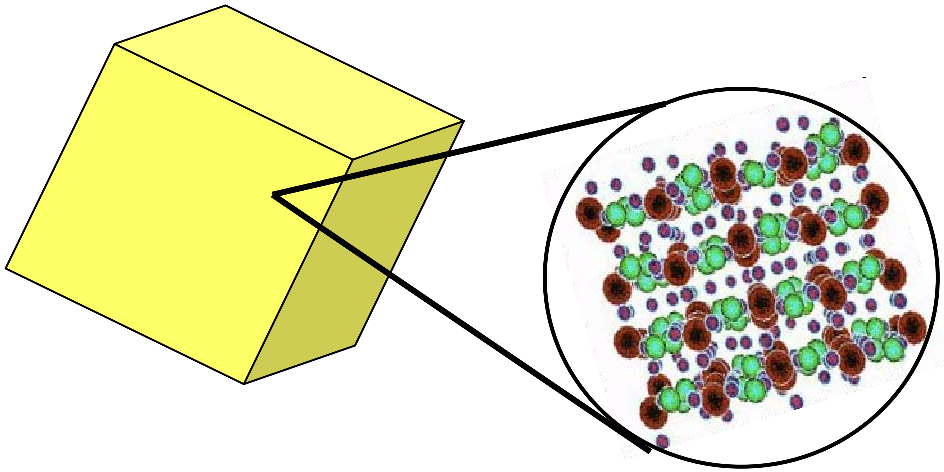


$$\vec{S}=\vec{E}\times\frac{\bar{B}}{\mu_0}$$

Materiales

visión macroscópica

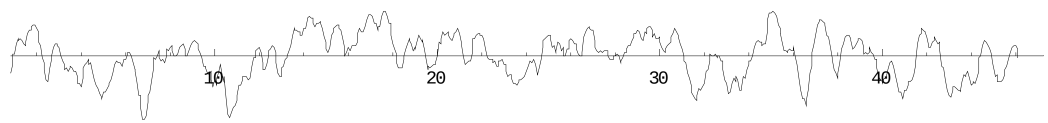
visión molecular



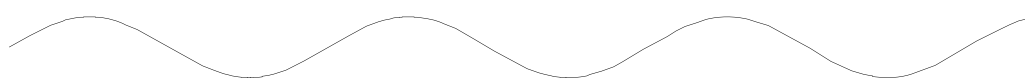
$$\rho(\vec{r},t)$$

$$\vec{J}(\vec{r},t)$$

enfoque macroscópico



promedio

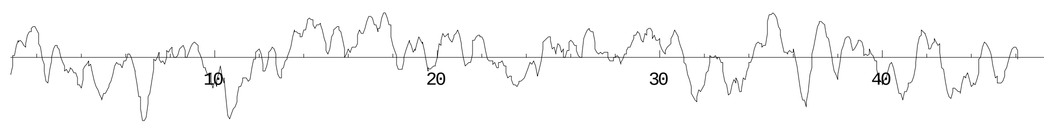


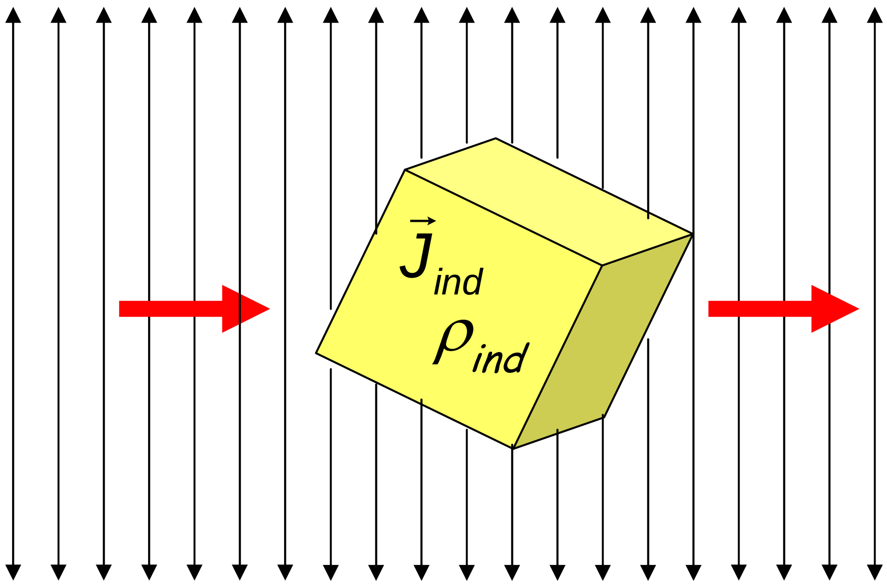
"suavizar"

NEUTRA

$$\left\langle \rho(\vec{r},t) \right\rangle = 0$$

$$\left\langle \vec{J}(\vec{r},t) \right\rangle = 0$$



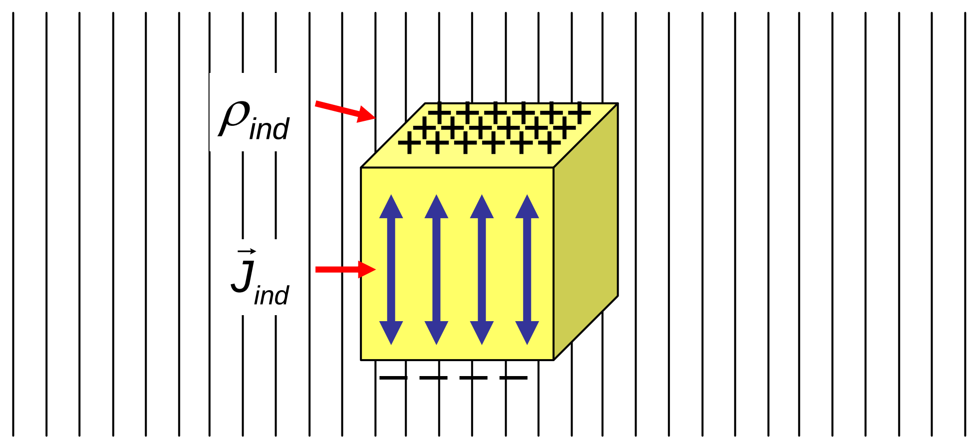


promedio

$$\rho_{ind} \rightarrow \langle \rho_{ind} \rangle \neq 0$$

$$\vec{J}_{ind} \rightarrow \langle \vec{J}_{ind} \rangle \neq 0$$

$$\nabla \cdot \langle \vec{J}_{ind} \rangle + \frac{\partial \langle \rho_{ind} \rangle}{\partial t} = 0$$



homogeneo

MODELO

(origen del magnetismo)

4 $\langle \vec{J}_{ind} \rangle$ $\langle \rho_{ind} \rangle$ $\rightarrow$ $\vec{P}$ Polarización, $\vec{M}$ Magnetización 6

definición

$$\boxed{\langle \rho_{ind} \rangle = -\nabla \cdot \vec{P}}$$

$\vec{P} = 0$ *fuera*

$$\nabla \cdot \langle \vec{J}_{ind} \rangle + \frac{\partial \langle \rho_{ind} \rangle}{\partial t} = 0$$

$$\nabla \cdot \underbrace{\left(\langle \vec{J}_{ind} \rangle - \frac{\partial \vec{P}}{\partial t} \right)}_{\nabla \times \vec{M}} = 0$$

$\vec{M} = 0$ *fuera*

$$\boxed{\langle \vec{J}_{ind} \rangle = \underbrace{\frac{\partial \vec{P}}{\partial t}}_{\vec{J}_P} + \underbrace{\nabla \times \vec{M}}_{\vec{J}_M}}$$

$\vec{J}_P$ "abiertas"

$\vec{J}_M$ "cerradas"

Ambigüedad

$$\langle \rho_{ind} \rangle = -\nabla \cdot \vec{P} \qquad \langle \vec{J}_{ind} \rangle = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

¿ significado físico ?

cambio de norma

$$\vec{P} \rightarrow \vec{P} + \nabla \times \Lambda$$

$$\vec{M} \rightarrow \vec{M} - \frac{\partial \Lambda}{\partial t}$$

$$\langle \vec{J}_{ind} \rangle \rightarrow \frac{\partial \vec{P}}{\partial t} + \cancel{\frac{\partial}{\partial t} \nabla \times \vec{\Lambda}} + \nabla \times \vec{M} - \cancel{\nabla \times \frac{\partial \vec{\Lambda}}{\partial t}} = \langle \vec{J}_{ind} \rangle$$

¿ significado físico ?

Campo "inducido"

"solución" tradicional

$$\langle \rho_{ind} \rangle (\vec{r},t) = \langle \rho_{ind} \rangle (\vec{r}) e^{-i\omega t}$$

$$\langle \vec{J}_{ind} \rangle (\vec{r},t) = \langle \vec{J}_{ind} \rangle (\vec{r}) e^{-i\omega t}$$

con retardamiento

$$\phi_{ind}(\vec{r},t) = \frac{1}{4\pi\varepsilon_0} \int \langle \rho_{ind} \rangle (\vec{r}\,') \frac{e^{ik|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} d^3r'$$

$$\vec{A}_{ind}(\vec{r},t) = \frac{\mu_0}{4\pi} \int \langle \vec{J}_{ind} \rangle (\vec{r}\,') \frac{e^{ik|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} d^3r'$$

Los campos

$$\vec{E}_{ind} = -\nabla \phi_{ind} + \frac{\partial \vec{A}_{ind}}{\partial t}$$

$$\vec{B}_{ind} = \nabla \times \vec{A}_{ind}$$

sustituimos:

$$\langle \rho_{ind} \rangle = -\nabla \cdot \vec{P}$$

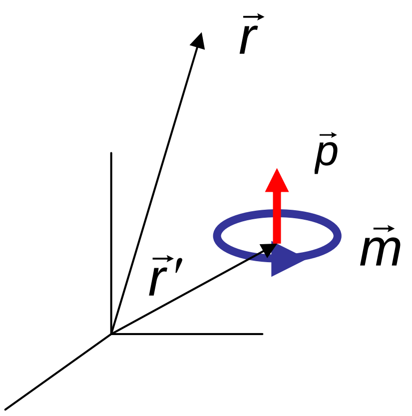
$$\langle \vec{J}_{ind} \rangle = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

obtenemos:

$$\phi_{ind}(\vec{r},t) = -\frac{1}{4\pi\varepsilon_0}\int \vec{P}(\vec{r}\,')\cdot\nabla\frac{e^{ikx}}{x}d^3r'$$

$$\vec{x} = \vec{r} - \vec{r}\,'$$

$$\vec{A}_{ind}(\vec{r},t) = \frac{\mu_0}{4\pi}\int\left[(-i\omega)\vec{P}(\vec{r}\,') - ik\,\hat{x}\times\vec{M}(\vec{r}\,')\left(1-\frac{1}{ikx}\right)\right]\frac{e^{ikx}}{x}d^3r'$$



comparamos:

$\vec{p}:$ $$\phi_{ind}(\vec{r},t) = -\frac{1}{4\pi\varepsilon_0}\vec{p}\cdot\nabla\frac{e^{ikx}}{x} \qquad \vec{A}_{ind} = -\frac{\mu_0}{4\pi}i\omega\vec{p}\frac{e^{ikx}}{x}$$

$\vec{m}:$ $$\vec{A}_{ind}(\vec{r},t) = -\frac{\mu_0}{4\pi}ik\hat{x}\times\vec{m}(\vec{r}\,')\frac{e^{ikx}}{x}\left(1-\frac{1}{ikx}\right) \qquad \phi_{ind} = 0$$

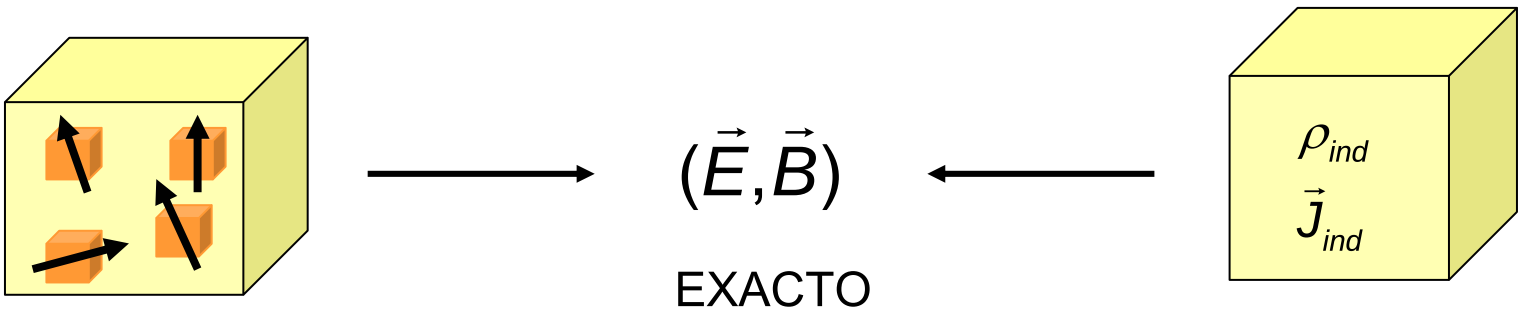
identificamos:

$$\vec{P}\,d^3r \to \langle\vec{p}\rangle \qquad \vec{M}\,d^3r \to \langle\vec{m}\rangle$$

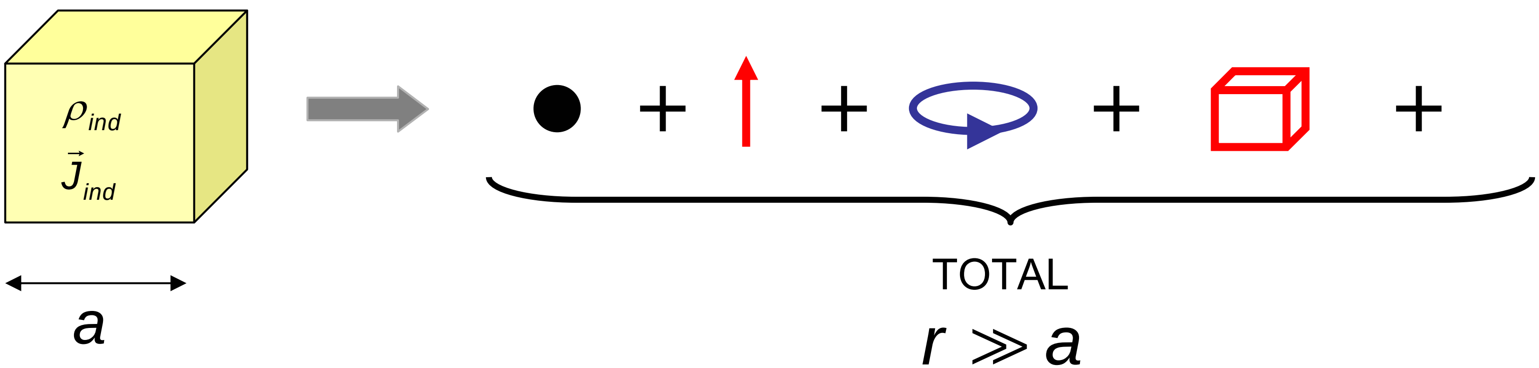
densidad de momento dipolar promedio

Interpretación

NO es un desarrollo multipolar



desarrollo multipolar



con retardamiento $r \gg \lambda \gg a$

Landau & Lifshitz

Electrodynamics of continuous media   p.252

field by equation (56.7):

$$\mathbf{curl\,B} = \frac{4\pi}{c}\overline{\rho\mathbf{v}} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}. \tag{60.2}$$

Subtracting the equation $\mathbf{curl\,H} = (1/c)\partial\mathbf{D}/\partial t$, we obtain

$$\overline{\rho\mathbf{v}} = c\,\mathbf{curl\,M} + \partial\mathbf{P}/\partial t. \tag{60.3}$$

The integral (60.1) can, as shown in §27, be put in the form $\int\mathbf{M}\,\mathrm{d}V$ only if $\overline{\rho\mathbf{v}} = c\,\mathbf{curl\,M}$ and $\mathbf{M} = 0$ outside the body.

Thus the physical meaning of **M**, and therefore of the magnetic susceptibility, depends on the possibility of neglecting the term $\partial\mathbf{P}/\partial t$ in (60.3). Let us see to what extent the conditions can be fulfilled which make this neglect permissible.

For a given frequency, the most favourable conditions for measuring the

Dudas…

¿cómo se calculan?

$$\vec{P} = \vec{r}'\langle \rho_{ind} \rangle(\vec{r}') \qquad \vec{M} = \frac{1}{2}\vec{r}' \times \langle \overleftarrow{J}_{ind} \rangle(\vec{r}')$$

Por ejemplo, si cambio de origen

$$\vec{r}' \to \vec{r}' - \vec{c} \qquad \vec{r}'\langle \rho_{ind} \rangle(\vec{r}') \to \vec{r}'\langle \rho_{ind} \rangle(\vec{r}') - \vec{c}\,\underline{\langle \rho_{ind} \rangle(\vec{r}')}$$

$$\vec{p}_{TOTAL} \to \vec{p}_{TOTAL} - \vec{c}\underbrace{\int \langle \rho_{ind} \rangle(\vec{r}')d^3r'}_{=0\ \text{(neutra)}} \qquad \phi_{ind} \to \phi_{ind} - \frac{1}{4\pi\varepsilon_0}\int \rho(\vec{r}')\vec{c}\cdot\nabla\frac{e^{ikx}}{x}d^3r'$$

Peor aún…

$$\nabla'\cdot\vec{P} = \nabla\cdot\left(\vec{r}'\langle \rho_{ind} \rangle\right) = 3\langle \rho_{ind} \rangle + \nabla\langle \rho_{ind} \rangle\cdot\vec{r}' \neq \langle \rho_{ind} \rangle$$

¿cómo se calculan?

…de todas maneras

$$\vec{P} \to \vec{P} + \nabla\times\Lambda$$

$$\vec{M} \to \vec{M} - \frac{\partial\Lambda}{\partial t}$$

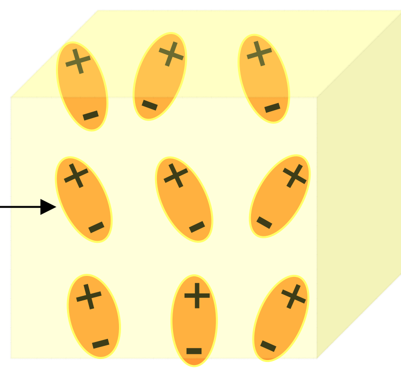
Densidad de momento dipolar

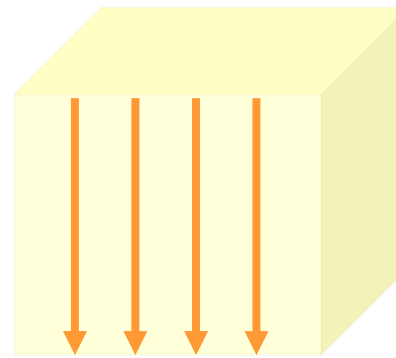
materiales moleculares

"homogeneo" $\langle \vec{p} \rangle = \frac{1}{N} \sum_i \vec{p}_i$

$$P = \frac{N}{V} \langle \vec{p} \rangle$$

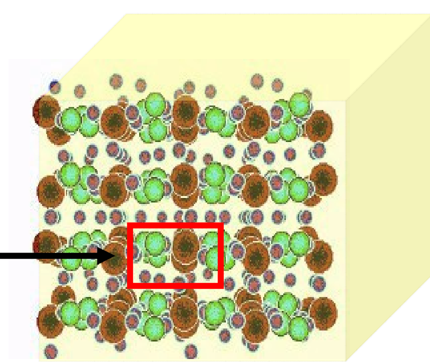
$\vec{p}_i$

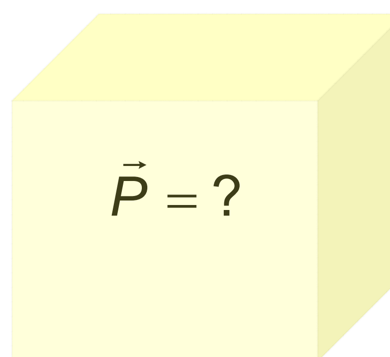




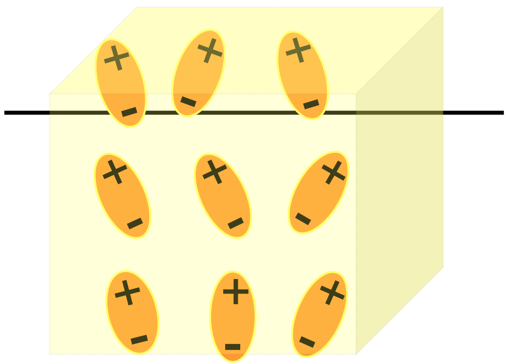
estado sólido

$\langle \vec{p} \rangle$

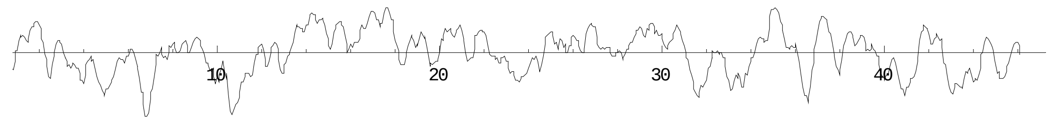




Campo eléctrico

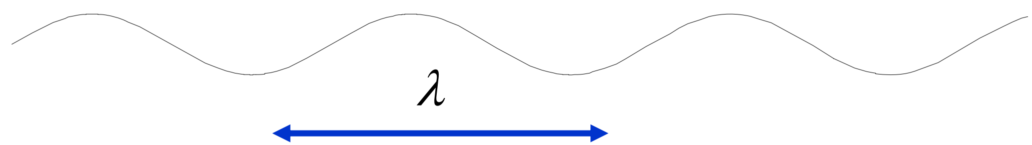


$$\hat{P}_a\vec{E} = \left\langle \vec{E} \right\rangle$$



$$\vec{E} = \left\langle \vec{E} \right\rangle + \delta\vec{E}$$

promedio



macroscopico… coherente

$$\vec{E} = \left\langle \vec{E} \right\rangle + \cancel{\delta\vec{E}}$$

promedio fluctuaciones

$$\left\langle \delta\vec{E} \right\rangle = 0$$

$$\hat{P}_a^{\,2}\vec{E} = \left\langle \vec{E} \right\rangle \qquad \hat{P}_a^{\,2} = \hat{P}_a$$

ondas planas

$$\vec{S} = \frac{1}{2}\mathrm{Re}\left(\vec{E}\times\vec{H}^{\,*}\right) = \frac{1}{2}\mathrm{Re}\left[\left(\left\langle\vec{E}\right\rangle + \delta\vec{E}\right)\times\left(\left\langle\vec{H}\right\rangle + \delta\vec{H}\right)^{*}\right] = \left\langle\vec{S}\right\rangle + \cancel{\delta\vec{S}}$$

no es suficiente

$$\left\langle\vec{S}\right\rangle = \underbrace{\left\langle\vec{E}\right\rangle\times\left\langle\vec{H}\right\rangle}_{\left\langle\vec{S}\right\rangle_{coh}} + \underbrace{\left\langle\delta\vec{E}\times\delta\vec{H}\right\rangle}_{\left\langle\vec{S}\right\rangle_{diffuse}}$$

$$\left\langle\vec{S}\right\rangle = \left\langle\vec{E}\right\rangle\times\left\langle\vec{H}\right\rangle$$

$$\left\langle\vec{S}\right\rangle_{coh} \gg \left\langle\vec{S}\right\rangle_{diffuse}$$

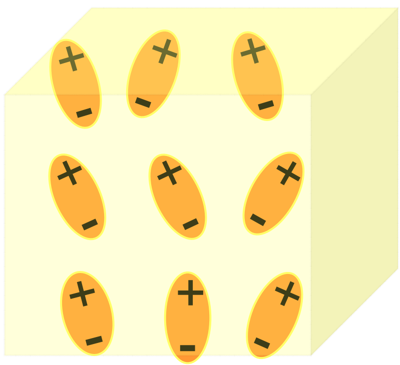
Guerra de las galaxias

materiales lineales

"homogeneo"

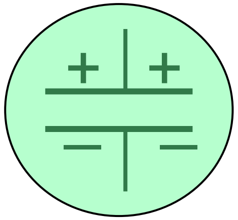
interpretación física ⟷ **MEDICION**

rompe la ambigúedad

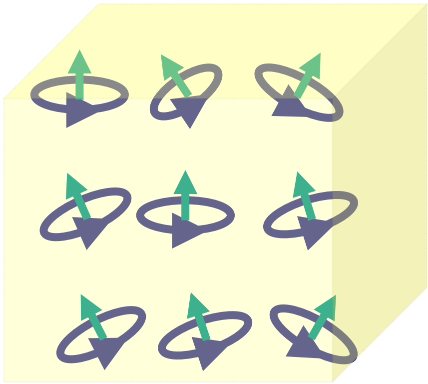


$$\vec{P} = \varepsilon_0 \chi_E \left\langle \vec{E} \right\rangle \qquad \left| \left\langle \vec{E} \right\rangle \right| \ll \left| \vec{E}_{mol} \right|$$

MEDICION $\nabla \cdot \vec{P} \rightarrow \vec{P} \cdot \hat{n} = -\sigma_{ind}$



C

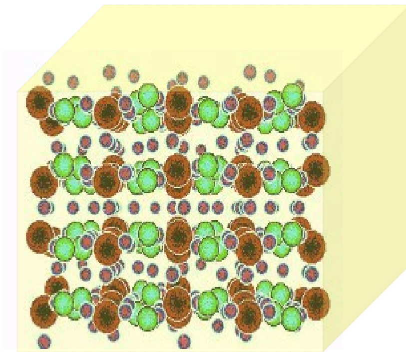


$$\vec{M} = \chi_B \left\langle \vec{B} \right\rangle \qquad \left| \left\langle \vec{B} \right\rangle \right| \ll \left| \vec{B}_{mol} \right|$$

nótese $\nabla \cdot \left\langle \vec{J}_{ind} \right\rangle + \dfrac{\partial \left\langle \rho_{ind} \right\rangle}{\partial t} \neq 0$

corrientes moleculares

origen "físico" del magnetismo



$$\left\langle \vec{J}_{ind} \right\rangle$$

LEY DE OHM GENERALIZADA

$$\left\langle \vec{J}_{ind} \right\rangle = \hat{\Sigma} \left\langle \vec{E} \right\rangle$$

TOTAL

$$\left\langle \vec{J}_{ind} \right\rangle = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$$

$$\vec{P} = \varepsilon_0 \hat{\chi}_E \left\langle \vec{E} \right\rangle \qquad \vec{M} = \frac{\hat{\chi}_B}{\mu_0} \left\langle \vec{B} \right\rangle$$

Espacio ω $\quad \sim e^{-i\omega t}$

$$\left\langle \vec{J}_{ind} \right\rangle = -i\omega \vec{P} + \nabla \times \vec{M} = -i\omega \varepsilon_0 \hat{\chi}_E \left\langle \vec{E} \right\rangle + \nabla \times \frac{\hat{\chi}_B}{\mu_0} \left\langle \vec{B} \right\rangle$$

$$\nabla \times \left\langle \vec{E} \right\rangle = i\omega \left\langle \vec{B} \right\rangle$$

$$\left\langle \vec{J}_{ind} \right\rangle = \underbrace{\left[-i\omega \varepsilon_0 \hat{\chi}_E + \frac{1}{i\omega} \nabla \times \frac{\hat{\chi}_B}{\mu_0} \nabla \times \right]}_{\hat{\Sigma}} \left\langle \vec{E} \right\rangle$$

La tradición… esquema $(\varepsilon\,\mu)$

El "eter"

$$\vec{D} = \varepsilon_0 \left\langle \vec{E} \right\rangle + \underbrace{\vec{P}}_{\varepsilon_0 \hat{\chi}_E \left\langle \vec{E} \right\rangle} \qquad \vec{H} = \frac{\left\langle \vec{B} \right\rangle}{\mu_0} - \underbrace{\vec{M}}_{\frac{\hat{\chi}_B}{\mu_0}\left\langle \vec{B} \right\rangle}$$

$$\vec{D} = \varepsilon_0 \left(1 + \hat{\chi}_E\right)\left\langle \vec{E} \right\rangle = \hat{\varepsilon}\left\langle \vec{E} \right\rangle \qquad \vec{H} = \frac{1}{\mu_0}\left(1 - \hat{\chi}_B\right)\left\langle \vec{B} \right\rangle = \hat{\mu}^{-1}\left\langle \vec{B} \right\rangle$$

$$\hat{\varepsilon} = \varepsilon_0 \left(1 + \hat{\chi}_E\right) \qquad \hat{\chi}_E = \frac{1}{\varepsilon_0}\hat{\varepsilon} - 1$$

$$\hat{\mu}^{-1} = \frac{1}{\mu_0}\left(1 - \hat{\chi}_B\right) \qquad \hat{\chi}_B = 1 - \mu_0 \hat{\mu}^{-1}$$

$$\hat{\Sigma} = -i\omega\left(\hat{\varepsilon} - \varepsilon_0\right) - \frac{1}{i\omega}\nabla \times \hat{\mu}^{-1}\nabla \times$$

Espacio de frecuencias $\sim e^{-i\omega t}$ $\vec{D} = \hat{\varepsilon}\langle \vec{E} \rangle$

homogeneo e isótropo

Respuesta no-local

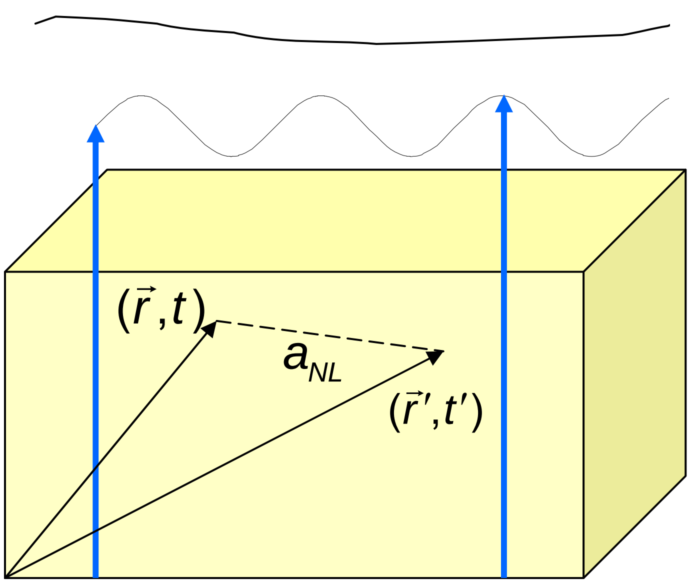
dispersión temporal

$$\vec{D}(\vec{r},\omega) = \int \varepsilon(|\vec{r}-\vec{r}'|;\omega)\langle \vec{E} \rangle(\vec{r}',\omega)d^3r'$$

$$\vec{H}(\vec{r},\omega) = \int d^3r'\mu^{-1}(|\vec{r}-\vec{r}'|;\omega)\langle \vec{B} \rangle(\vec{r}',\omega)$$

permeabilidad magnética

permitividad eléctrica



Respuesta local

$$\vec{D}(\vec{r},\omega) = \underbrace{\left[\int \varepsilon(|\vec{r}-\vec{r}'|;\omega)d^3r'\right]}_{\varepsilon(\omega)}\langle \vec{E} \rangle(\vec{r},\omega)$$

$$\vec{D}(\vec{r},\omega) = \varepsilon(\omega)\langle \vec{E} \rangle(\vec{r},\omega)$$

Dispersión espacial $(\varepsilon\,\mu)$ ó $(L\,T)$

Invariancia translacional … material sin fronteras

$$\vec{D}(\vec{r},\omega)=\int \varepsilon(|\vec{r}-\vec{r}'|;\omega)\left\langle \vec{E}\right\rangle(\vec{r}',\omega)d^3r' \qquad \vec{H}(\vec{r},\omega)=\int d^3r'\mu^{-1}(|\vec{r}-\vec{r}'|;\omega)\left\langle \vec{B}\right\rangle(\vec{r}',\omega)$$

Espacio $(\vec{k},\omega)$

$$\vec{D}(\vec{k},\omega)=\varepsilon(k,\omega)\left\langle \vec{E}\right\rangle(\vec{k},\omega) \qquad \vec{H}(\vec{k},\omega)=\frac{1}{\mu(k,\omega)}\left\langle \vec{B}\right\rangle(\vec{k},\omega)$$

dispersión espacial

$$\hat{\Sigma}=-i\omega\left(\hat{\varepsilon}-\varepsilon_0\right)-\frac{1}{i\omega}\nabla\times\hat{\mu}^{-1}\nabla\times$$

$$\overline{\overline{\Sigma}}(\vec{k},\omega)=-i\omega\left(\varepsilon(k,\omega)-\varepsilon_0\right)\overline{\overline{1}}+\frac{1}{i\omega\,\mu(k,\omega)}\vec{k}\times\vec{k}\times$$

$$\overline{\overline{1}}=\hat{k}\hat{k}-\hat{k}\times\hat{k}\times=\hat{P}^L+\hat{P}^T$$

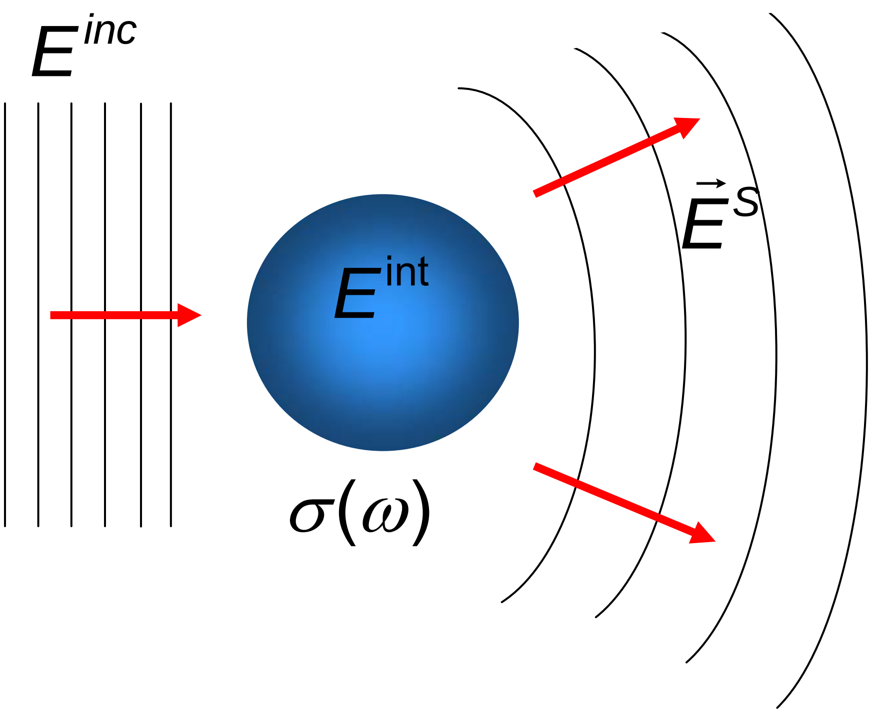
$$\overline{\overline{\Sigma}}(\vec{k},\omega)=\underbrace{-i\omega\left(\varepsilon(k,\omega)-\varepsilon_0\right)}_{\Sigma^L(k,\omega)}\hat{k}\hat{k}+\underbrace{(-i\omega)\left(\frac{k^2}{\omega^2}\frac{1}{\mu(k,\omega)}-\left(\varepsilon(k,\omega)-\varepsilon_0\right)\right)}_{\Sigma^T(k,\omega)}\hat{k}\times\hat{k}\times$$

Límite local

Límite "local"

$$\varepsilon(\omega) = \varepsilon(k \to 0, \omega) \qquad \mu(\omega) = \mu(k \to 0, \omega)$$

local o no-local



BC

$$\vec{E}^{out} = \vec{E}^{inc} + \vec{E}^{S} \longleftrightarrow E^{int}$$

$$\vec{J}_{ind}(\vec{r};\omega) = \underline{\sigma(\omega)}\vec{E}^{int}(\vec{r};\omega) \qquad \text{local}$$

$$= \int_{V_S} \bar{\bar{\sigma}}_{NL}(\vec{r},\vec{r}';\omega) \cdot \underline{\vec{E}^{inc}}(\vec{r}';\omega) d^3r'$$

región de no-localidad

$$a_{NL} \sim a$$

En el óptico…

materiales comunes

indice de refracción

$$a_{NL} \sim a_{mol}$$

$$a_{NL} \sim \frac{v_F}{c}\lambda$$

RESPUESTA “LOCAL”

$$\varepsilon(\omega)$$

$$\mu(\omega)$$

$$n(\omega) = \sqrt{\varepsilon(\omega)\mu(\omega) / \varepsilon_0\mu_0}$$

$$\varepsilon(\omega) = \varepsilon'(\omega) + \underline{i\,\varepsilon''(\omega)}$$

disipación

$$\mu(\omega) \approx \mu_0$$

no hay magnetismo óptico

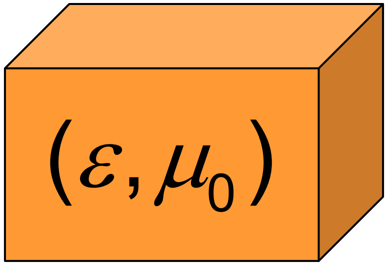
Electrodinámica de medios Continuos. Landau & Lifshitz Parágrafo 60

Así pues, es evidente que carece de sentido utilizar la permeabilidad magnética no bien se alcanza el dominio de las frecuencias ópticas, y al considerar los correspondientes fenómenos es necesario hacer $\mu = 1$. Distinguir entre **B** y **H** en dicho dominio equivaldría a excederse en la precisión aceptable. Es más, de hecho, tener en cuenta la diferencia entre μ y la unidad equivale a un exceso de precisión para la mayoría de los fenómenos incluso para frecuencias mucho más bajas que las ópticas.

indice de refracción

$$n(\omega) = \sqrt{\varepsilon(\omega)/\varepsilon_0}$$

$$= n'(\omega) + i\,n''(\omega)$$



“continuo”

PHYSICAL REVIEW B VOLUME 32, NUMBER 8 15 OCTOBER 1985

Electromagnetic response of systems with spatial fluctuations. I. General formalism

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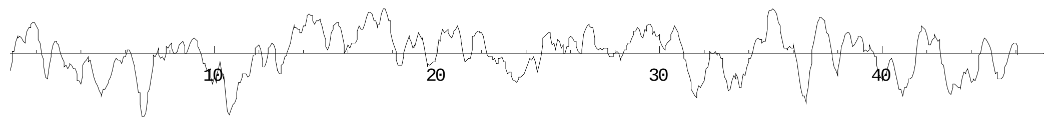
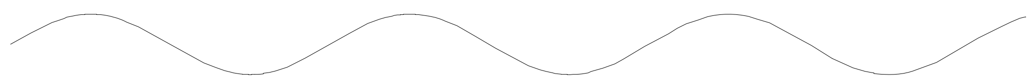
(Received 3 July 1984; revised manuscript received 8 February 1985)

PROMEDIOS

$$F$$





$$\hat{P}_a F = F_a$$

$$\hat{P}_f F = (\hat{1} - \hat{P}_a)F = \delta F \qquad \hat{P}_a + \hat{P}_f = \hat{1} \qquad F = F_a + \delta F$$

idempotencia

$$\hat{P}_a^2 = \hat{P}_a \qquad \hat{P}_f^2 = \hat{P}_f \qquad \hat{P}_a \hat{P}_f = 0 \qquad \hat{P}_f \hat{P}_a = 0$$

operadores de proyección

$$F \to \begin{pmatrix} F_a \\ \delta F \end{pmatrix} \qquad \left[\hat{P}_a, \partial_t\right] = \left[\hat{P}_a, \nabla\right] = 0$$

esfera

Promedio espacial

$$F_a(\mathbf{r}) = \int d^3r'\, P_a(\mathbf{r}-\mathbf{r}')F(\mathbf{r}')$$

$$P_a = \begin{cases} \dfrac{3}{4\pi R^3}, & r < R \\ 0, & r > R \end{cases}$$

“suave”

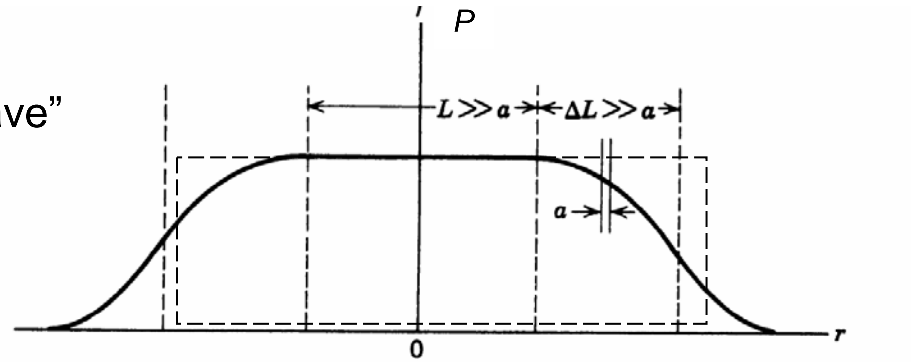
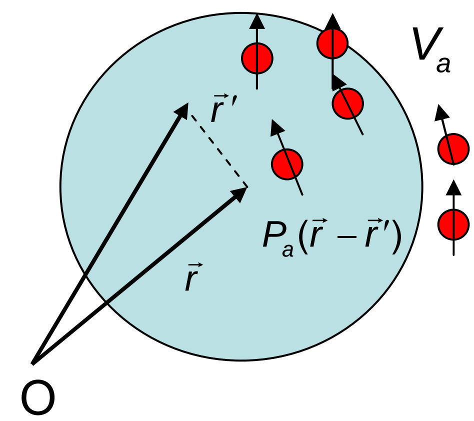


Figure 6.1 Schematic diagram of test function $f(\mathbf{x})$ used in the spatial averaging



$\lambda \gg a$ Independiente de R

$L = 2R \sim \lambda$

Idem-potente

$$\int d^3r'\, P_a(\mathbf{r}-\mathbf{r}')P_a(\mathbf{r}') = P_a(\mathbf{r})$$

NO se satisface con P_a positiva definida

Truncamiento

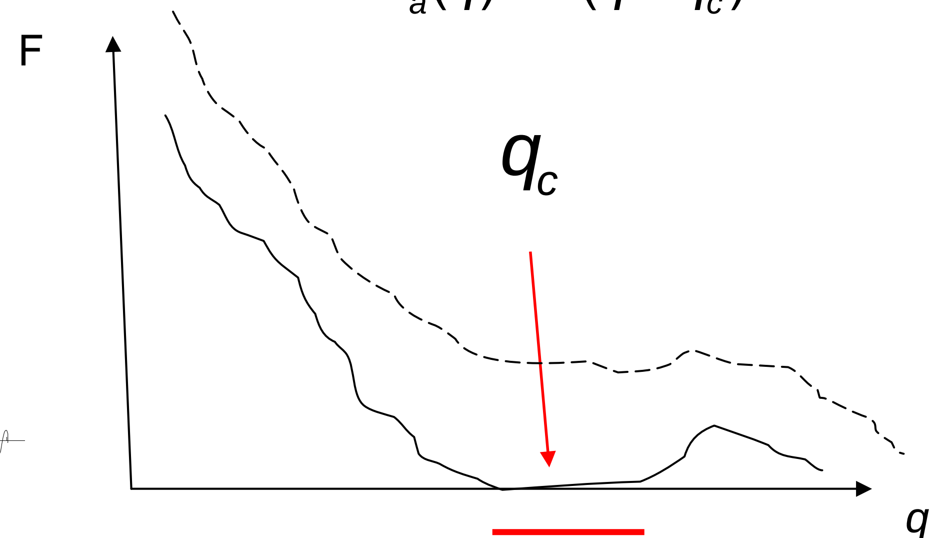
Truncamiento en el espacio q

$$\boldsymbol{F}(\mathbf{q}) \equiv \boldsymbol{P_a}(\mathbf{q})\boldsymbol{F}(\mathbf{q})$$

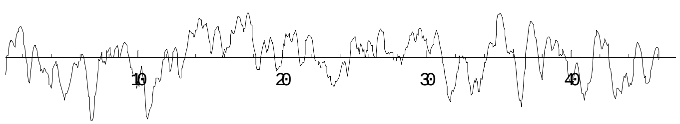
$$P_a^2(\vec{q}) = P_a(\vec{q})$$

i.e.

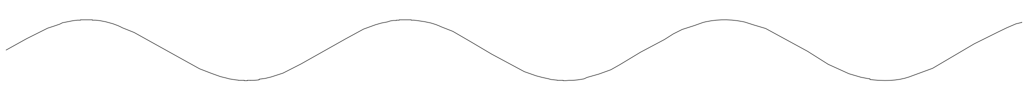
$$P_a(q) = \theta(q - q_c)$$



$$F(\vec{r}\,) \rightarrow$$



$$\langle F \rangle(\vec{r}\,) = \int F(\vec{q}\,)\theta(q - q_c)\exp[i\vec{q}\cdot\vec{r}\,]\frac{d^3q}{(2\pi)^3}$$



$$1/q_c$$

Truncamiento y promedio espacial

$$\langle F\rangle(\vec{r}) = \int F(\vec{q})\,\theta(q-q_c)\exp[i\vec{q}\cdot\vec{r}]\frac{d^3q}{(2\pi)^3}$$

$$F(q) = \int F(\vec{r}\,')\exp[i\vec{q}\cdot\vec{r}\,']d^3r'$$

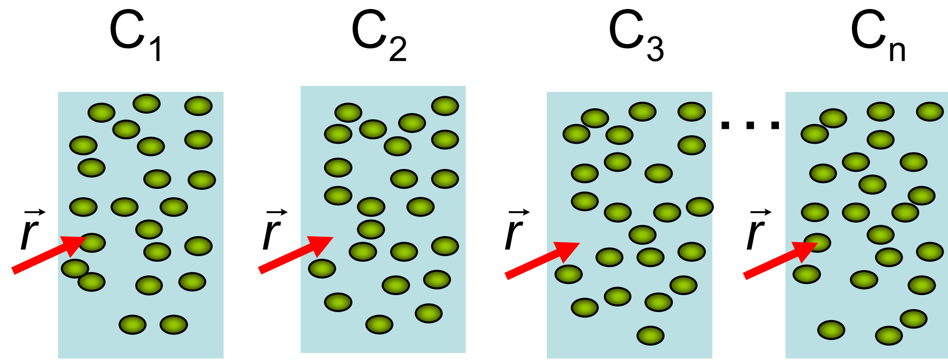
$$\langle F\rangle(\vec{r}) = \int d^3r' F(\vec{r}\,')\left[\int \theta(q-q_c)\exp[i\vec{q}\cdot(\vec{r}-\vec{r}\,')]\frac{d^3q}{(2\pi)^3}\right]$$

$$P_a(|\vec{r}-\vec{r}\,'|) = \frac{q_c^3}{2\pi^2}\frac{j_1(q_c|\vec{r}-\vec{r}\,'|)}{q_c|\vec{r}-\vec{r}\,'|}$$

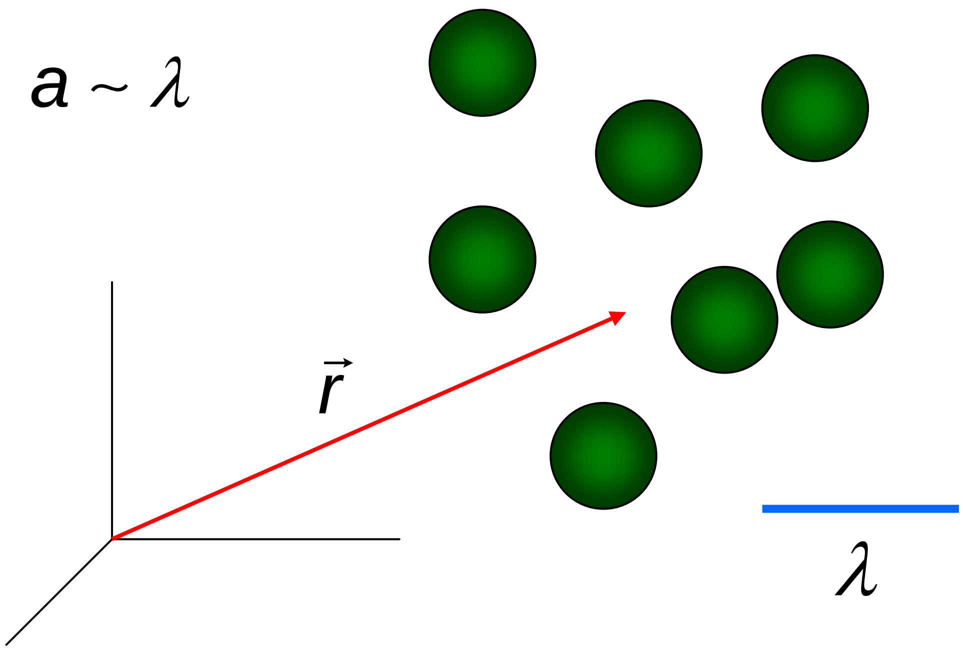
Ensamble

Promedio de ensamble (configuración)

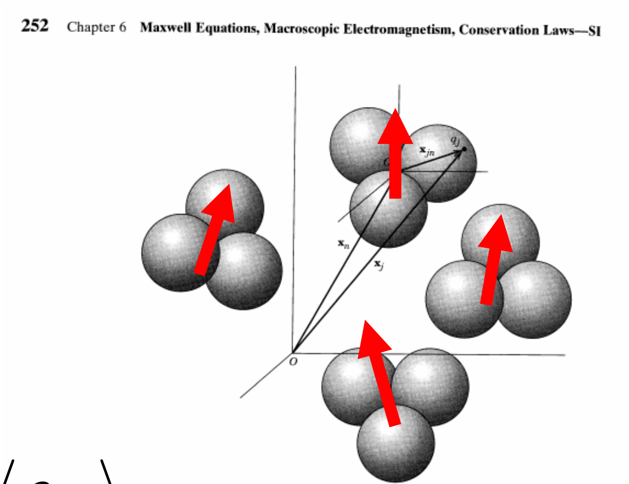
$$F_a(\lambda) = \sum_c P_c F_c(\lambda)$$



$a \sim \lambda$



micro → macro

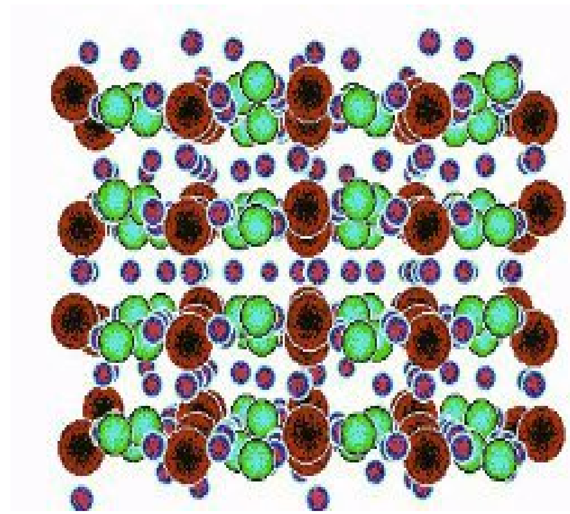


$$\nabla \cdot \langle \vec{E} \rangle = \langle \rho_{ind} \rangle$$

$$\langle \rho_{ind} \rangle = \langle q_n \delta(\mathbf{x} - \mathbf{x}_n) \rangle - \nabla \cdot \langle \mathbf{p}_n \delta(\mathbf{x} - \mathbf{x}_n) \rangle + \frac{1}{6} \sum_{\alpha\beta} \frac{\partial^2}{\partial x_\alpha \, \partial x_\beta} \langle (Q_n')_{\alpha\beta} \delta(\mathbf{x} - \mathbf{x}_n) \rangle + \cdots$$

$$D_\alpha = \epsilon_0 E_\alpha + P_\alpha - \sum_\beta \frac{\partial Q'_{\alpha\beta}}{\partial x_\beta} + \cdots$$

Pero… se definió… $\langle \rho_{ind} \rangle = -\nabla \cdot \vec{P}$



FORMULA DE KUBO

$$\langle \vec{J}_{ind} \rangle = \hat{\Sigma} \langle \vec{E} \rangle$$

(no magnético)

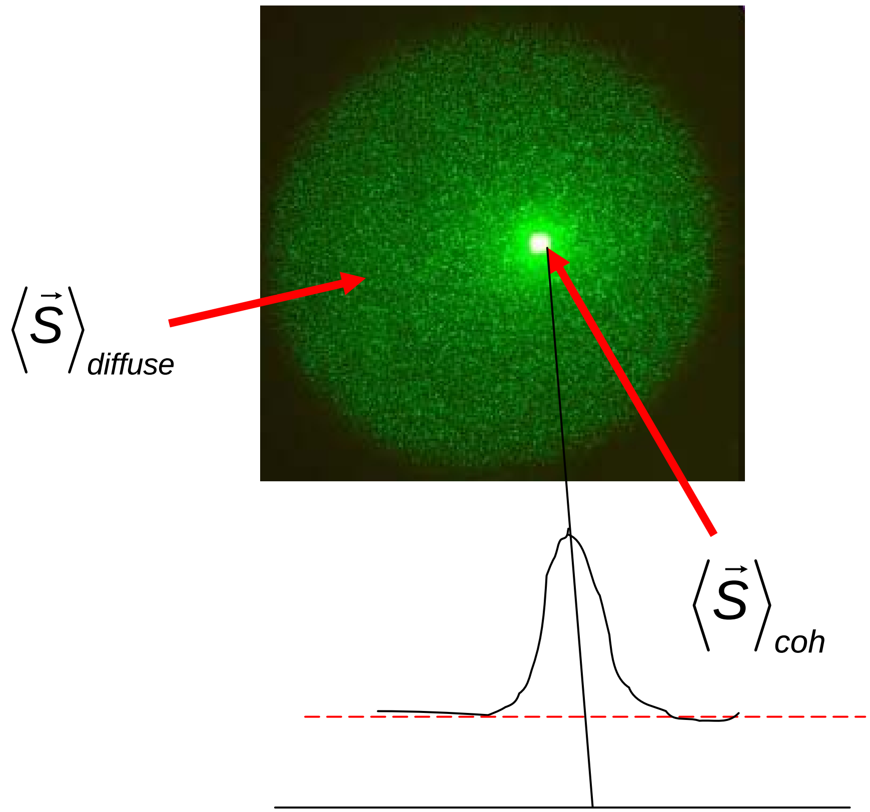
Promedio correcto

¿Quién hace el promedio?... nuestros aparatos

$$\left\langle \vec{S} \right\rangle$$

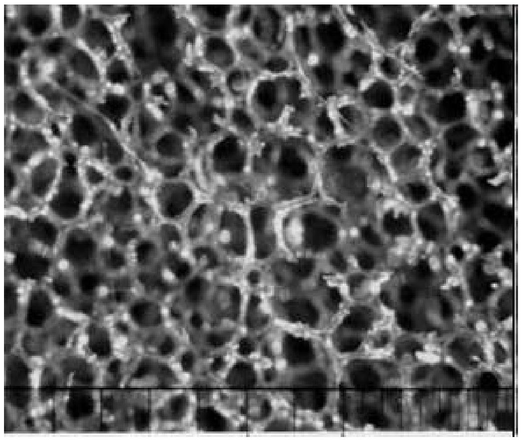
Experimento

$$\left\langle \vec{S} \right\rangle = \underbrace{\left\langle \vec{E} \right\rangle \times \left\langle \vec{H} \right\rangle}_{\left\langle \vec{S} \right\rangle_{coh}} + \underbrace{\left\langle \delta\vec{E} \times \delta\vec{H} \right\rangle}_{\left\langle \vec{S} \right\rangle_{diffuse}}$$

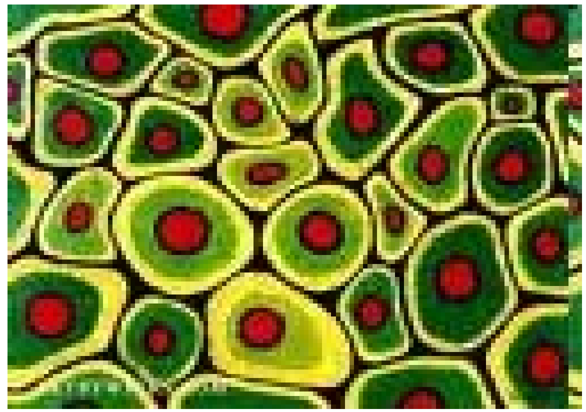


Materiales Inhomogeneos

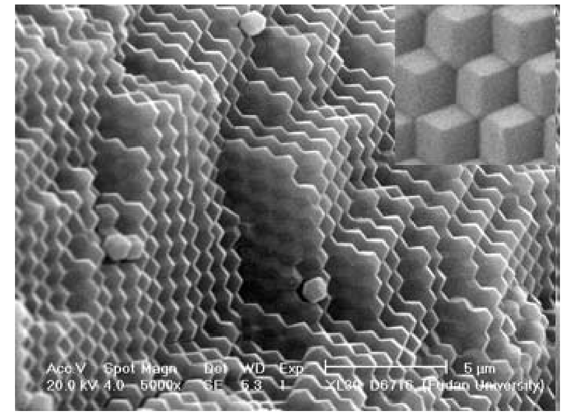
Materiales inhomogeneos



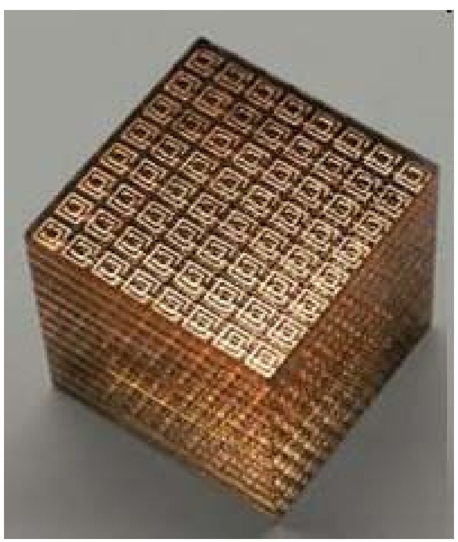
porosos



tejidos



Cristales fotónicos



metamateriales

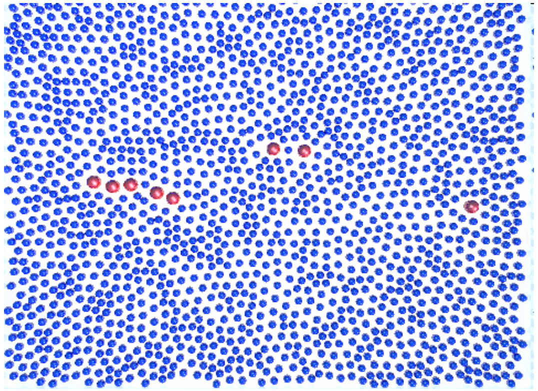
Alas de mariposa

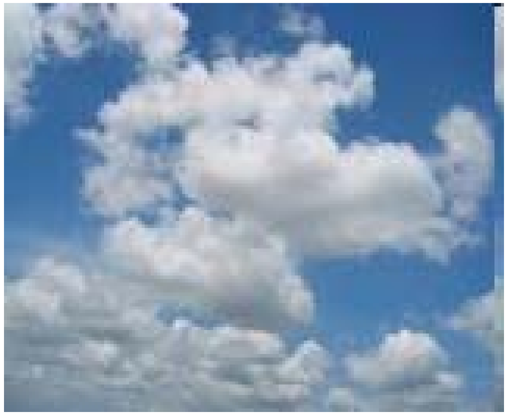
espuma

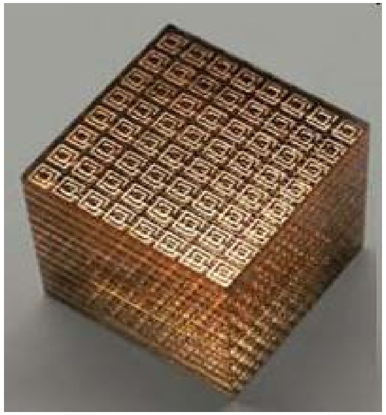
coloides

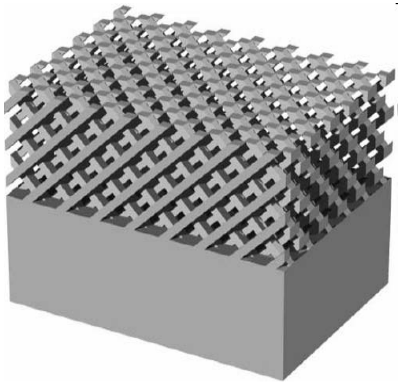
fase dispersa / fase homogenea

EJEMPLOS









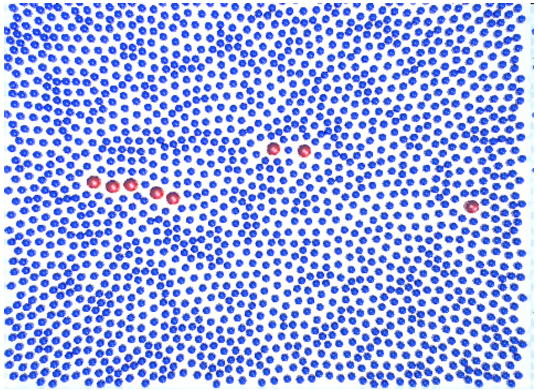
colloidal particles / matrix

“ordered” colloids

Teorías del medio efectivo…en general

Reglas de mezclado

$$\langle n \rangle = c\, n_1 + (1-c)\, n_2$$



$$n = \sqrt{\varepsilon\, \mu}$$

$$\langle n \rangle = \sqrt{\frac{\langle \varepsilon \rangle \langle \mu \rangle}{\varepsilon_0 \mu_o}} \rightarrow \langle n \rangle = \sqrt{\frac{\langle \varepsilon \rangle}{\varepsilon_0}}$$

$$\langle s \rangle = c\, s_1 + (1-c)\, s_2$$

adtividad $\langle \vec{J} \rangle$ $\langle \vec{E} \rangle$

$$\langle \sigma \rangle = c\, \sigma_1 + (1-c)\, \sigma_2$$

Función respuesta $\langle \vec{J} \rangle = \sigma_{eff} \langle \vec{E} \rangle$

percolación

$$n_{eff} \qquad\qquad n = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$$

$$\nabla \times \nabla \times \vec{E} - \omega^2 \underline{\varepsilon(\vec{r}\,)}\, \mu_0 \vec{E} = i\omega\mu_0 \vec{J}_{ext}$$

$$\langle ... \rangle \;\rightarrow\; \langle \vec{E} \rangle \;\rightarrow\; \exp[i\,\vec{k}_{eff} \cdot \vec{r}\,] \;\rightarrow\; k_{eff} = k_0 n_{eff}$$

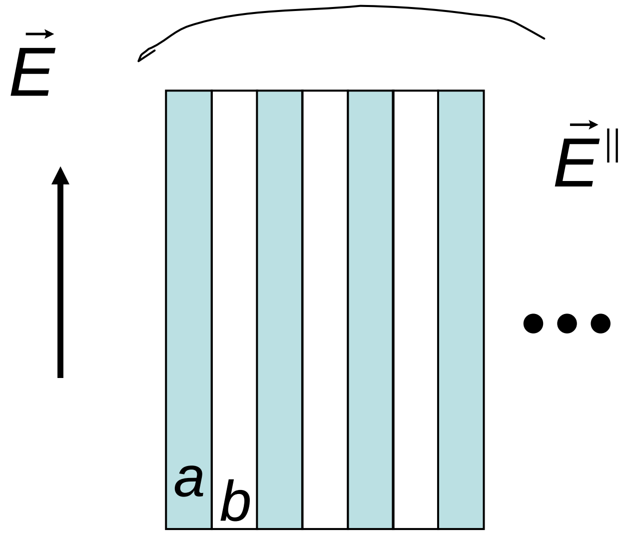
Líquido / gas

$$s^2 = \frac{1}{\rho\,\chi} \qquad s_{eff}^2 = \frac{1}{\underline{\langle\rho\rangle\langle\chi\rangle}} \qquad \left\langle \frac{\delta V}{V} \right\rangle = \chi_{eff} \langle \delta P \rangle$$

correlacionados

D Eq, $\longrightarrow$ k_{eff}

La geometría… la forma

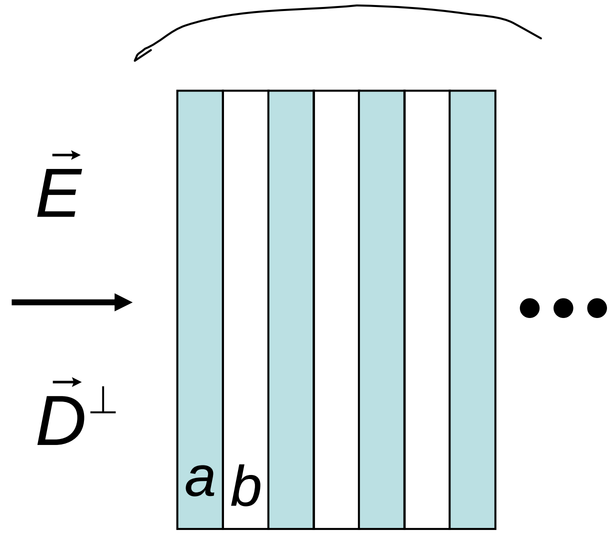


$$\vec{J} = -i\omega\vec{P}$$

$$\left\langle \vec{J} \right\rangle = -i\omega\varepsilon_0 \underbrace{[f_a\chi_a + (1-f_a)\chi_b]}_{\chi_{eff}^{\|}} \left\langle \vec{E} \right\rangle$$

$$\boxed{\varepsilon_{eff}^{\|} = f_a\varepsilon_a + (1-f_a)\varepsilon_b}$$

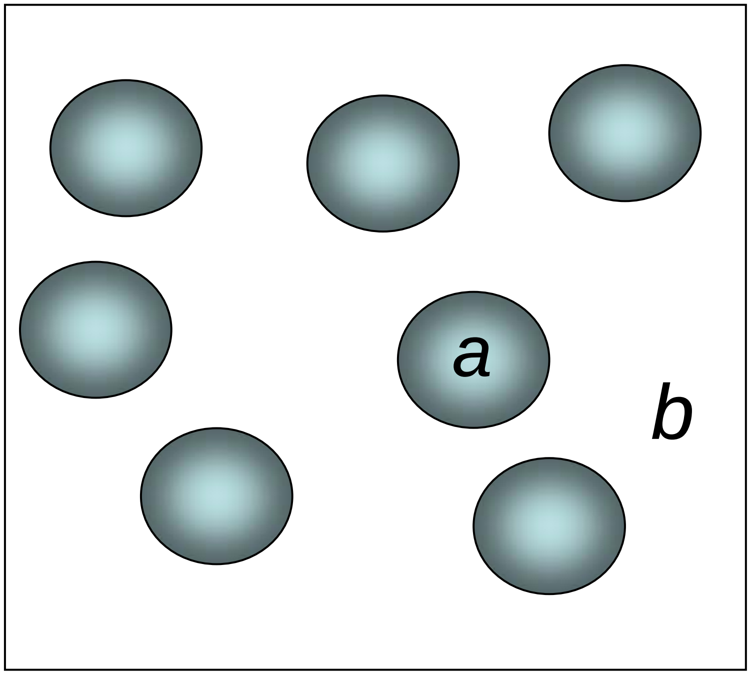
La geometría… la forma

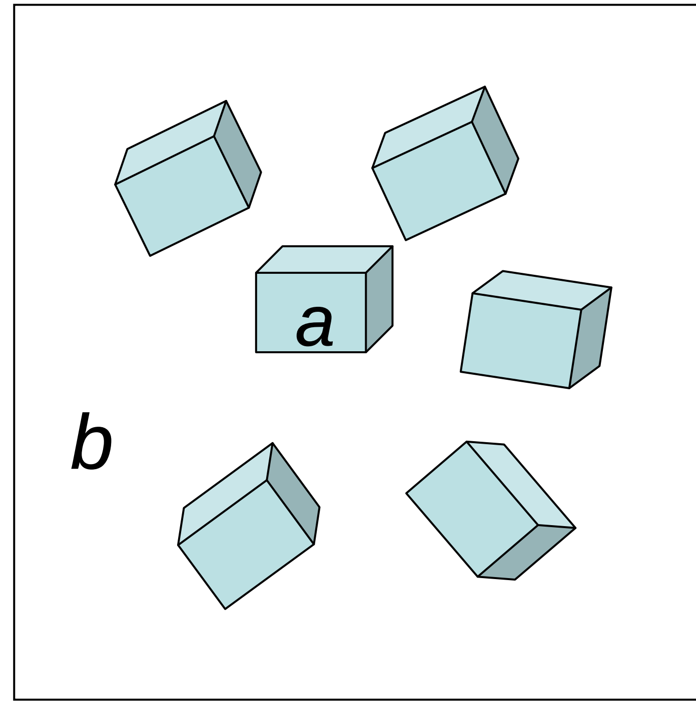


$$\left\langle \vec{J}_{\perp} \right\rangle = -i\omega\varepsilon_0 \underbrace{[f_a \frac{\chi_a}{\varepsilon_a} + (1-f_a)\frac{\chi_b}{\varepsilon_b}]}_{\chi_{eff}^{\perp}\,/\,\varepsilon_{eff}^{\perp}} \left\langle D_{\perp} \right\rangle$$

$$\boxed{\frac{1}{\varepsilon_{eff}^{\perp}} = \frac{f_a}{\varepsilon_a} + \frac{(1-f_a)}{\varepsilon_b}}$$

La geometría… la forma





Límite diluido

$$\frac{\varepsilon_{eff}}{\varepsilon_b} = f_a \frac{\varepsilon_a - \varepsilon_b}{\varepsilon_a + 2\varepsilon_b} + 1$$

$$\frac{\varepsilon_{eff}}{\varepsilon_b} = ?$$

La geometría… el tamaño

parámetro de tamaño

$$x = ka = \frac{2\pi a}{\lambda}$$

óptico

$$200 \leq \lambda \leq 1000 \ \text{ nm}$$

$x \ll 1$ $\quad$ $a \ll 100$ nm

$$Q_{abs} = \frac{C_{abs}}{\pi a^2} = 4x \operatorname{Im} \frac{\varepsilon_p - \varepsilon_M}{\varepsilon_p + 2\varepsilon_M}$$

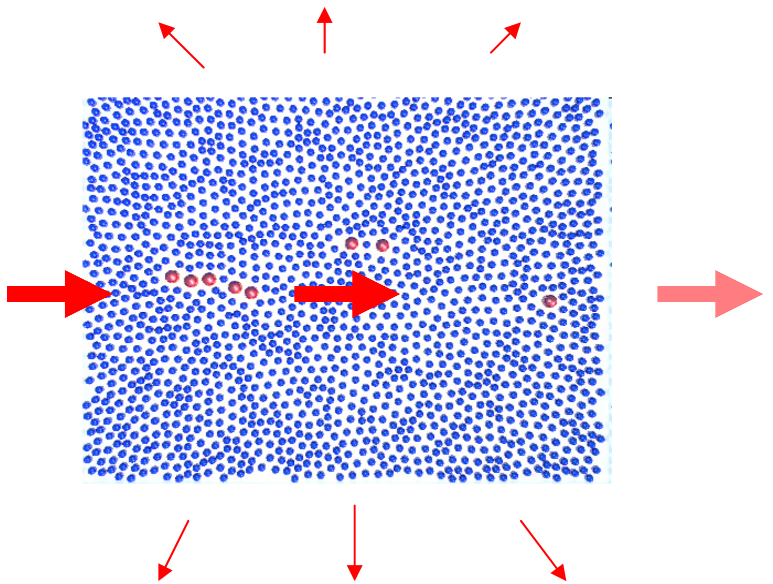
$$Q_S = \frac{C_{sca}}{\pi a^2} = \frac{8}{3} x^4 \left| \frac{\varepsilon_p - \varepsilon_M}{\varepsilon_p - \varepsilon_M} \right|^2$$

(Rayleigh)

pequeñas $\quad$ grandes

$x \ll 1$ $\quad$ $x \sim 1$

$$C_{sca} \ll C_{abs}$$



$$\left\langle \vec{S} \right\rangle_{coh} \gg \left\langle \vec{S} \right\rangle_{diffuse}$$

J. Phys. Chem. B **2003,** *107,* 6269−6275

ARTICLES

Optical Properties of Metal Nanoparticles with Arbitrary Shapes

Iván O. Sosa, Cecila Noguez,* and Rubén G. Barrera

$a_{eq} = 50$ nm

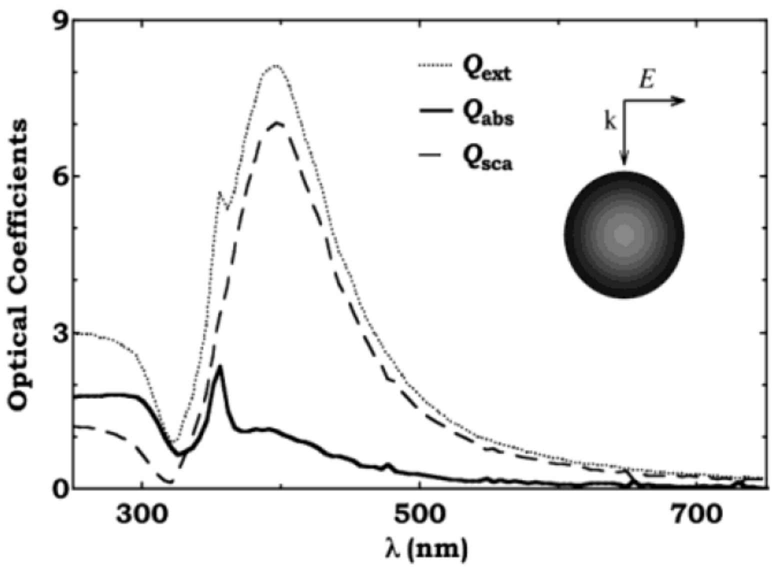


Figure 1. Optical coefficients for a silver nanosphere.

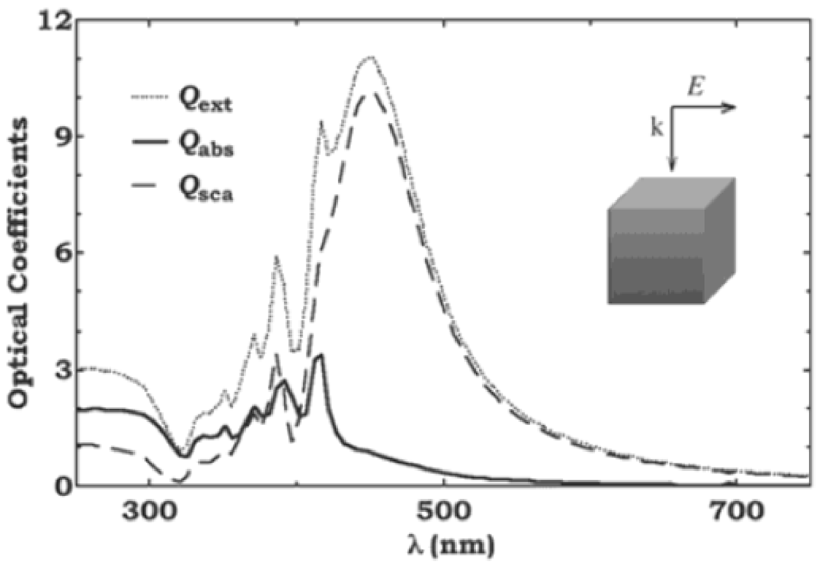
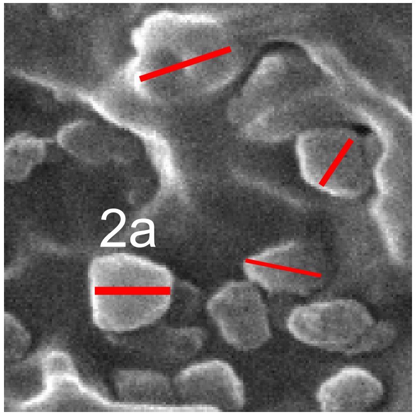


Figure 2. Optical coefficients for a silver nanocube.

Optical properties

SIZE



gold colloids

transparency

turbidity

size parameter

$$ka = \frac{2\pi a n}{\lambda_0}$$

$$ka \ll 1$$

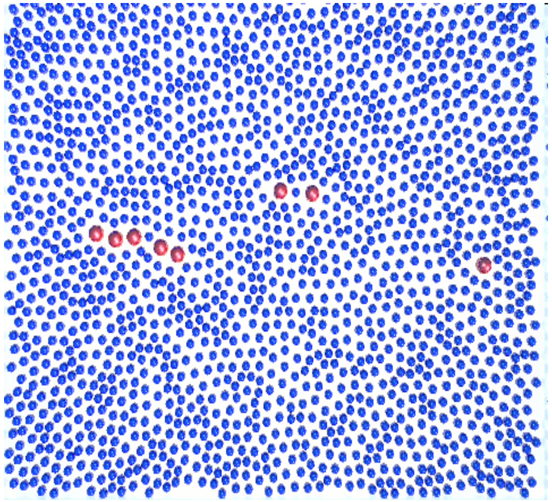
$$\langle \vec{S} \rangle_{diffuse} \ll \langle \vec{S} \rangle_{coh}$$

$$ka \sim 1$$

$$\langle \vec{S} \rangle_{diffuse} \sim \langle \vec{S} \rangle_{coh}$$

Effective-medium approach

homogenization



$$(\tilde{\varepsilon}_{eff}, \mu_{eff})$$

continuum

$$n_{eff} = \sqrt{\tilde{\varepsilon}_{eff}\,\mu_{eff}}$$

small particles $ka \ll 1$

advantage

electrodynamics
of
continuous media

always possible…
although it may be difficult

optical properties

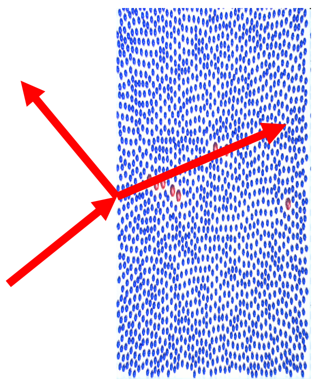
UNRESTRICTED

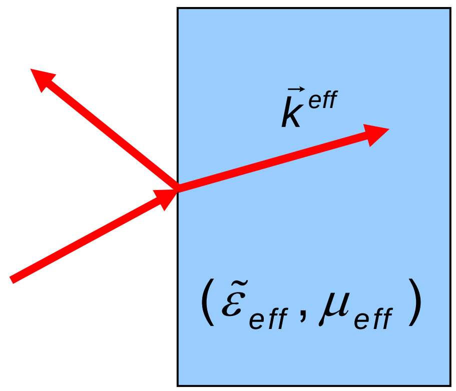
Reflection & refraction

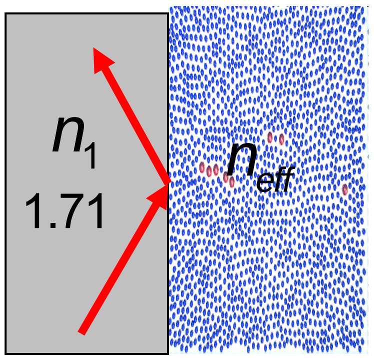
unrestricted

measurement

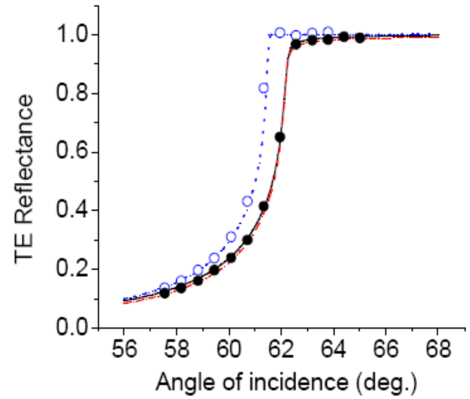
critical-angle
refractometry







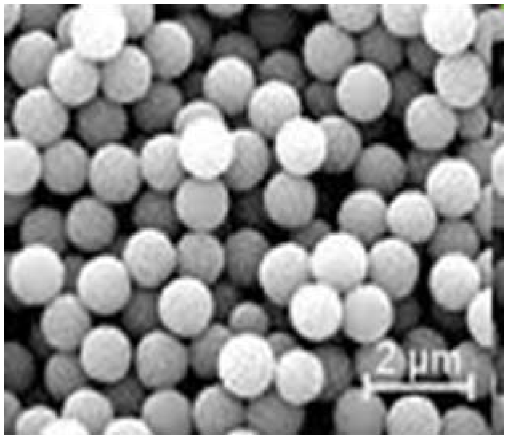
$$k^{eff} = k_0 n_{eff} \approx k_0 \sqrt{\varepsilon_{eff}}$$



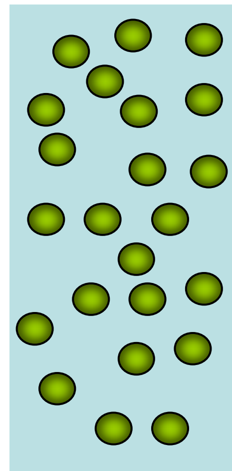
Fresnel´s relation

$$r_p = \frac{\varepsilon_{eff} k_z^i - \varepsilon_0 k_z^{eff}}{\varepsilon_{eff} k_z^i + \varepsilon_0 k_z^{eff}}$$

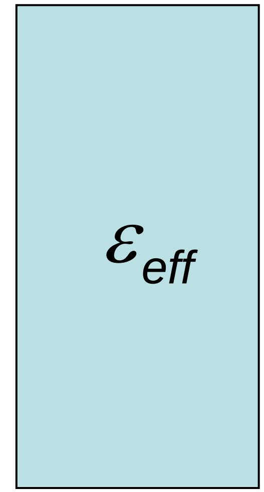
$x \ll 1$



partículas coloidales



modelo

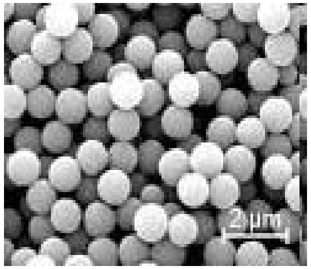


Medio efectivo

Problema

$$\varepsilon_{eff}(\overbrace{n_p, n_M, \lambda}^{\text{ópticos}}; \overbrace{a, f, \rho^{(2)}, \rho^{(3)}, \ldots}^{\text{microestructura}})$$

"reglas de mezclado"



Modelo

parámetros geométricos

- esferas idénticas de radio *a*
- ubicadas al azar
- $$f = \frac{N}{V}\frac{4\pi a^3}{3}$$
- $$W(\vec{R}_1, \vec{R}_2 \ldots \vec{R}_N)$$

parámetros ópticos

- no magnéticas
- $$n_p = \sqrt{\varepsilon_p / \varepsilon_0} = n'_p + i n''_p$$

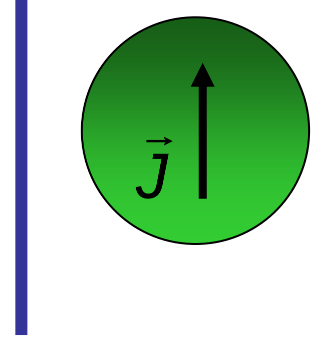
matriz (homogenea)

$$n_M = \sqrt{\varepsilon_M / \varepsilon_0}$$

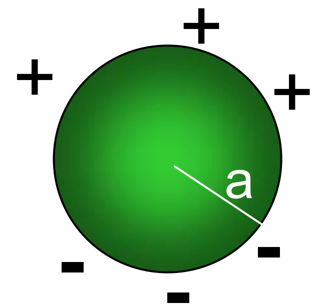
Problema: Excitar $\longrightarrow \vec{P} \sim \langle \vec{E} \rangle$

Esfera aislada

$\vec{E}^{ext} \sim e^{-i\omega t}$



$$\langle \vec{J} \rangle = \frac{1}{V_a} \int \vec{J}\, dV = \frac{1}{V_a} (-i\omega) \vec{p}$$



$a \ll \lambda$

$$\vec{p} = \varepsilon_0 \alpha \vec{E}$$

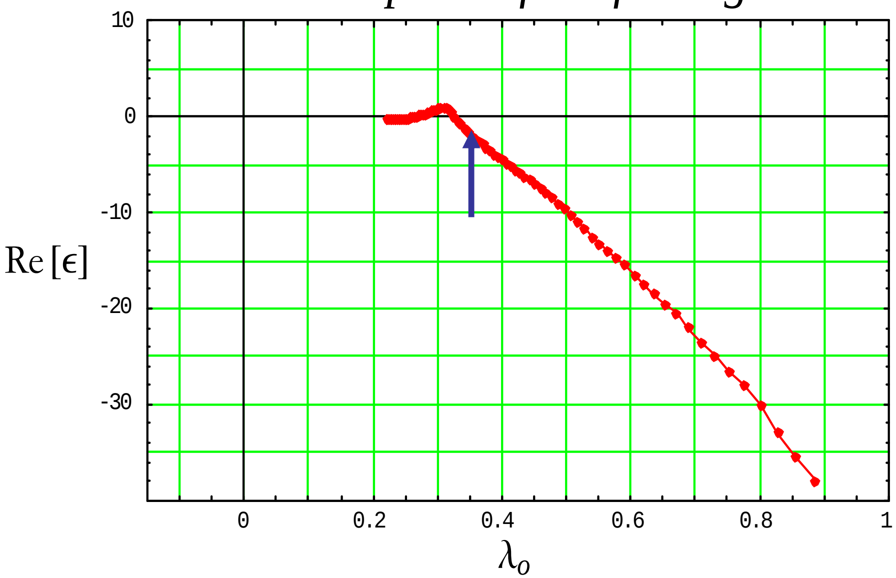
polarizabilidad (cuasi-estático)

$$\alpha = 4\pi a^3 \frac{\varepsilon_p - \varepsilon_M}{\varepsilon_p + 2\varepsilon_M}$$

resonancia

$$\varepsilon_p(\omega) = -2\varepsilon_M$$

metales



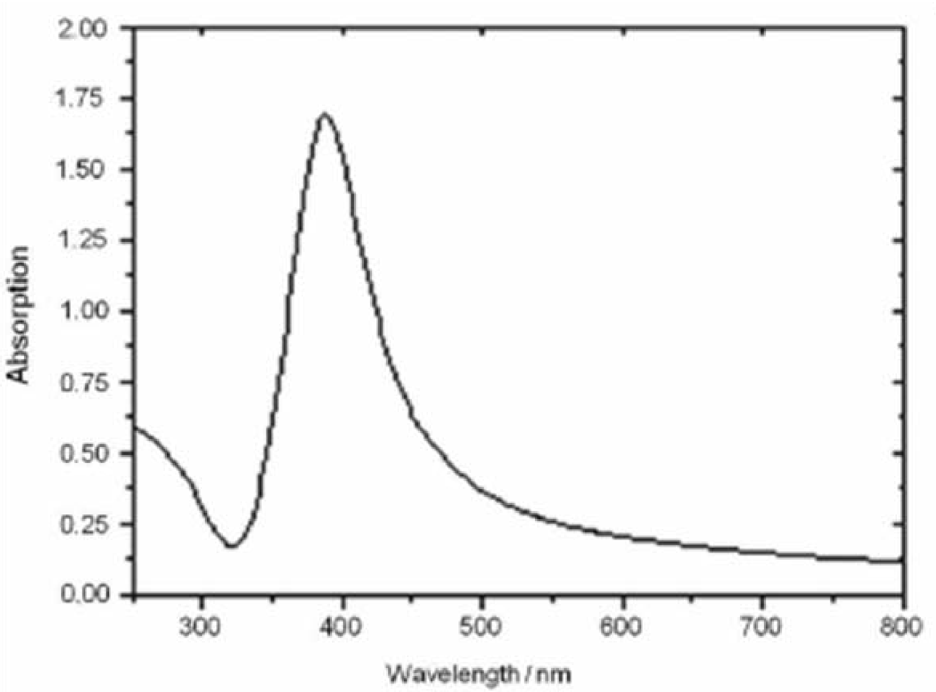
Absorción…esfera aislada

Absorción

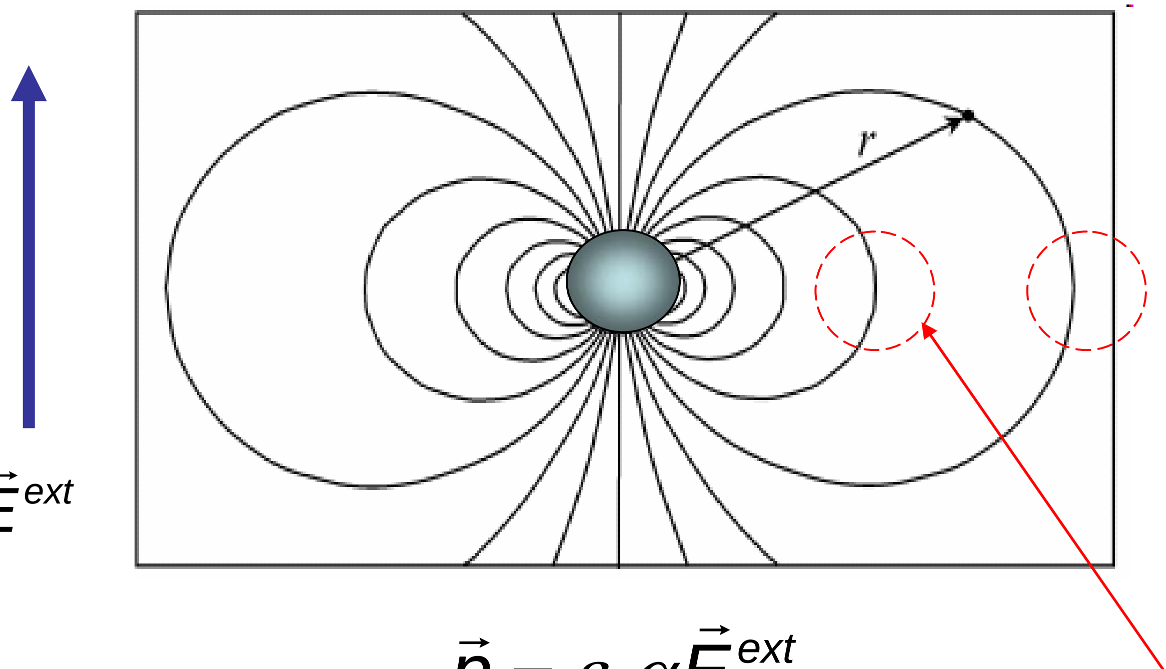
$$C_{abs} = \frac{2\pi}{\lambda} \mathrm{Im}\,\alpha$$

polarizabilidad (cuasi-estático)

$$\alpha = 4\pi a^3 \frac{\varepsilon_p - \varepsilon_M}{\varepsilon_p + 2\varepsilon_M}$$



Campo eléctrico… esfera aislada



$$\vec{p} = \varepsilon_0 \alpha \vec{E}^{ext}$$

multipolos

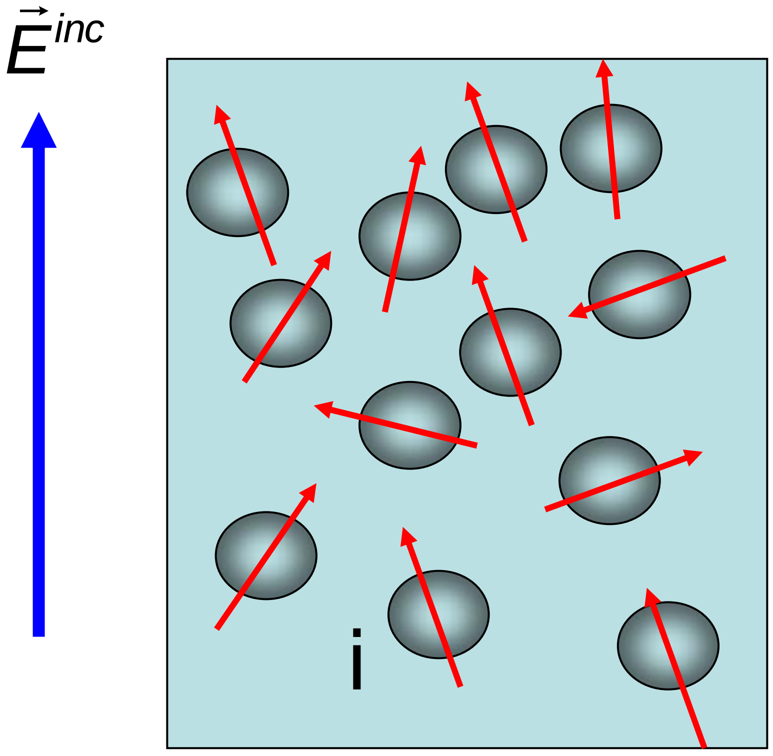
sistema diluido

$$\vec{E}_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \overline{\overline{T}} \cdot \vec{p}$$

(cuasi-estática)

$$\overline{\overline{T}} = \frac{3\hat{r}\hat{r} - \overline{\overline{1}}}{r^3}$$

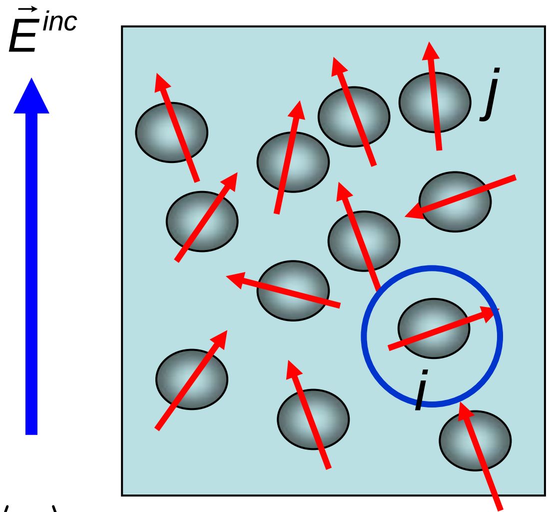
Dipolo inducido



Depende de la presencia de los vecinos

Coloide… dipolos puntuales

homogeneo e isótropo
"en promedio"



$\langle \cdots \rangle$ Promedio de ensamble

Problema

$$\vec{p}_i = \varepsilon_0 \, \alpha \, \vec{E}^{loc}$$

$$\vec{E}^{loc} = \vec{E}^{inc} + \vec{E}'_{dip}$$

Aproximación de dipolos puntuales

$$\vec{p}_i = \varepsilon_0 \alpha \left(\vec{E}^{inc} + \frac{1}{4\pi\varepsilon_0} \sum_{j \neq i} \overline{\overline{T}}_{ij} \cdot \vec{p}_j \right)$$

$j \neq i$ campo local

$$\langle \vec{p} \rangle = \varepsilon_0 \alpha \left(\vec{E}^{inc} + \frac{1}{4\pi\varepsilon_0} \left\langle \sum_{j \neq i} \overline{\overline{T}}_{ij} \cdot \vec{p}_j \right\rangle \right)$$

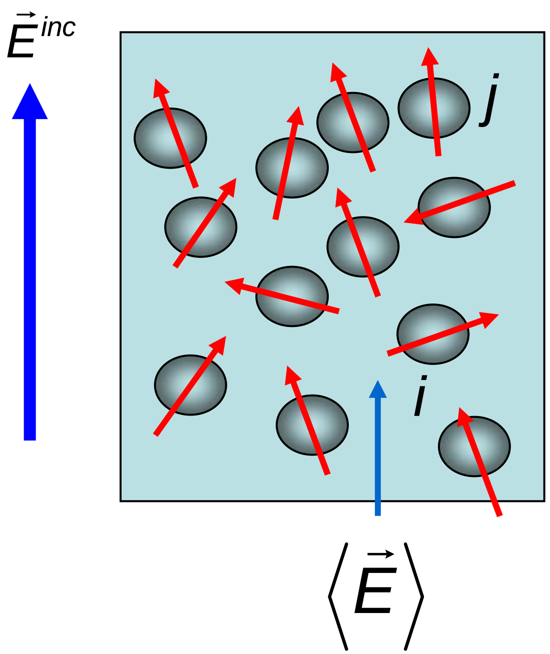
$$\vec{P} = \frac{N}{V_T} \langle \vec{p} \rangle$$

$$\langle \vec{E} \rangle = \vec{E}^{inc} + \frac{1}{4\pi\varepsilon_0} \left\langle \sum_{j} \overline{\overline{T}}_{ij} \cdot \vec{p}_j \right\rangle$$

$$\boxed{P = \varepsilon_0 \chi_{eff} \langle \vec{E} \rangle}$$

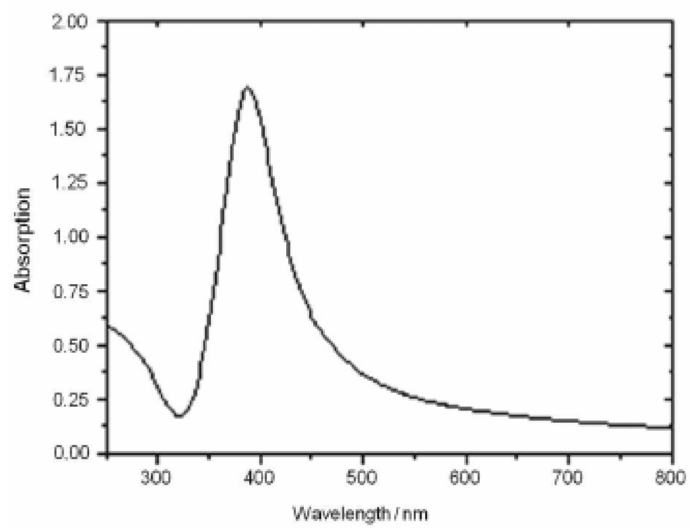
Límite diluido

$$\langle \vec{p}\rangle = \varepsilon_0\alpha\left(\vec{E}^{inc} + \frac{1}{4\pi\varepsilon_0}\left\langle \sum_{j\neq i}\overline{\overline{T}}_{ij}\cdot\vec{p}_j\right\rangle\right) \longrightarrow \langle \vec{p}\rangle = \varepsilon_0\alpha\underbrace{\left(\vec{E}^{inc} + \frac{1}{4\pi\varepsilon_0}\left\langle \sum_{j}\overline{\overline{T}}_{ij}\cdot\vec{p}_j\right\rangle\right)}_{\langle\vec{E}\rangle}$$



$$\vec{P} = \varepsilon_0 n\alpha\langle\vec{E}\rangle$$

$$\chi_{eff} = n\alpha = \frac{N}{V_T}\frac{4\pi a^3}{3}3\tilde{\alpha} = 3f\tilde{\alpha}$$



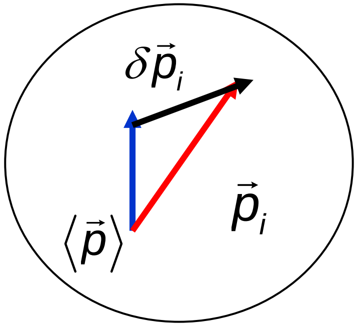
$$\boxed{\tilde{\alpha} = \frac{\varepsilon_p - \varepsilon_M}{\varepsilon_p + 2\varepsilon_M}}$$

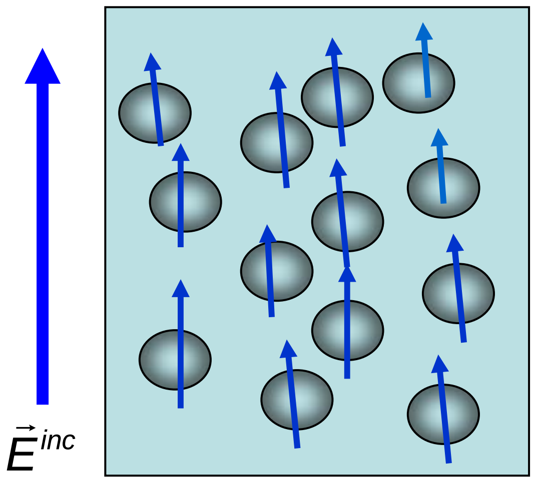
$$C_{abs} = \frac{2\pi}{\lambda}\mathrm{Im}\,\alpha$$

Campo medio

Aproximación de campo medio (MFT)

$$\vec{p}_i \rightarrow \langle \vec{p} \rangle + \cancel{\delta \vec{p}_i}$$





Resulta que:

$$\vec{P} = \varepsilon_0 \underbrace{\frac{n\alpha}{1 - \frac{n\alpha}{3}}}\langle \vec{E} \rangle$$

$$\chi_{eff} = \frac{n\alpha}{1 - \frac{n\alpha}{3}} = \frac{3f\tilde{\alpha}}{1 - f\tilde{\alpha}}$$

Clausius-Mossotti

$$\frac{\varepsilon_{eff} - \varepsilon_M}{\varepsilon_{eff} + 2\varepsilon_M} = f\tilde{\alpha}$$

$$\frac{\varepsilon_{eff}}{\varepsilon_M} = \frac{1 + 2f\tilde{\alpha}}{1 - f\tilde{\alpha}}$$

Polo

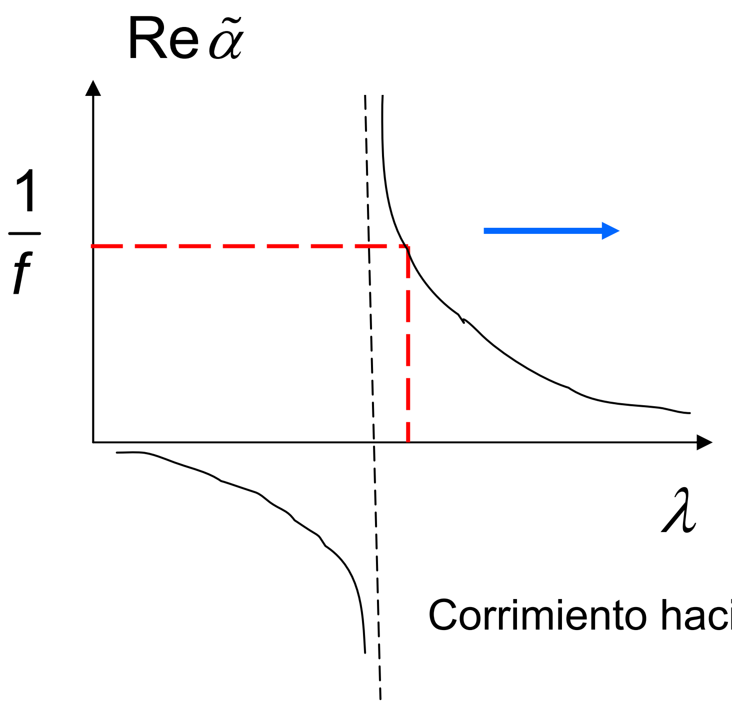
JC Maxwell Garnett

$$\frac{\varepsilon_{eff} - \varepsilon_M}{\varepsilon_{eff} + 2\varepsilon_M} = f\frac{\varepsilon_p - \varepsilon_M}{\varepsilon_p + 2\varepsilon_M}$$

Absorción

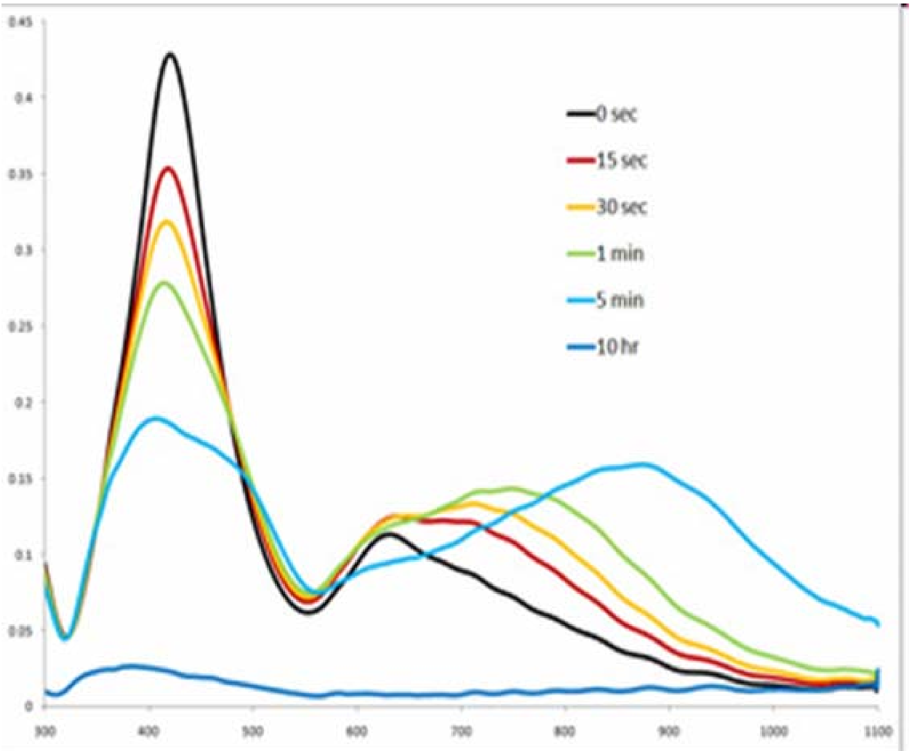
$$\frac{\varepsilon_{eff}}{\varepsilon_0} = \frac{1 + 2f\,\tilde{\alpha}}{1 - f\,\tilde{\alpha}}$$

Polo: $\tilde{\alpha}(\omega) = \dfrac{1}{f}$

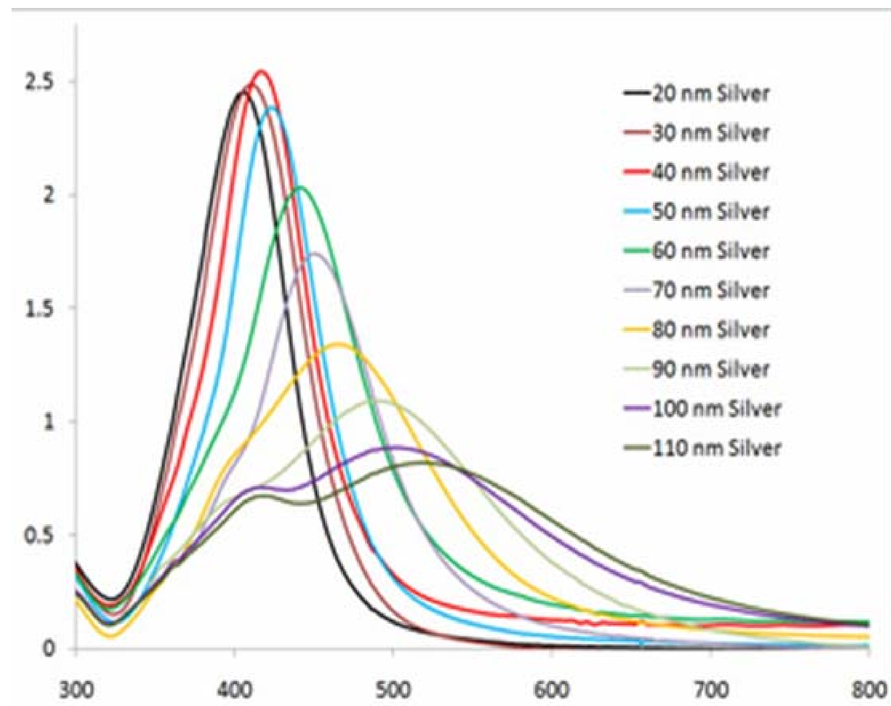


Corrimiento hacia el rojo

Plasmónica



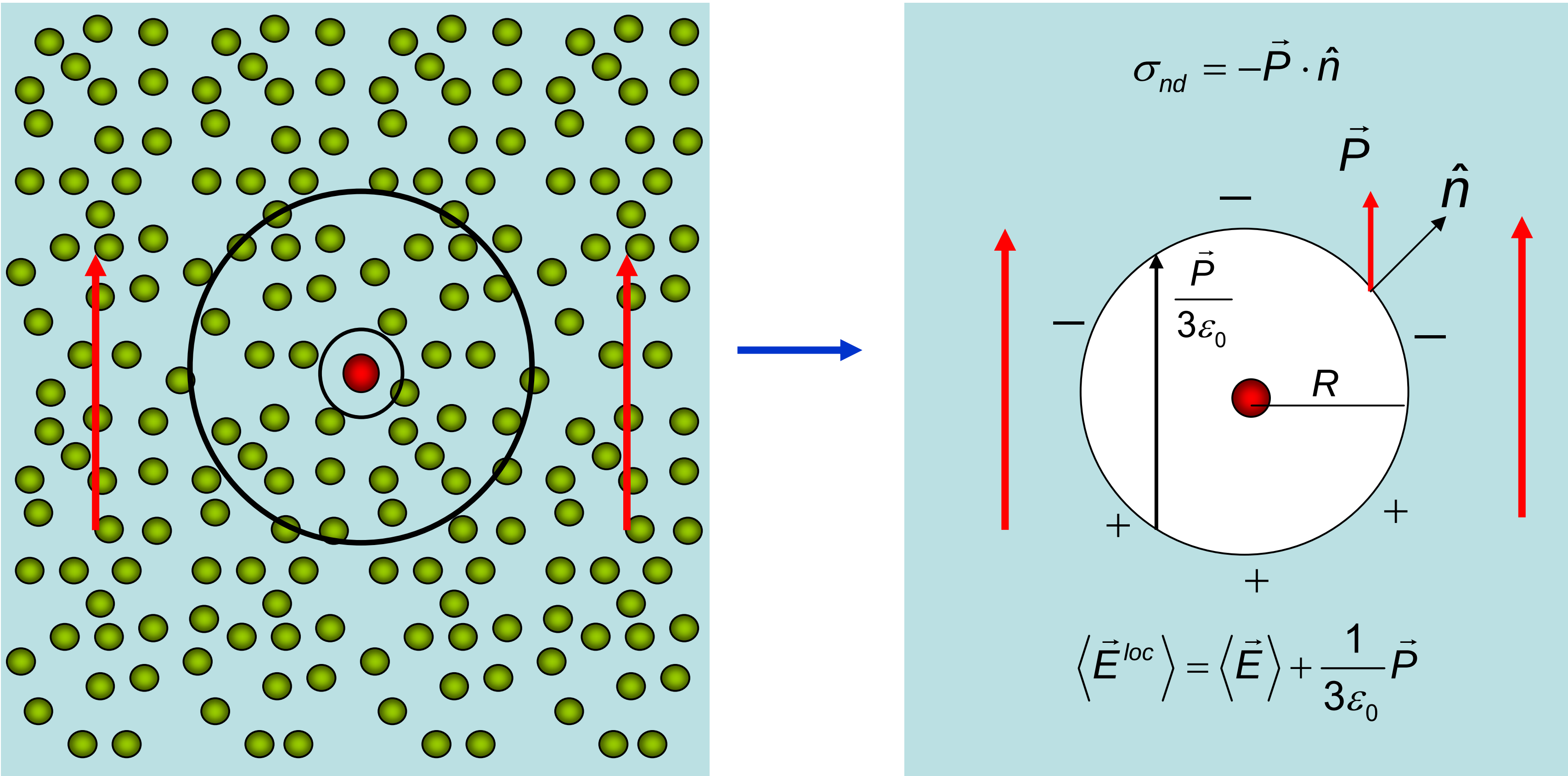
0 sec
15 sec
30 sec
1 min
5 min
10 hr
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
300 400 500 600 700 800 900 1000 1100



20 nm Silver
30 nm Silver
40 nm Silver
50 nm Silver
60 nm Silver
70 nm Silver
80 nm Silver
90 nm Silver
100 nm Silver
110 nm Silver
2.5
2
1.5
1
0.5
0
300 400 500 600 700 800

Método de Lorentz… el más popular

se congela



$$\vec{P} = \varepsilon_0 n\alpha \left\langle \vec{E}^{loc} \right\rangle$$

$$\boxed{\vec{P} = \varepsilon_0 \frac{n\alpha}{1 - \frac{n\alpha}{3}} \left\langle \vec{E} \right\rangle}$$

Independiente de R

Extensiones al método de Lorentz

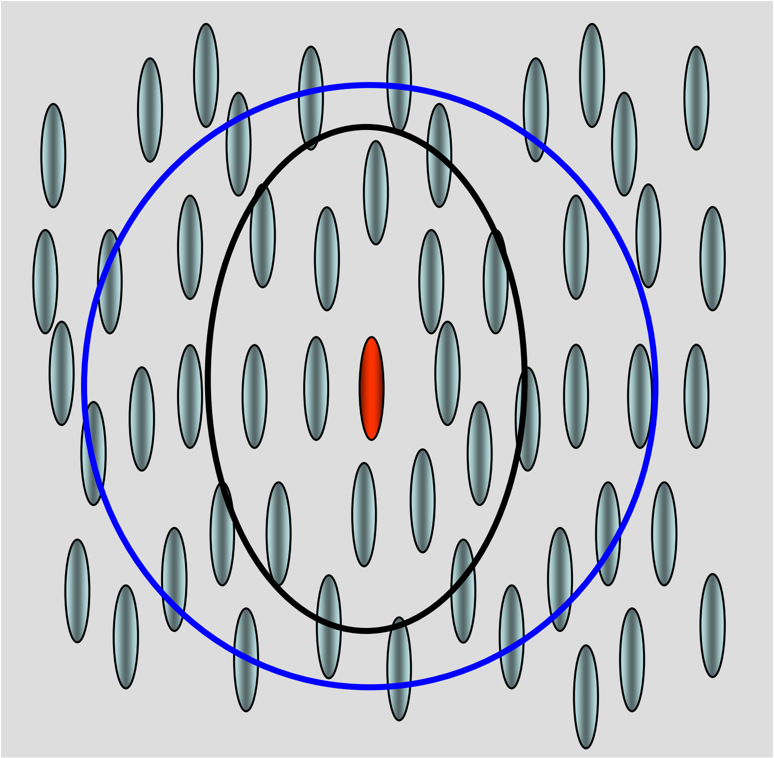
Maxwell Garnett

$$\frac{\varepsilon_{eff} - \varepsilon_0}{\varepsilon_{eff} + 2\varepsilon_0} = \frac{n\alpha}{3}$$

esferas en 3D

$$\frac{\varepsilon_{eff} - \varepsilon_0}{\varepsilon_{eff} + 2\varepsilon_0} = n\frac{4\pi a^3}{3}\frac{\varepsilon_p - \varepsilon_M}{\varepsilon_p + 2\varepsilon_M}$$

Elsferoides alineados



… ha habido muchos intentos por extender el método de Lorentz…

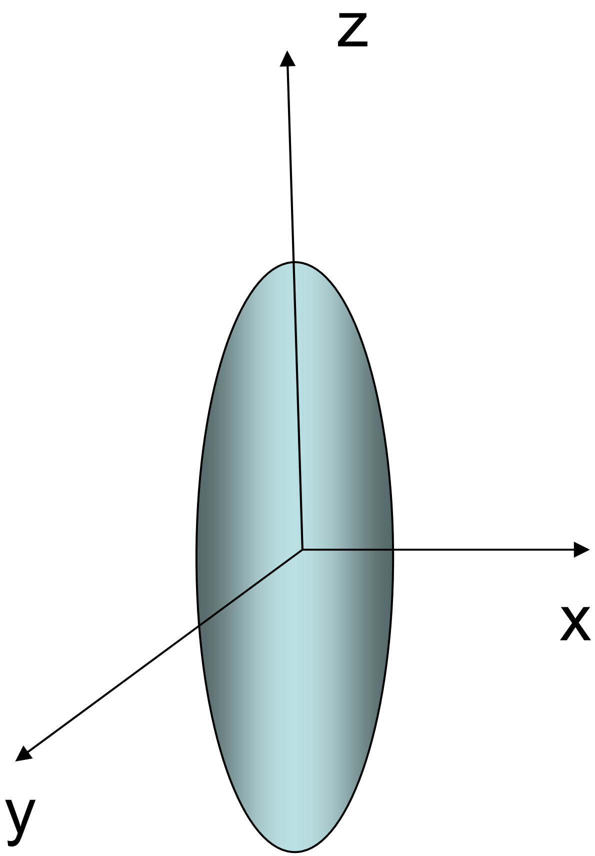
Pregunta:

¿cuál debe ser la forma de la cavidad?

esfera elipsoide

¿con qué eccentricidad?

Esferoide aislado



Tensor de polarizabilidad

(aproximación cuasi-estática)

$$\alpha^{\gamma} = 4\pi ab^2 \frac{\varepsilon_p - \varepsilon_M}{3L_\gamma \varepsilon_p + 3(1-L_\gamma)\varepsilon_M} \qquad \sum_\gamma L_\gamma = 1$$

$$L_z(e) = \begin{cases} \dfrac{1}{g^2(e)}\left[\dfrac{1}{2e}\ln\dfrac{1+e}{1-e} - 1\right] & (P)\,, \\ \dfrac{1}{e^2}\left[1 - \dfrac{1}{g(e)}\tan^{-1} g(e)\right] & (O)\,, \end{cases}$$

$$L_x = L_y = \tfrac{1}{2}(1 - L_z)\,,$$

esfera

$$L_\gamma = \frac{1}{3}$$

$$e = (1 - r_<^2 / r_>^2)^{1/2}$$

$$g(e) = e/(1-e^2)^{1/2}\,.$$

Cavidad de Lorentz

esferas en 3D

$$\frac{\varepsilon_{eff} - \varepsilon_0}{\varepsilon_{eff} + 2\varepsilon_0} = f\,\frac{\varepsilon_p - \varepsilon_M}{\varepsilon_p + 2\varepsilon_M}$$

esferoides en 3D

cavidad esférica

$$\frac{\varepsilon_{eff} - \varepsilon_0}{\varepsilon_{eff} + 2\varepsilon_0} = \frac{f}{3}\,\frac{\varepsilon_p - \varepsilon_M}{L_\gamma \varepsilon_p + (1 - L_\gamma)\varepsilon_M}$$

cavidad esferoidal

$$\frac{\varepsilon_{eff} - \varepsilon_0}{L_\gamma \varepsilon_{eff} + (1 - L_\gamma)\varepsilon_0} = f\,\frac{\varepsilon_p - \varepsilon_M}{L_\gamma \varepsilon_p + (1 - L_\gamma)\varepsilon_M}$$

PHYSICAL REVIEW B VOLUME 8, NUMBER 8 15 OCTOBER 1973

Optical Properties of Granular Silver and Gold Films

R. W. Cohen, G. D. Cody, M. D. Coutts, and B. Abeles
RCA Laboratories, Princeton, New Jersey 08540
(Received 22 March 1973)

istic depolarization factor L_m. Galeener's result is equivalent to substituting for $\alpha(\omega)$ on the right side of Eq. (2) the expression[10] for the polarizability of an isolated metallic ellipsoid immersed in a dielectric medium. Equation (2) then becomes

cavidad esférica

$$\frac{\epsilon(\omega)-\epsilon_i(\omega)}{\epsilon(\omega)+2\epsilon_i(\omega)} = \tfrac{1}{3}(1-x)\,\frac{\epsilon_m(\omega)-\epsilon_i(\omega)}{L_m\epsilon_m(\omega)+(1-L_m)\epsilon_i(\omega)}\,. \quad (4)$$

Although, as noted by Galeener, the above equation is valid for x close to unity, inconsistencies arise if one applies Eq. (4) to larger concentrations of metal. For example, for the case $L_m=0$ (flat me-

$$1-x=f$$

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The inconsistencies arising from Eq. (4) can be avoided if, in the calculation of the modification of the electric field by the dipole fields of the metal particles, one employs a cavity whose shape is congruent to that of the metal particles; e.g., the cavity is ellipsoidal with depolarization factor L_m associated with the principal axis that is parallel to the electric field. We shall adopt this mathematical construction. The generalized Clausius-Mosotti equation (2) is then modified, and, in place of Eq. (4), we obtain

cavidad elipsoidal

$$\frac{\epsilon(\omega)-\epsilon_i(\omega)}{L_m\epsilon(\omega)+(1-L_m)\epsilon_i(\omega)}=(1-x)\,\frac{\epsilon_m(\omega)-\epsilon_i(\omega)}{L_m\epsilon_m(\omega)+(1-L_m)\epsilon_i(\omega)}$$

obviamente todo esto o está mal… o… no se entiende


Para saber la forma correcta
dela cavidad de Lorentz,
se requiere… un análisis
más profundo

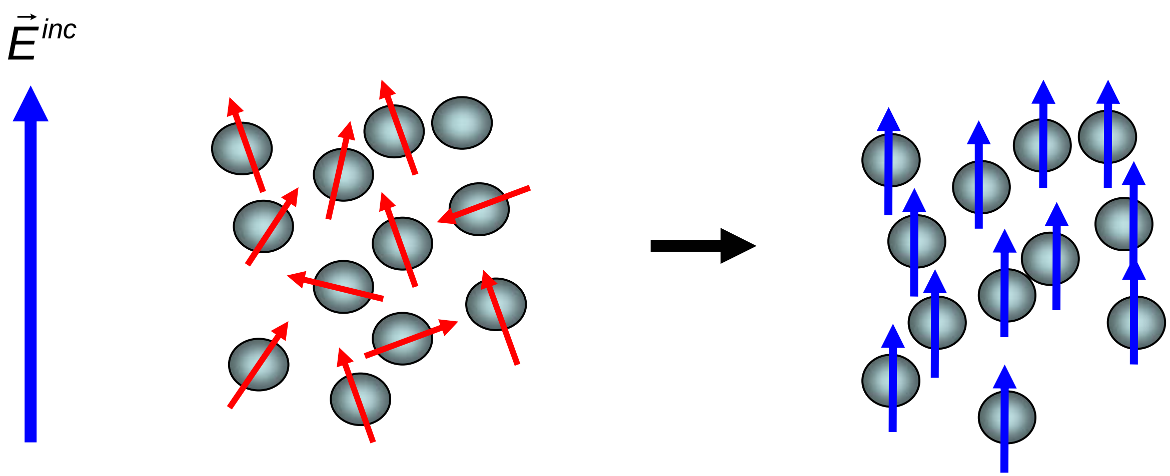
Análisis más… profundo

$$\frac{\varepsilon_{eff}}{\varepsilon_0} = \frac{1 + 2f\,\tilde{\alpha}(\omega)}{1 - f\,\tilde{\alpha}(\omega)}$$

$$\alpha(\omega) = 4\pi a^3 \frac{\varepsilon(\omega) - \varepsilon_M(\omega)}{\varepsilon(\omega) + 2\varepsilon_M(\omega)}$$

esferas

Aproximación de campo medio $\vec{p}_j \rightarrow \langle \vec{p} \rangle + \cancel{\delta\vec{p}_j}$



$$\langle \vec{p} \rangle = \varepsilon_0 \alpha \left(\vec{E}^{ext} + \frac{1}{4\pi\varepsilon_0} \left\langle \sum_{j \neq i} \overline{\overline{T}}_{ij} \cdot \vec{p}_j \right\rangle \right) \approx \varepsilon_0 \alpha \left(\vec{E}^{ext} + \frac{1}{4\pi\varepsilon_0} \left\langle \sum_{j \neq i} \overline{\overline{T}}_{ij} \right\rangle \cdot \langle \vec{p} \rangle \right)$$

Promedio condicional

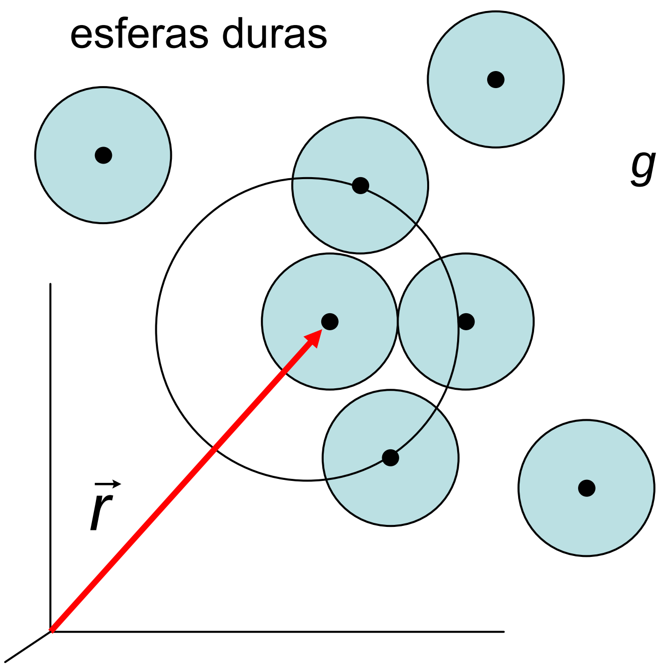
$$\langle \vec{p} \rangle \approx \varepsilon_0 \alpha \left(\vec{E}^{ext} + \frac{1}{4\pi\varepsilon_0} \left\langle \sum_{j \neq i} \overline{\overline{T}}_{ij} \right\rangle \cdot \langle \vec{p} \rangle \right)$$

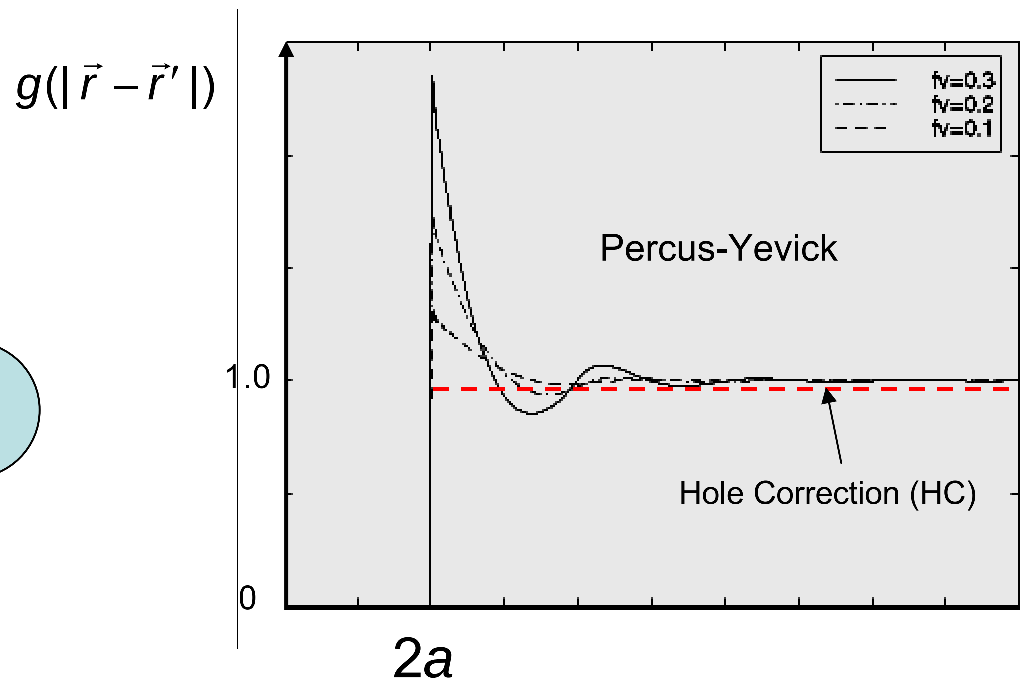
Se incluye el tamaño de la esfera, pero se excluyen los multipolos. Válido en el límite diluido

$$\vec{P} = \frac{N}{V_T} \langle \vec{p} \rangle \equiv n \langle \vec{p} \rangle$$

$$\vec{P} = \varepsilon_0 \alpha n \left[E^{ext} + \frac{1}{4\pi\varepsilon_0} \int_{V_T} \overline{\overline{T}}(\vec{r} - \vec{r}\,') g(\vec{r}, \vec{r}\,')\, d^3 r' \cdot \vec{P} \right]$$

esferas duras





Correlación de pares

El campo de polarización

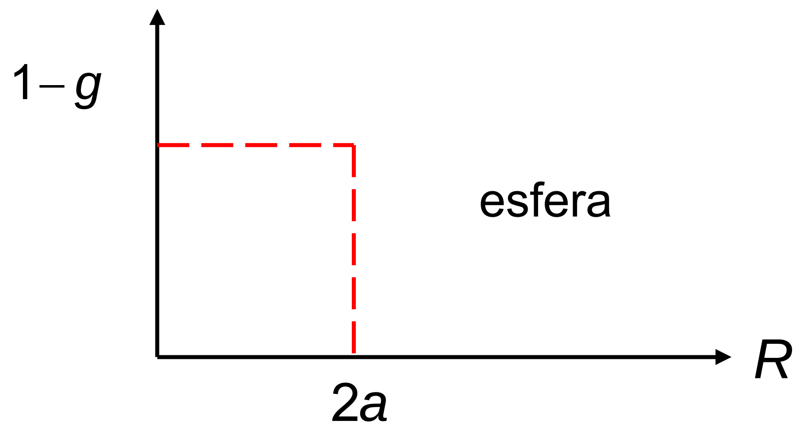
$$\vec{P} = \varepsilon_0 \alpha n \left[E^{ext} + \frac{1}{4\pi\varepsilon_0} \int_{V_T} \overline{\overline{T}}(\vec{r} - \vec{r}') \cdot g(\vec{r} - \vec{r}') d^3 r' \cdot \vec{P} \right]$$

$$\langle \vec{E} \rangle = E^{ext} + \frac{1}{4\pi\varepsilon_0} \int_{V_T} \overline{\overline{T}}(\vec{r} - \vec{r}') \cdot \vec{P}\, d^3 r'$$

… la integral es singular en el origen

$$\vec{P} = \varepsilon_0 n \alpha \left[\langle E \rangle - \frac{1}{4\pi\varepsilon_0} \int_{V_T} \overline{\overline{T}}(\vec{r} - \vec{r}') \cdot [1 - g(\vec{r} - \vec{r}')] d^3 r' \cdot \vec{P} \right]$$

… la integral es singular en el origen



$$\boxed{\vec{R} \equiv \vec{r} - \vec{r}'}$$

El tensor de Lorentz

$$\vec{P} = \varepsilon_0 n \alpha \left[\langle E \rangle - \frac{1}{4\pi\varepsilon_0} \int_{V_T} \overline{\overline{T}}(\vec{R}) \cdot [1 - g(\vec{R})]\, d^3R \cdot \vec{P} \right]$$

integral impropia
singular en $R = 0$

pero

$$\overline{\overline{T}}(\vec{R}) = \nabla_R \nabla_R \left(\frac{1}{R} \right)$$

Tensor de Lorentz

$$\overline{\overline{L}} \equiv -\frac{1}{4\pi\varepsilon_0} \int_{V_T} \overline{\overline{T}}(\vec{R}) \cdot [1 - g(\vec{R})]\, d^3R$$

$$= -\frac{1}{4\pi\varepsilon_0} \int_{V_T} \nabla_R \nabla_R \left(\frac{1}{R} \right) \cdot [1 - g(\vec{R})] d^3R$$

por tanto

$$\boxed{\overline{\overline{L}} = -\frac{1}{4\pi\varepsilon_0} \int_{V_T} \nabla_R \left(\frac{1}{R} \right) \nabla g(\vec{R}) d^3R}$$

La integral es no singular

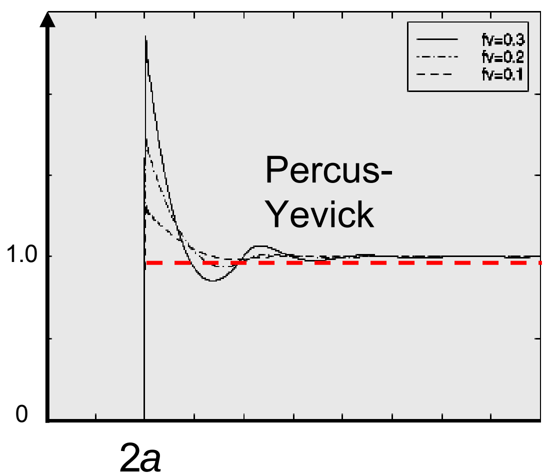
El campo de polarización

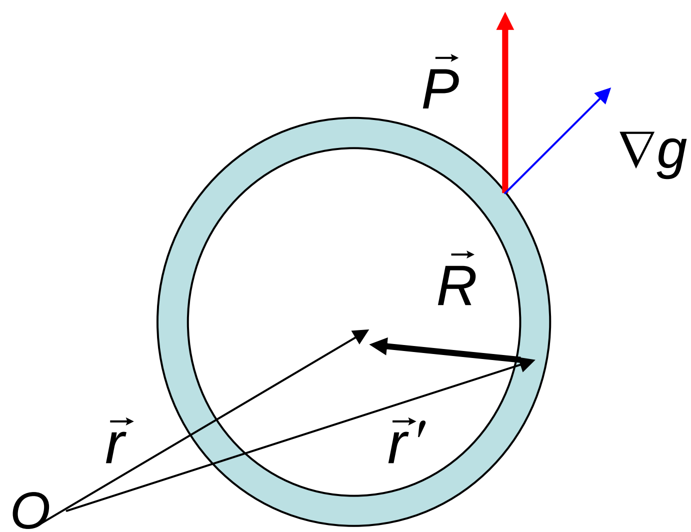
$$\vec{P} = \varepsilon_0 n\,\alpha\left[\langle E\rangle + \underline{\overline{\overline{L}}\cdot\vec{P}}\right]$$

correcciones de campo local

$$\overline{\overline{L}}\cdot\vec{P} = -\frac{1}{4\pi\varepsilon_0}\int_{V_T}\nabla_R\left(\frac{1}{R}\right)\nabla g(\vec{R})\cdot\vec{P}\,d^3R$$

$$= \frac{1}{4\pi\varepsilon_0}\int_{V_T}\frac{\hat{R}}{R^2}\underbrace{\nabla g(\vec{R})\cdot\vec{P}}_{-\rho_{pol}}\,d^3R = \frac{1}{4\pi\varepsilon_0}\int_{V_T}\frac{\hat{R}}{R^2}\,\nabla\cdot\left[\underline{g(\vec{R})\vec{P}}\right]d^3R$$



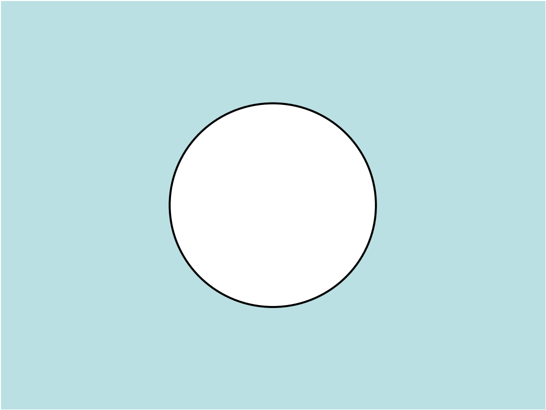


$$\boxed{\vec{R} \equiv \vec{r} - \vec{r}'}$$

La esfera de Lorentz

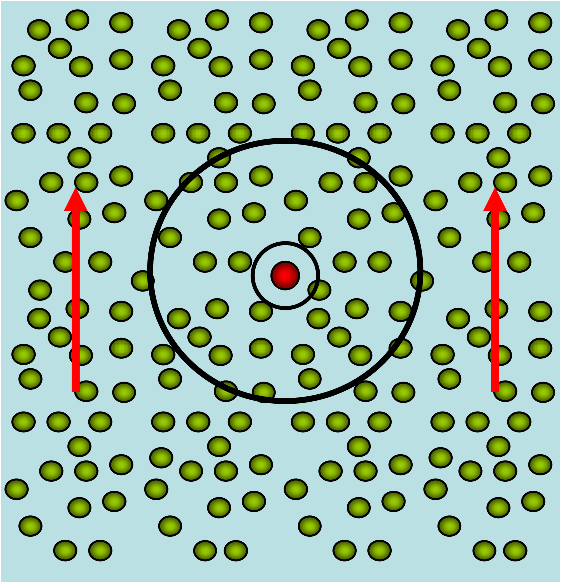
Si

$$g(R) = \begin{cases} 0 & R < 2a \\ 1 & R > 2a \end{cases}$$



$$\nabla_R g(R) = \hat{R}\,\delta(R - 2a)$$

$$\overline{\overline{L}} \cdot \vec{P} = \frac{1}{4\pi\varepsilon_0} \int_{V_T} \frac{\hat{R}}{R^2} \underbrace{\left[\nabla g(\vec{R}) \cdot \vec{P}\right]}_{-\sigma_{pol}} R^2\, dR\, d\Omega = \frac{\vec{P}}{3\varepsilon_0}$$



…en el método de Lorentz se aproxima g(R) por una cavidad esférica…pero no se dice…

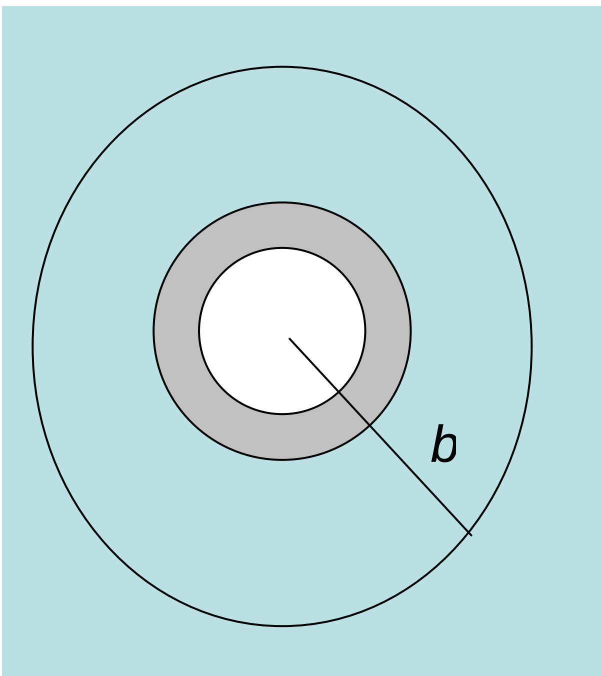
$$\left\langle \vec{E}^{loc} \right\rangle = \left\langle \vec{E} \right\rangle + \frac{\vec{P}}{3\varepsilon_0}$$

independiente de R

Simetría esférica

El resultado es más general

3D



simetría esférica

$$\overline{\overline{L}} = -\frac{1}{4\pi\varepsilon_0}\int_{V_T} \nabla_R\left(\frac{1}{R}\right)\nabla g(\vec{R})d^3R$$

$$Tr\,\varepsilon_0\overline{\overline{L}} = -\frac{1}{4\pi}\int_{V_T} \nabla_R\left(\frac{1}{R}\right)\cdot\nabla g(\vec{R})d^3R$$

usando

$$\nabla^2\frac{1}{R} = 4\pi\,\delta(R)$$

$$= -\frac{1}{4\pi}\int_{S_b} \nabla_R\left(\frac{1}{R}\right)\cdot d\vec{a} - \int_{V_b} g(\vec{R})\delta(R)d^3R = 1$$

$$g(b) = 1 \qquad g(0) = 0$$

$$Tr\,\varepsilon_0\overline{\overline{L}} = 1 \rightarrow \varepsilon_0\overline{\overline{L}} = \frac{1}{3}\overline{\overline{1}} \rightarrow \overline{\overline{L}}\cdot\vec{P} = \frac{\vec{P}}{3\varepsilon_0}$$

La función de correlación

Maxwell Garnett

Clausius-Mossotti

$$\boxed{\frac{\varepsilon_{eff} - \varepsilon_0}{\varepsilon_{eff} + 2\varepsilon_0} = f\tilde{\alpha}}$$

$g(R)$ simetría esférica

$g(\infty) = 1$

$g(0) = 0$

… y no quedó ningún rastro de g… como si nunca hubiera existido… como si la tierra se la hubiera tragado… que cosa…

sólo f

Extensiones

PHYSICAL REVIEW B VOLUME 47, NUMBER 14 1 APRIL 1993-II

Effective dielectric response of a composite with aligned spheroidal inclusions

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(Received 26 September 1991; revised manuscript received 17 December 1992)

The effective dielectric response ϵ_M of a composite with aligned spherodial inclusions is calculated. Using the dipolar and the mean-field approximation (MFA) an analytical expression for ϵ_M as a functional of the two-particle distribution function $\rho^{(2)}$ is obtained. It is shown that previous expressions reported in the literature correspond to different choices of $\rho^{(2)}$, thus, clarifying the origin of their discrepancies. The theory is further extended beyond the MFA by including the dipolar fluctuations through a renormalization of the polarizability tensor of the inclusions. The absorption peaks are diminished and broadened by the spatial disorder, which also yields an easily identified coupling among electromagnetic modes with perpendicular polarizations.

I. INTRODUCTION

The study of the linear electromagnetic response of

at a reference inclusion. The MFA is obtained when the contribution to the local field due to the other inclusions contained in the cavity is neglected and the contribution

Nuestro método

$$p_i^{\gamma} = \alpha^{\gamma}\left[E_{0i}^{\gamma} + \sum_{j,\delta} s_{ij}^{\gamma\delta} p_j^{\delta}\right]$$

$$E_i^{\gamma} = s_{ij}^{\gamma\delta} p_j^{\delta}$$

$$s^{xx}(\mathbf{R}) = 3(\eta_0/a)^3 \left\{ \frac{\pm\eta}{\eta^2 \mp 1} \left[\frac{1-\xi^2}{\eta^2 \mp \xi^2} \right] \cos^2\phi + \frac{1}{2} Q'_{10}(\eta) \right\}, \tag{A3a}$$

$$s^{xy}(\mathbf{R}) = 3(\eta_0/a)^3 \frac{\pm\eta}{\eta^2 \mp 1} \left[\frac{1-\xi^2}{\eta^2 \mp \xi^2} \right] \cos\phi \sin\phi, \tag{A3b}$$

$$s^{xz}(\mathbf{R}) = 3(\eta_0/a)^3 \frac{\xi}{\eta^2 \mp \xi^2} \left[\frac{1 \mp \xi^2}{\eta^2 \mp 1} \right]^{1/2} \cos\phi, \tag{A3c}$$

$$s^{zz}(\mathbf{R}) = -3(\eta_0/a)^3 \left[Q_0(\eta) - \frac{\eta}{\eta^2 \mp \xi^2} \right], \tag{A3d}$$

$$\frac{\epsilon_M^\zeta - \epsilon_h}{\mathcal{L}_\zeta \epsilon_M^\zeta + (1-\mathcal{L}_\zeta)\epsilon_h} = 3f\tilde{\alpha}^\zeta = f\frac{\epsilon_m - \epsilon_h}{L_\gamma \epsilon_m + (1-L_\gamma)\epsilon_h} \, , \tag{9}$$

where

$$4\pi n(\mathcal{L}_\zeta - 1) \equiv \lim_{q\to 0} \left\langle \sum_j S_{ij}^{\zeta\zeta} \right\rangle = \lim_{q\to 0} \left\langle \sum_j s_{ij}^{\zeta\zeta} \exp(-iqR_{ij}^\zeta) \right\rangle$$

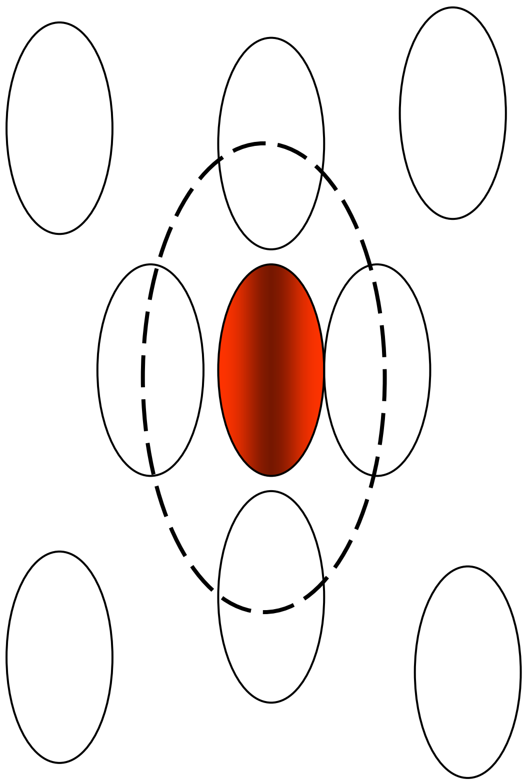
is the longitudinal average of the particle-particle interaction. $\mathcal{L}_\zeta$ is independent of i due to the homogeneity of the ensemble. Here $f = 4\pi nab^2/3$ is the volume fraction of spheroids and $\tilde{\alpha}^\gamma = \alpha^\gamma / ab^2$.

The average interaction is now calculated as[20]

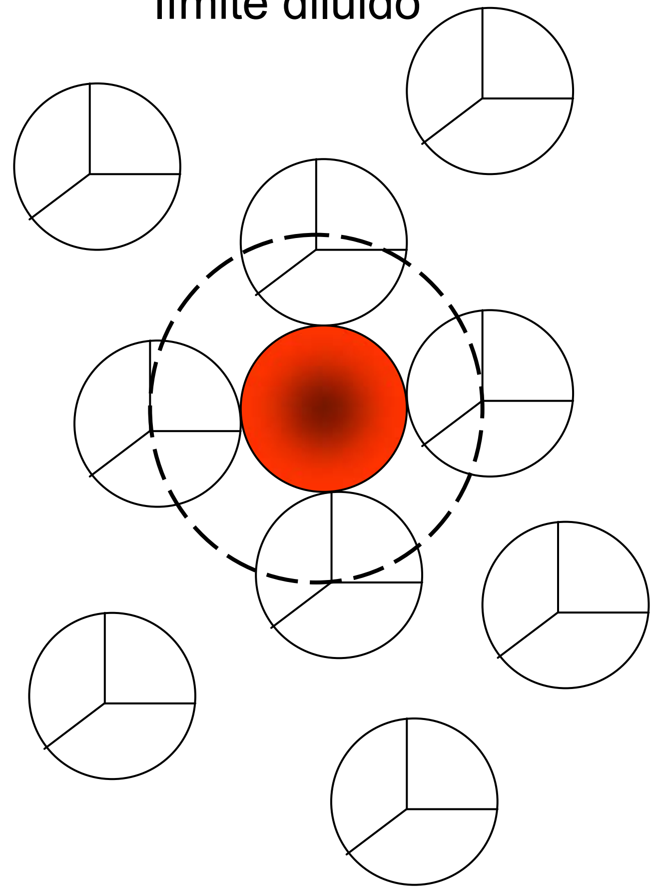
$$\mathcal{L}_\zeta - 1 = \lim_{q\to 0} \frac{1}{4\pi} \int s^{\zeta\zeta}(\mathbf{R}) e^{-iqR^\zeta} \rho^{(2)}(\mathbf{R}) d^3R \, , \tag{10}$$

which contains the two-particle distribution function $\rho^{(2)}(\mathbf{R})$ of the spheroids. In the very special case of

Función de correlación



límite diluido

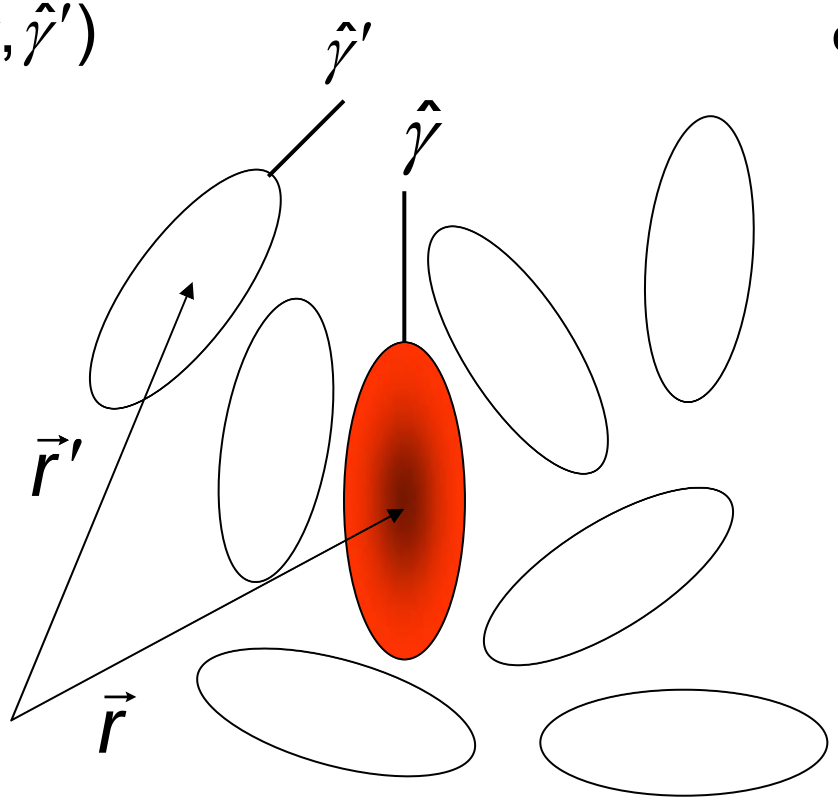


esferas anisotrópicas

Esferoides no alineados

$$\rho^{(2)}(\vec{r},\vec{r}\,';\hat{\gamma},\hat{\gamma}')$$

orden orientacional



$\int d\hat{\gamma} =$

$$\rho^{(2)}(\vec{r}-\vec{r}\,')=\frac{1}{\Omega^2}\int \rho^{(2)}(\vec{r}-\vec{r}\,';\hat{\gamma},\hat{\gamma}')\,d\hat{\gamma}\,d\hat{\gamma}'$$

$$\Omega=\int d\hat{\gamma}=\int d\hat{\gamma}'$$

PHYSICAL REVIEW B VOLUME 43, NUMBER 17 15 JUNE 1991-I

Optical properties of two-dimensional disordered systems on a substrate

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We calculate the dielectric response of free-standing and supported two-dimensional layers of polarizable entities, such as metallic particles or adsorbed molecules. We take into account dipole-dipole and the image interaction and investigate the effects of disorder within a two-dimensional renormalized polarizability theory. The behavior of the resonances arising from both the single particle's and the substrate's surface plasmon is studied.

I. INTRODUCTION

The macroscopic dielectric function of granular materials made up of a mixture of substances with different individual response functions depends on the morphology of the sample. The most simple effective-medium theory

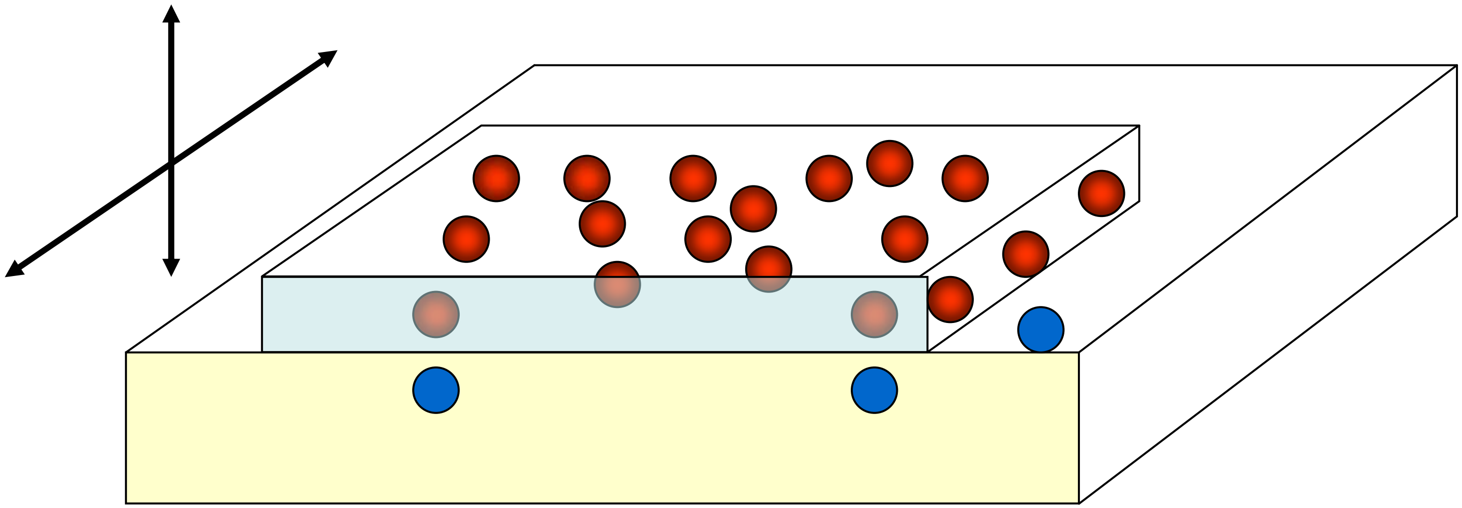
tigate the effects of disorder, taking into account the fluctuations in the dipole moments and the influence of the substrate for supported films.

There are many ways to approach the problem of the macroscopic response of granular materials in both two and three dimensions. The topology may be incorporated

¿cuál es la expresión equivalente a Maxwell Garnett en 2D

$$\chi_{eff} = \frac{n\alpha}{1 - \frac{n\alpha}{3}}$$

pues… no hay una expresión equivalente



en primer lugar…la respuesta es anisotrópica…

tween the entities. Within the dipolar approximation, the induced dipole $\mathbf{p}_i$ at R_i obeys

$$\mathbf{p}_i = \alpha(\omega)\left[\mathbf{E}^{\text{ext}} + \sum_j \mathbf{v}_{ij} \cdot \mathbf{p}_j\right], \tag{1}$$

where $\alpha(\omega)$ is the isotropic polarizability of each entity, $\mathbf{v}_{ij} = \mathbf{t}_{ij} + \mathbf{t}_{ij}^I \cdot \mathbf{M}$,

$$\mathbf{t}_{ij} = (1-\delta_{ij})\nabla_i \nabla_i (1/R_{ij}) \tag{2}$$

is the dipole-dipole interaction tensor in the quasistatic limit, with $\mathbf{R}_{ij} = \mathbf{R}_i - \mathbf{R}_j$,

$$t_{ij}^I = \nabla_i \nabla_i (1/R_{ij}^I) \tag{3}$$

is the corresponding dipole-image dipole interaction tensor with $\mathbf{R}_{ij}^I = \mathbf{R}_{ij} - 2d\hat{e}_z$ the vector from the image of the jth particle to the ith particle,

$$\mathbf{M} = A \operatorname{diag}(-1, -1, 1), \tag{4}$$

and $A = (\epsilon_s - 1)/(\epsilon_s + 1)$ is the strength of the image of a

Using the continuity of the normal component of the displacement field and the tangential component of the electric field, which allows us to identify $\mathbf{E}^{\mathrm{ext}}$ with the macroscopic fields E_x, E_y, and D_z, we have

$$P_x = \frac{1}{4\pi}(\epsilon_x - 1)E_x^{\mathrm{ext}} = \chi_x^{\mathrm{ext}} E_x^{\mathrm{ext}} , \tag{5a}$$

$$P_z = \frac{1}{4\pi}(1 - \epsilon_z^{-1})E_z^{\mathrm{ext}} = \chi_z^{\mathrm{ext}} E_z^{\mathrm{ext}} , \tag{5b}$$

B. Mean-field theory

The mean-field theory (MFT) is obtained by neglecting completely the contributions to the field due to the dipole fluctuations in Eq. (6), and yields

$$\epsilon_x^{\mathrm{MFT}}-1=\frac{2f\tilde{\alpha}}{1-\frac{1}{2}f\tilde{\alpha}(g+AG^I)}\ , \tag{13a}$$

$$1-(\epsilon_z^{\mathrm{MFT}})^{-1}=\frac{2f\tilde{\alpha}}{1+f\tilde{\alpha}(g-AG^I)}\ , \tag{13b}$$

where we identified the diameter $2a$ as the width of the layer,

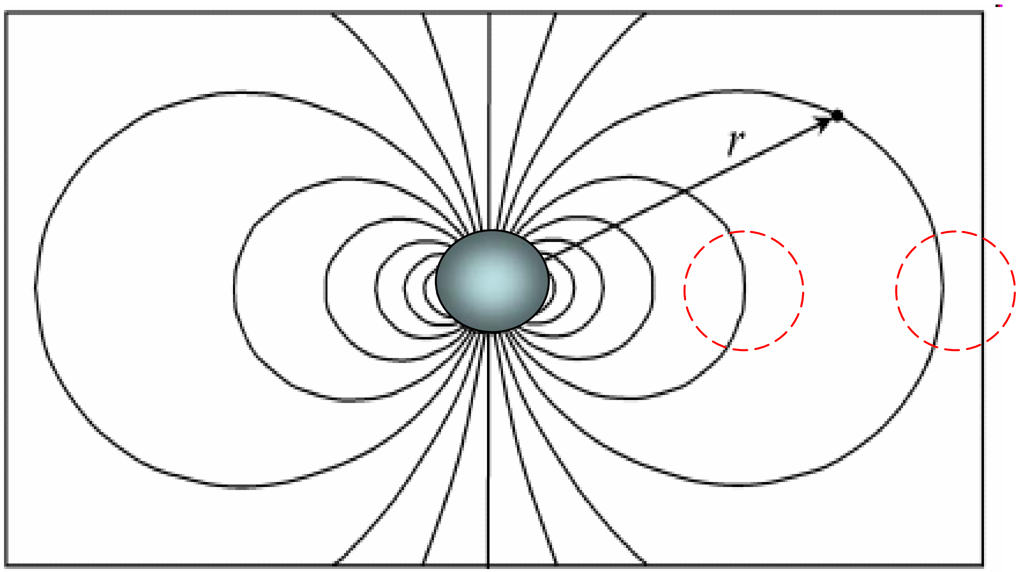
$$g\equiv\int_0^\infty\frac{\rho^{(2)}(2ax)}{x^2}dx\ , \tag{12a}$$

$$G^I\equiv\frac{1}{4fr^3}-g^I\ , \tag{12b}$$

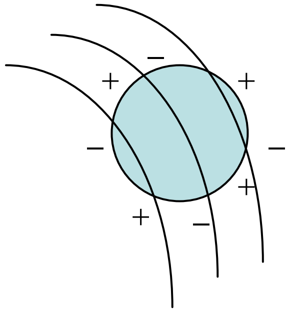
$$g^I\equiv\int_0^\infty\rho^{(2)}(2ax)\frac{x(x^2-2r^2)}{(x^2+4r^2)^{5/2}}dx\ , \tag{12c}$$

$$A=(\epsilon_s-1)/(\epsilon_s+1)$$

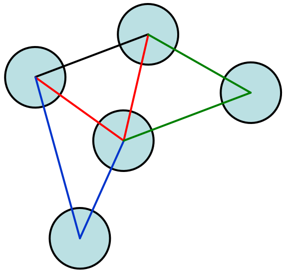
Area de avances



MULTIPOLOS



CORRELACIONES ESTADISTICAS



PHYSICAL REVIEW B VOLUME 38, NUMBER 8 15 SEPTEMBER 1988-I

Renormalized polarizability in the Maxwell Garnett theory

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(Received 10 July 1987; revised manuscript received 11 February 1988)

We develop a simple theory for the macroscopic dielectric function of a system of identical spheres embedded in a homogeneous matrix within the dipolar long-wavelength approximation. We obtained a relationship similar to the Clausius-Mossotti relation, but with a renormalized polarizability for the spheres instead of the bare polarizability. This renormalized polarizability obeys a second-order algebraic equation and it is given in terms of the bare polarizability, the volume fraction, and a functional of the two-particle correlation function of the spheres. We calculate the optical properties of metallic spheres within an insulating matrix and we compare our results with previous theories and with experiment.

$$\mathbf{P}_i = \alpha \left[\hat{\mathbf{q}} E^{\mathrm{ex}} / \epsilon_b + \sum_j \overleftrightarrow{\mathrm{T}}_{ij} \cdot \langle \mathbf{P} \rangle + \sum_j \overleftrightarrow{\mathrm{T}}_{ij} \cdot (\mathbf{P}_j - \langle \mathbf{P} \rangle) \right] \tag{9a}$$

$$\equiv \alpha \left[\mathbf{E}_i' + \sum_j \overleftrightarrow{\mathrm{T}}_{ij} \cdot \Delta \mathbf{P}_j \right] , \tag{9b}$$

$$\mathbf{P}_i = \overleftrightarrow{\alpha}_i^{\,*} \cdot \mathbf{E}_i' ,$$

$$\boxed{\frac{\epsilon_M - \epsilon_b}{\epsilon_M + 2\epsilon_b} = f \tilde{\alpha}^{\,*}}$$

$$\tfrac{1}{4} f_e \tilde{\alpha} (\tilde{\alpha}^{\,*})^2 - \tilde{\alpha}^{\,*} + \tilde{\alpha} = 0 ,$$

where we introduced the effective filling fraction

$$f_e = 3f \int_0^\infty \frac{\rho^{(2)}(2a_0 X)}{X^4} dX .$$

Resultados

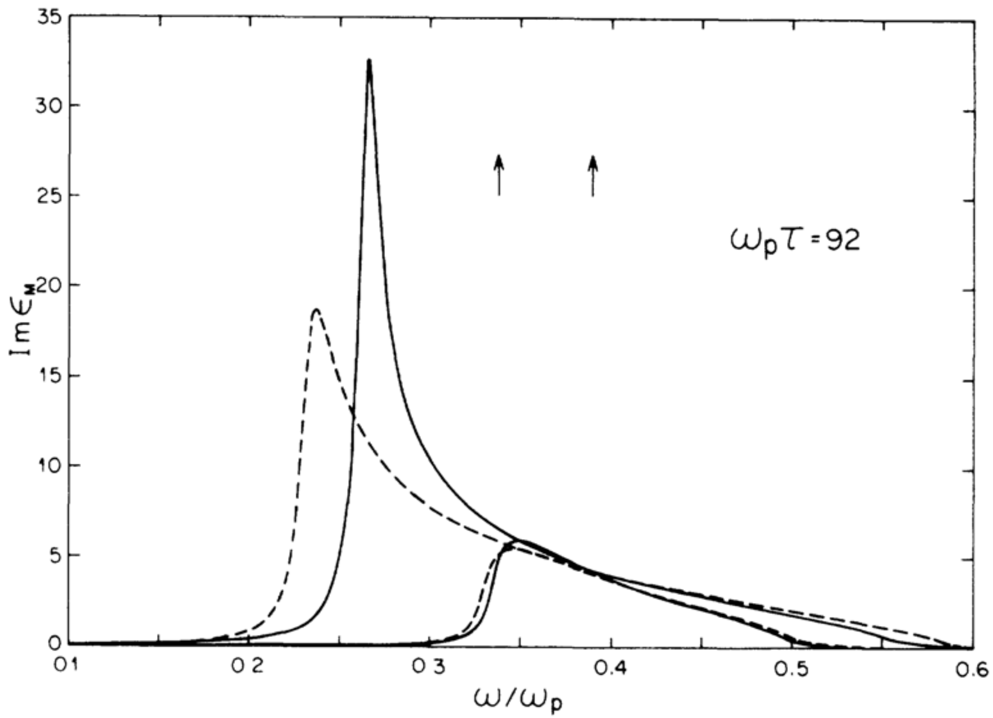

FIG. 3. Imaginary part of ϵ_M as a function of ω for Drude spheres in gelatin ($\epsilon_b = 2.37$) and two different volume fractions ($f = 0.1$ and 0.3). Here $\omega_p\tau = 92$. The solid (dashed) lines correspond to HC (PY) correlation function and the arrows indicate the position of the peaks of MGT. The curves for $f = 0.3$ are red shifted with respect to the ones for $f = 0.1$.

A new diagrammatic summation for the effective dielectric response of composites

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(Received 4 April 1991; accepted 11 October 1991)

We extended a previously developed diagrammatic formulation for the calculation of the effective dielectric response of composites prepared as a random, homogeneous, and isotropic distribution of small spherical inclusions in an otherwise homogeneous matrix. This is done within the long-wavelength, dipolar approximation in the low-density regime of inclusions. We propose a new diagrammatic summation and we compare our results with two recently reported computer simulations.

J. Chem. Phys. **96** (2), 15 January 1992

$$\epsilon_M(\omega) = \frac{1+2f\tilde{\alpha}\xi}{1-f\tilde{\alpha}\xi},$$

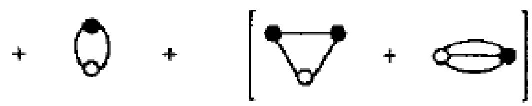
$$n\alpha\xi = \chi^{L,l}(q\to 0,\omega) = \lim_{q\to 0}\left\langle \sum_j (\mathbf{V}^{-1})^{l}_{ij} \right\rangle.$$

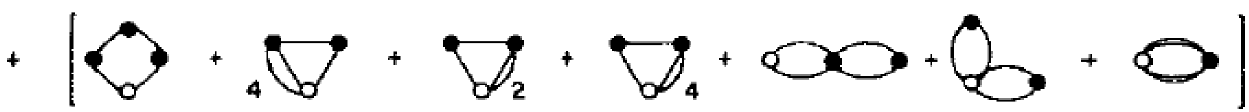
$$\sum_j (\mathbf{V}^{-1})_{ij} = 1 + \alpha \sum_j \Delta\mathbf{T}_{ij} + \alpha^2 \sum_{jk} \Delta\mathbf{T}_{ij}\cdot\Delta\mathbf{T}_{jk} + \cdots.$$

$$\equiv n^2\alpha^3 \lim_{q\to 0} \int \hat{q}\cdot\mathbf{T}_{12}\cdot\mathbf{T}_{23}\cdot\mathbf{T}_{31}\cdot\hat{q}$$

$$\times \rho^{(2)}(R_{12})\rho^{(2)}(R_{23})\, d^3R_2\; d^3R_3 .$$

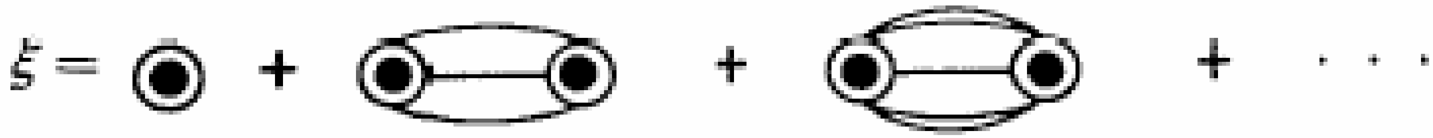
$$\xi = \sum_r \sum_s I(r,s) \equiv o \; +$$





$$+ \cdots, \qquad (11)$$

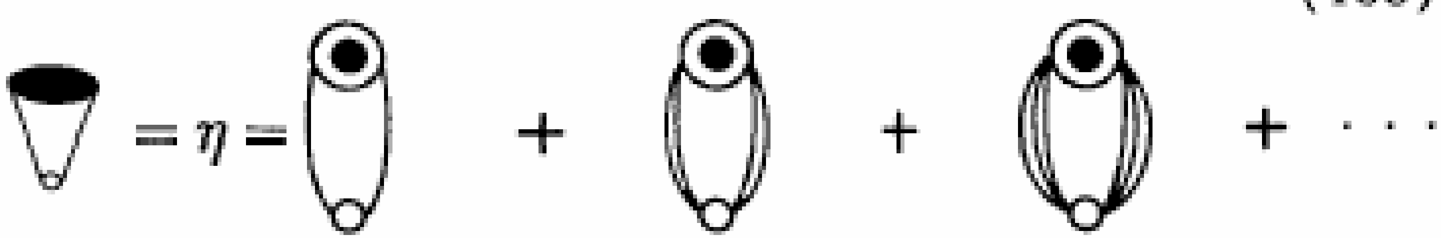
Suma de procesos de polarización elementales



(18a)

where the renormalized vertex $\odot \equiv \Delta$ is given by the self-consistent solution of the following diagrammatic equations:

(18b)



(18c)

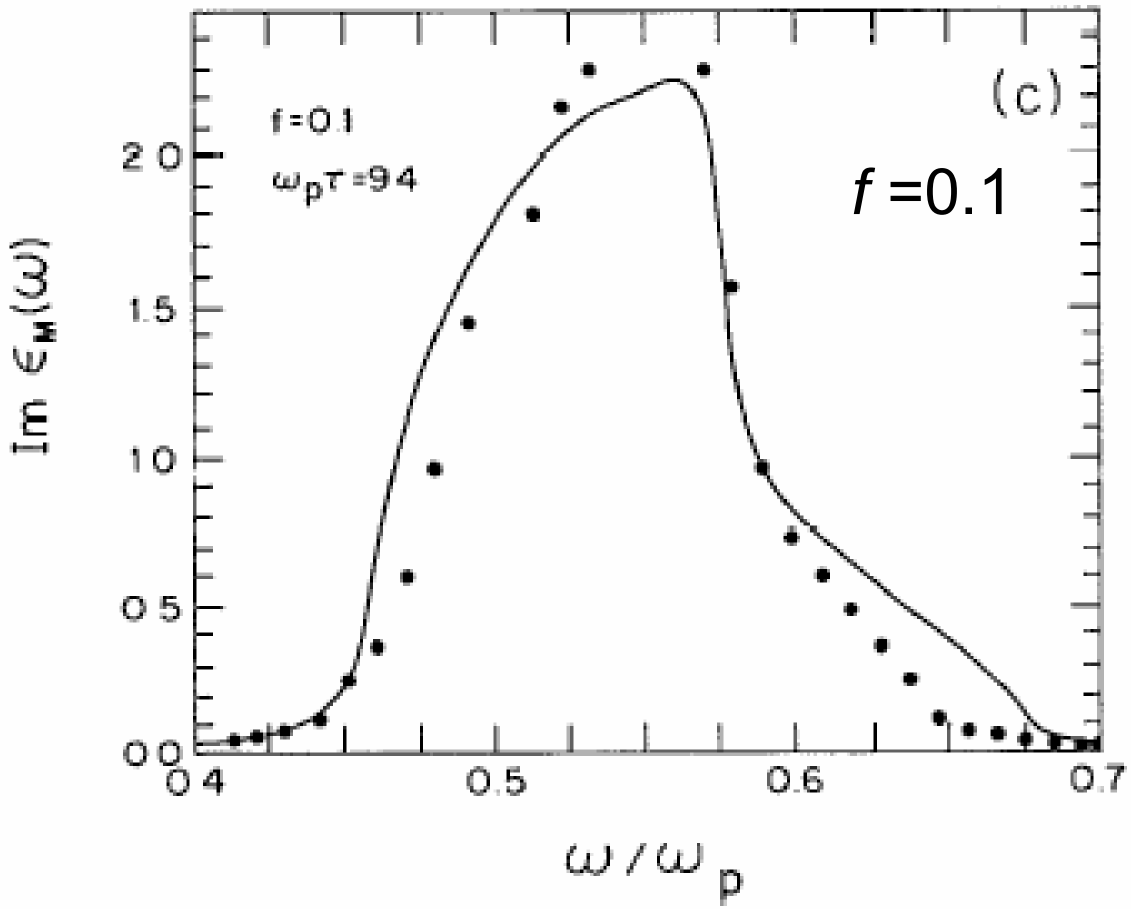
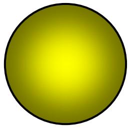


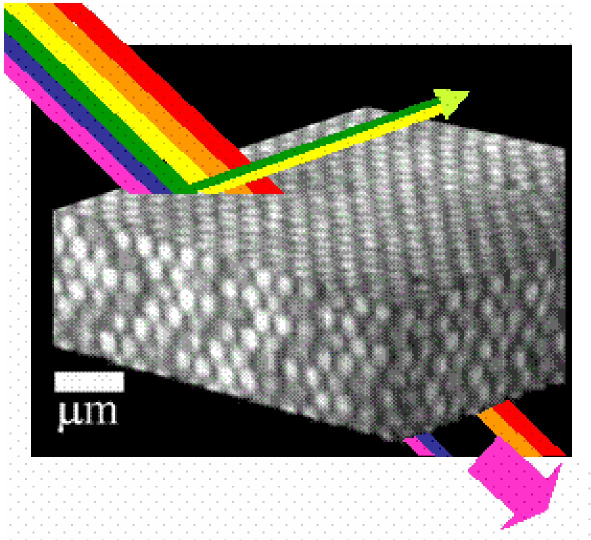
FIG. 2. Im ϵ_M as a function of ω/ω_p for filling fractions 0.01 (a), 0.03 (b), and 0.1 (c). The solid line corresponds to Eq. (8a) with ξ given by Eq. (25) and the dots are the results of the computer simulation of Ref. 11.



Drude

S. Kumar and R.I. Cukier, J. Phys. Chem, 93, 4334 (1989)

$$ka \sim 1$$



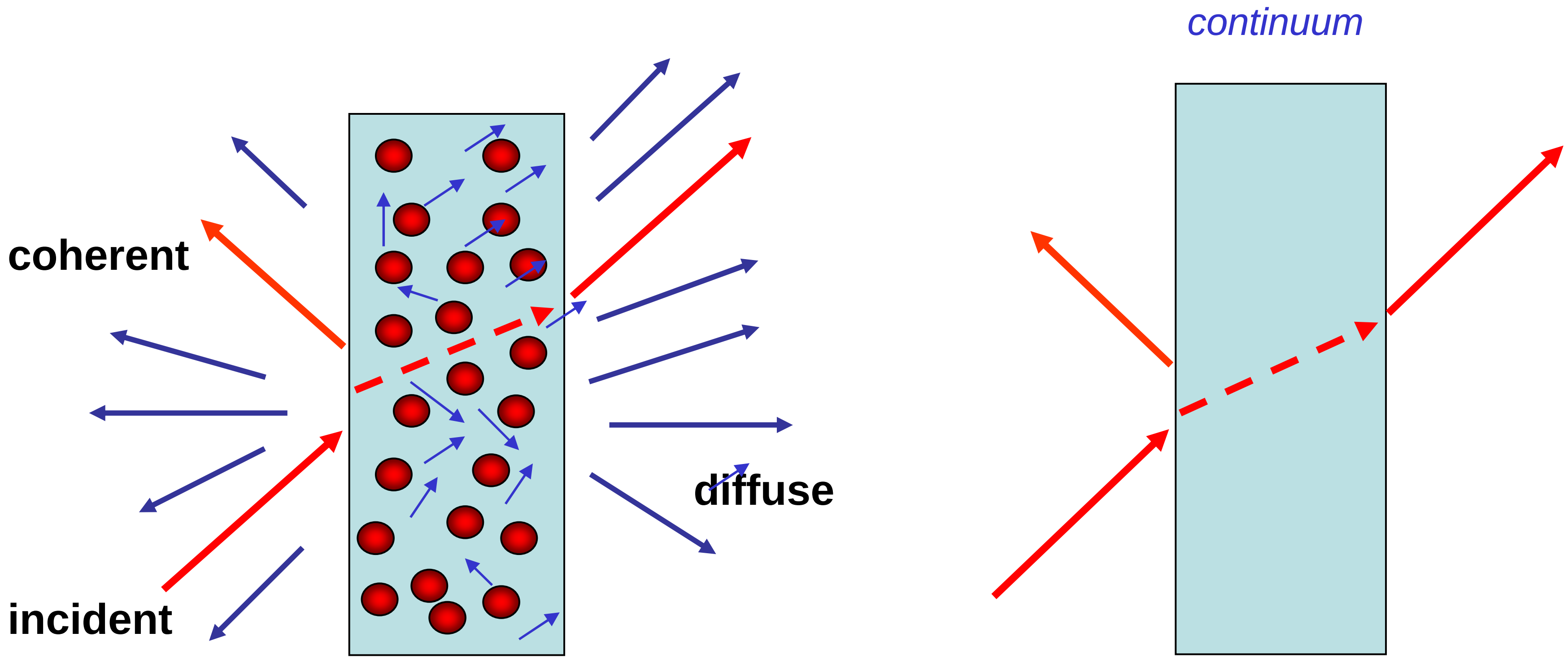
turbidity

diffraction

$$\left\langle \vec{S} \right\rangle_{diffuse} \sim \left\langle \vec{S} \right\rangle_{coh}$$

Effective medium

Is there an effective medium?

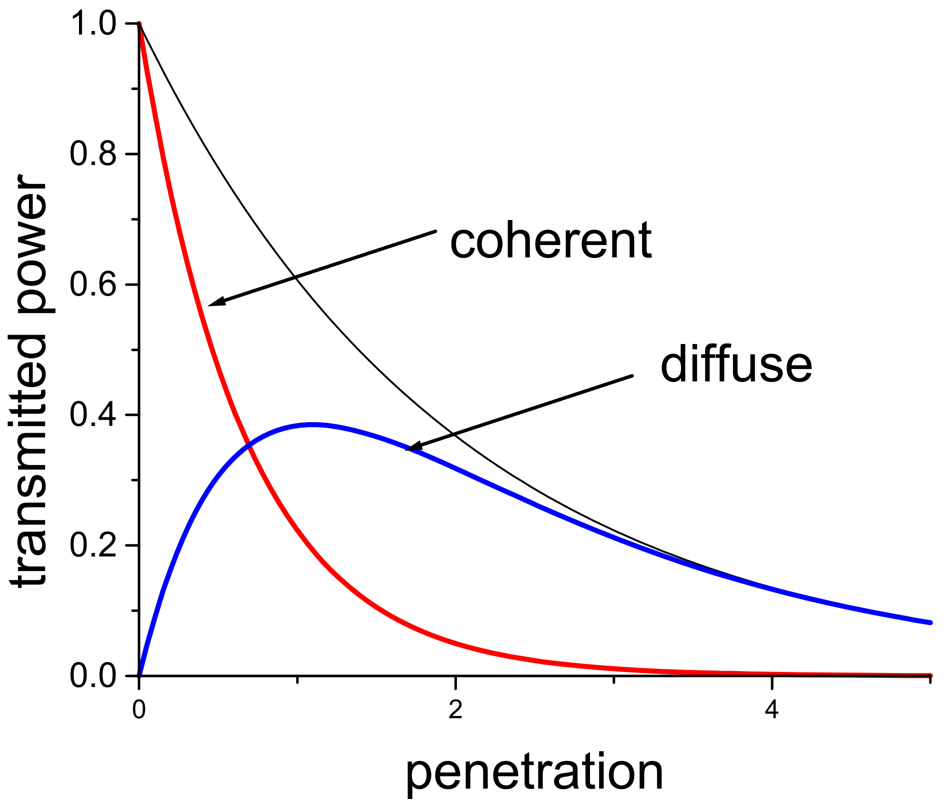


If there is one, it should be …. **for the coherent beam**

If there is one, the theory should be… **incomplete**

Energy conservation

$$Power \propto |E|^2 = |E_{AV}|^2 + |E_{fluc}|^2$$



effective properties… coherent beam… scattering… as... dissipation

Effective index of refraction

first attempts

van de Hulst

dilute limit

Light scattering by small particles (1957)

complex

$$n_{eff} = 1 + \underbrace{i\,\gamma\,S(0)}_{\delta n_{eff}}$$

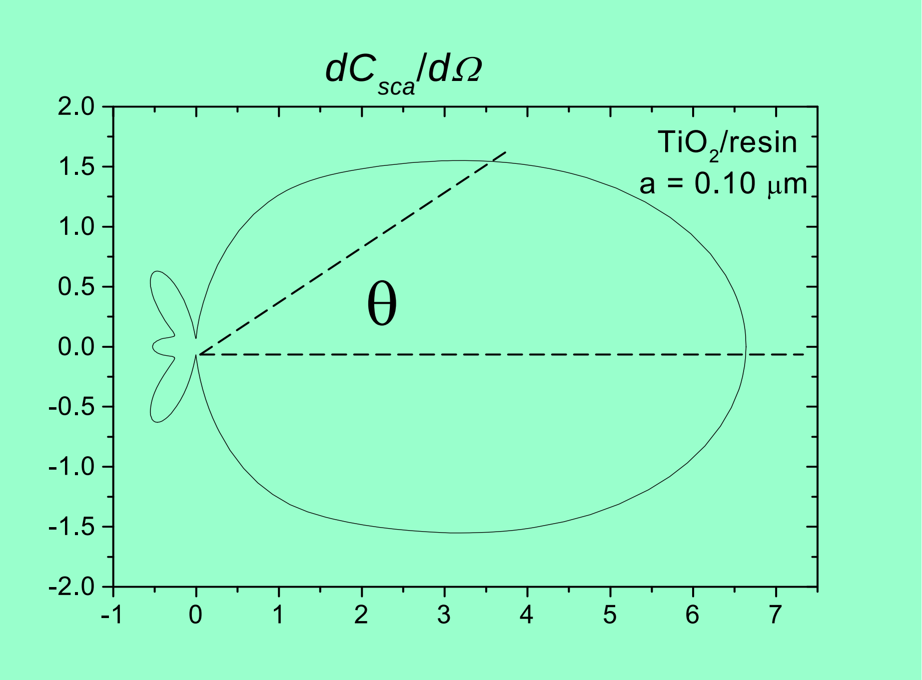
$$\gamma = \frac{3}{2}\frac{f}{(k_0 a)^3} \qquad \gamma \ll 1$$

sphere

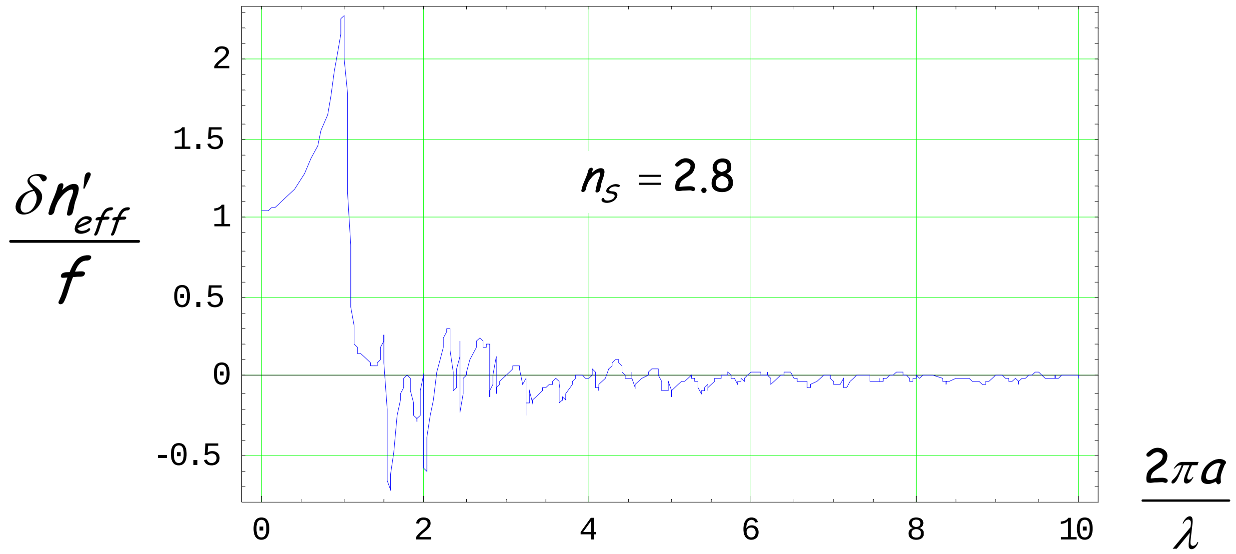
$$\begin{pmatrix} E_{\parallel}^{s} \\ E_{\perp}^{s} \end{pmatrix} = \frac{e^{ikr}}{-ikr} \begin{pmatrix} S_2(\theta) & 0 \\ 0 & S_1(\theta) \end{pmatrix} \begin{pmatrix} E_{\parallel}^{inc} \\ E_{\perp}^{inc} \end{pmatrix}$$

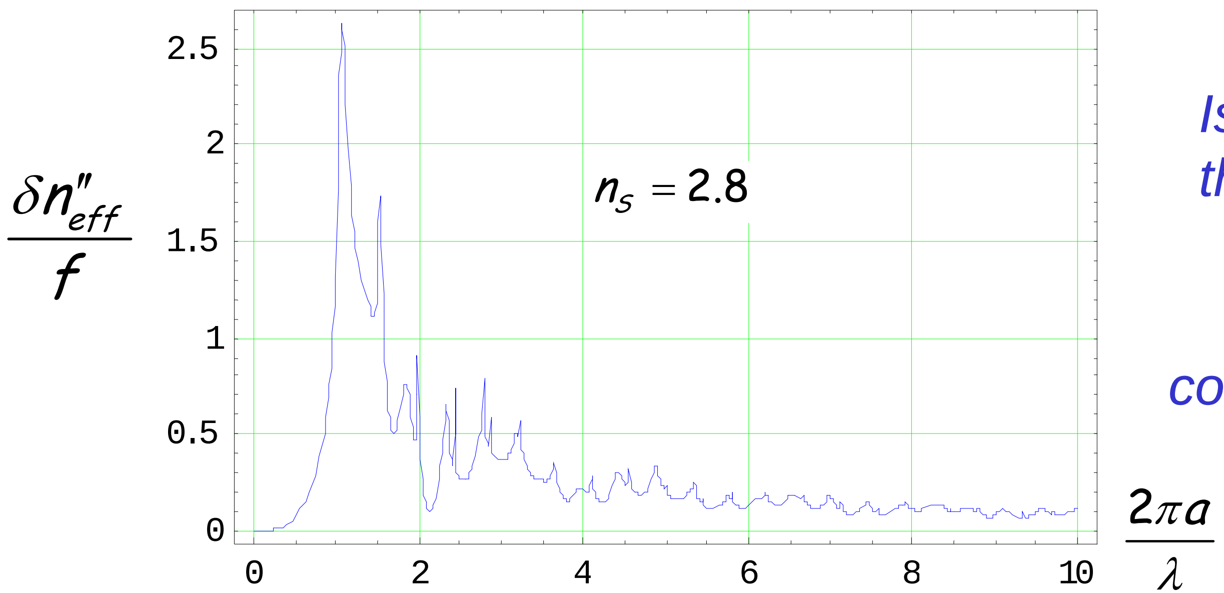
MIE

$$S_1(0) = S_2(0) = S(0)$$



Effective index of refraction





Van de Hulst

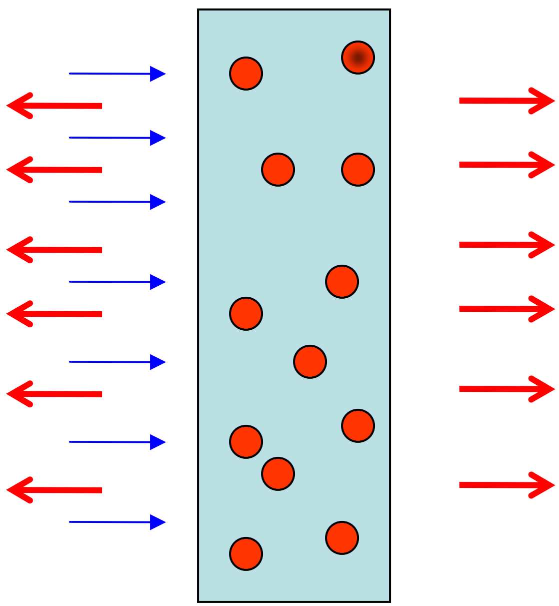
$$\delta n_{eff} = i \frac{3}{2} \frac{S(0)}{(k_0 a)^3} f$$

*Is this effective-medium theory **unrestricted**?*

compare with experiment

Craig Bohren

J. Atmos Sci. 43, 468 (85)



transmission $n_{eff} = 1 + i\gamma S(0)$

reflection $n_{eff} = 1 + i\gamma S_1(\pi)$

Proposition

$$\varepsilon_{eff} = 1 + i\gamma\left[S(0) + S_1(\pi)\right]$$

$$\mu_{eff} = 1 + i\gamma\left[S(0) - S_1(\pi)\right]$$

MAGNETIC ?

$$r = \frac{\sqrt{\mu} - \sqrt{\varepsilon}}{\sqrt{\mu} + \sqrt{\varepsilon}}$$

...It might be expected that a composite medium is nonmagnetic if its components are, but this is not correct... which was recognized as long ago as 1909 by Gans and Happel...

Our new result

IN COLLOIDAL SYSTEMS WITH BIG COLLOIDAL PARTICLES THE EFFECTIVE MEDIUM ***EXISTS*** *BUT IT IS* ***NONLOCAL***

ELECTROMAGNETIC RESPONSE

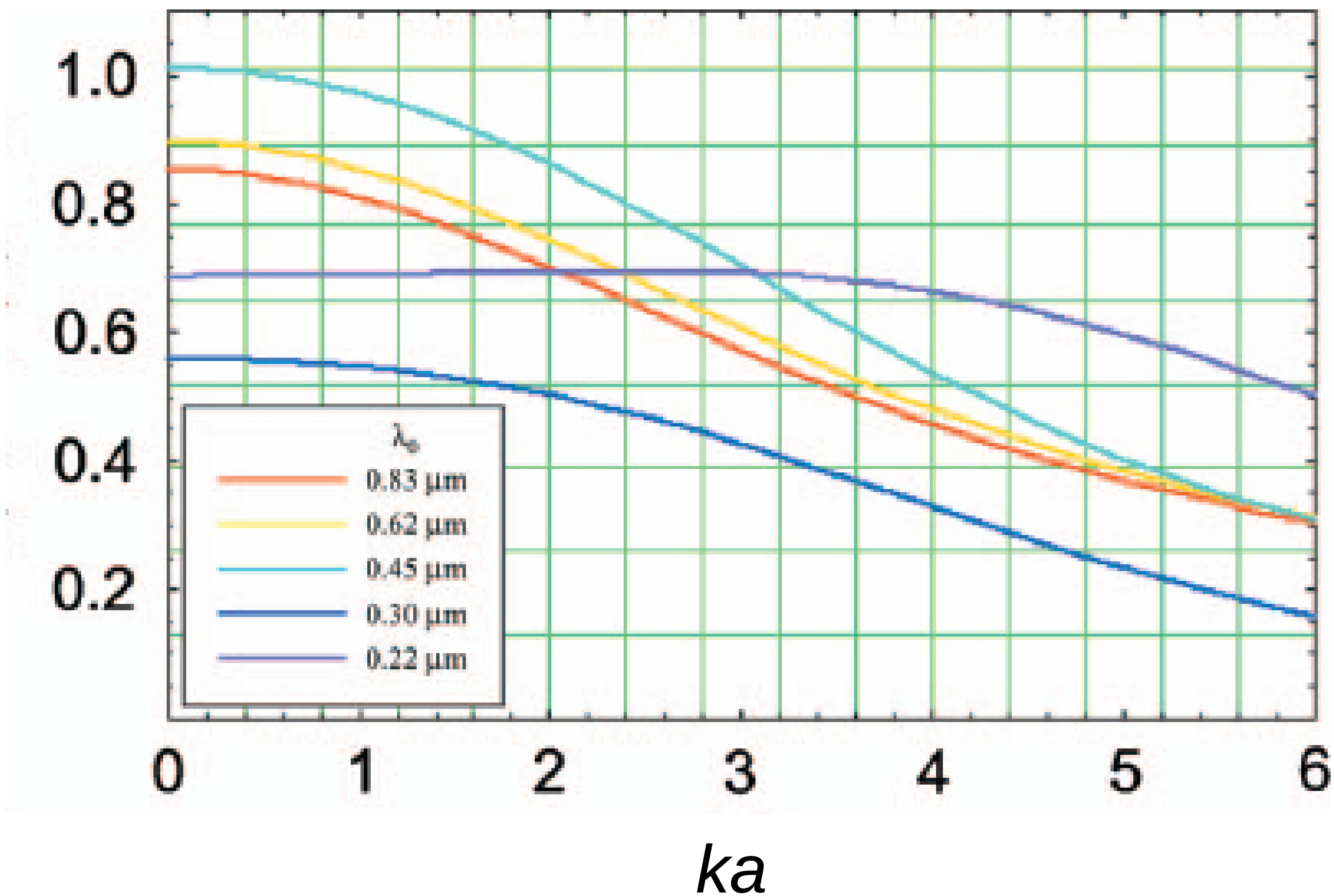
GENERALIZED EFFECTIVE CONDUCTIVITY

$$\left\langle \vec{J}_{ind} \right\rangle = \hat{\sigma}_{eff} \left\langle \vec{E} \right\rangle$$

TOTAL

LINEAR OPERATOR

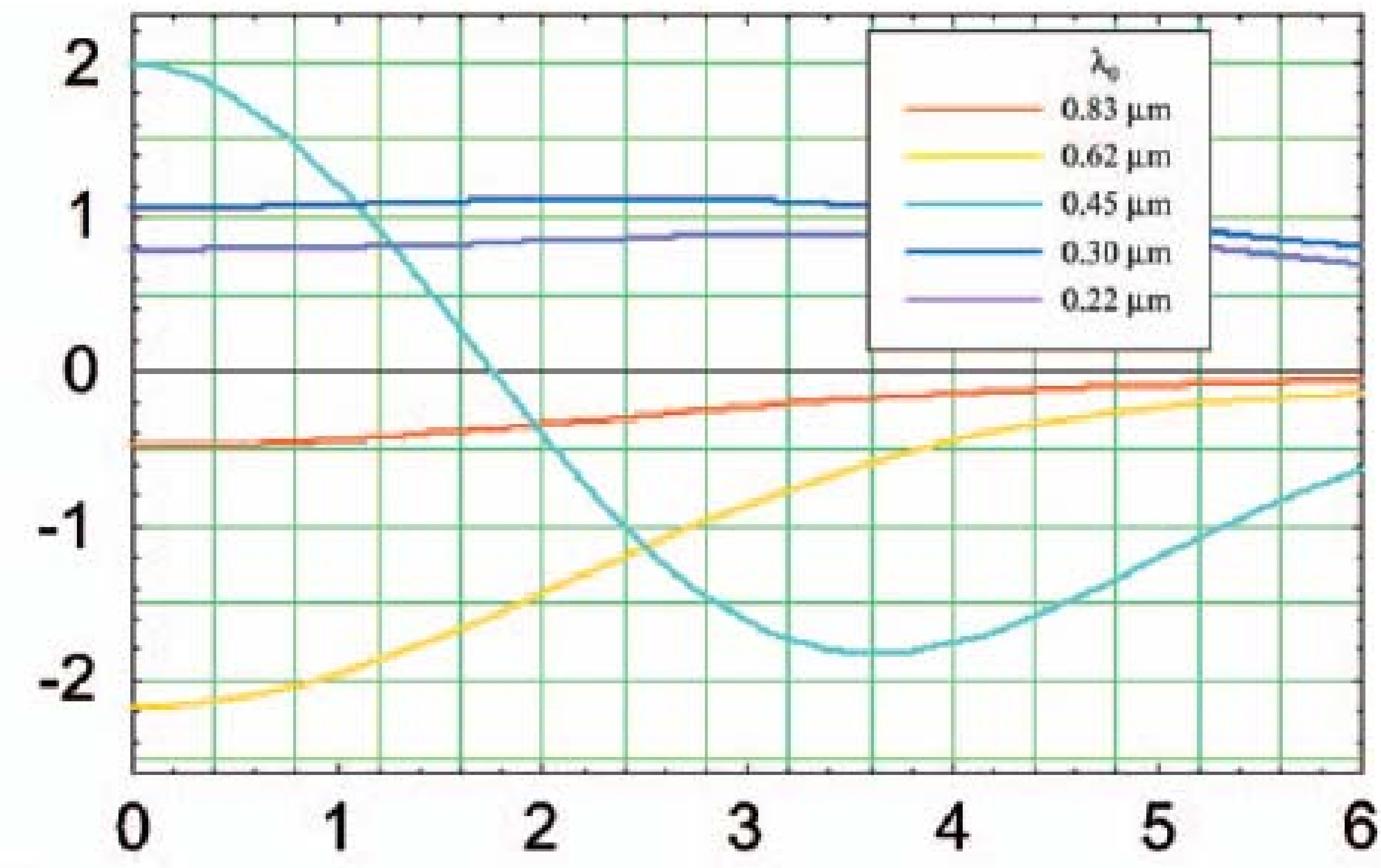
$$\mathrm{Re}\left[\frac{1}{\tilde{\mu}(k,\omega)}-1\right]\frac{1}{f}$$ spheres Ag / vacuum $a = 0.1\,\mu m$



$$\mathrm{Re}\left[\frac{1}{\tilde{\mu}(k,\omega)}-1\right]\frac{1}{f}$$

spheres TiO_2 / vacuum

$a = 0.1\,\mu m$



ka


Ven y Conoce
Plata & Oro
coloidal
y sus productos maravillosos
Ven y Conoce


Blau
Rot
Grün
20 ml