ON THE VAN DER WAALS ENERGY OF TWO HALF-SPACES AT SMALL SEPARATIONS

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It is shown that the introduction of a cut-off wavenumber corresponding to electron–hole excitations leads to a 10% reduction of the Lifshitz van der Waals attraction of low density metallic half-spaces at small separations.

In order to explain experiments on the adhesion of solid particles in terms of van der Waals forces, the Lifshitz formula [1] has been assumed to be valid for extremely small separations [2] (about 4 Å). In this range, however, the lattice structure and the overlap of the electrons are expected to become important. The influence of these effects, in a most qualitative way, can be incorporated into the theory of the non-retarded van der Waals forces, as derived, e.g., by van Kampen et al. [3] by introducting a cut-off wavenumber K_c into the surface excitation spectrum. For wavenumbers greater than K_c the surface excitations decay into electron–hole pairs. As is shown below the cut-off leads to a modification of the $1/d^2$ law at small separations. This indicates that in this range phenomena different from surface excitations, presumably direct overlap [4], are of non-negligible influence. (Effects of the lattice structure and size effects [5] are not considered in the present paper.) The quantity K_c, unfortunately, is not as well defined as for bulk plasmons, except for the case $Ka \ll 1$, where K is the surface plasmon wavenumber and a is the thickness of the surface profile [6].

Now, following the simple treatment of van Kampen [3, 7] we obtain for the van der Waals energy U of two metallic half-spaces separated by a gap of width d

$$U(d) = \tfrac{1}{2}\hbar \sum_{K}^{K_c} \{ [\omega_K^+(d) - 2^{-1/2}\omega_p] + [\omega_K^-(d) - 2^{-1/2}\omega_p] \}, \tag{1}$$

with

$$\omega^\pm = 2^{-1/2}\omega_p (1 \pm e^{-Kd})^{1/2}, \tag{2}$$

where ω_p is the plasma frequency.

Insertion of (2) into (1) leads to

$$U(d) = -\frac{\hbar\omega_p}{4\pi\sqrt{2}d^2} \times \int_0^{K_c d} dx\, x [2 - (1+e^{-x})^{1/2} - (1-e^{-x})^{1/2}], \tag{3}$$

which has the following limits

$$U(d \gg K_c^{-1}) = -\frac{\hbar\omega_p}{64\pi\sqrt{2}d^2} [1.1 - (1+2K_c d)\, e^{-2K_c d}] \tag{4}$$

$$U(d \ll K_c^{-1}) = -\frac{\hbar\omega_p}{4\pi\sqrt{2}} [\tfrac{1}{2}K_c^2(2-\sqrt{2}) - \tfrac{2}{5}K_c^{5/2} d^{1/2}]. \tag{5}$$

Eq. (4) is obtained by expanding the integrand of (3) with respect to e^{-x}. The error appearing for small values of x is approximately compensated by the fac-

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tor x in front of the bracket.

For K_c we suggest an expression which has been used in the calculation of surface energies [8]

$$K_c = \frac{\omega_p}{v_F\sqrt{2}} = \left(\frac{2}{3\pi^2}\right)^{1/6} \frac{1}{a_B\sqrt{r_s}} , \tag{6}$$

where v_F is the Fermi velocity, a_B is Bohr's radius and r_s is defined by $n^{-1} = 4\pi(a_B r_s)^3/3$. Using this value for K_c in eq. (4) leads to a reduction of Lifshitz's results [1] which was derived for $K_c \to \infty$. In the range of separations d studied in ref. [2] this reduction is about 10% for low density metals ($r_s \approx 6$) and about 1% for high density metals ($r_s \approx 2$).

The limit (5) has little physical relevance for our problem. Nevertheless, it shows that the introduction of a finite K_c leads to a value of $U(d)$ that remains finite for $d \to 0$ [8].

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