''Superluminal'' transmission of light pulses through optically opaque barriers

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Using simple considerations of causal electrodynamics we analyze the occurrence of superluminal transmission of light pulses through optically opaque barriers. We find that the phenomenon appears whenever the main frequency components of the pulse are confined to frequency regions where the presence of the barrier decreases the density of states of the electromagnetic modes of the system. We also show that these frequency regions correspond to the transmission gaps of sufficiently wide barriers. We discuss a simple theory for the density of states of the barrier system and compare the results of such a theory with exact numerical calculations.

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Despite the lack of consensus as to the proper definition and physical interpretation of ''superluminal'' propagation [1–4] and tunneling times [5,6], there is already a host of experiments and proposals [2,7–14] that address those issues and whose understanding deserves full attention. In the context of the present report we recall, in particular, the experiments with light pulses traveling across a dielectric barrier [7] or across media with anomalous dispersion near an absorption or gain line [2] or between two gain lines [13,14]. In all these experiments it is shown that the coincidental arrival of a pair of pulses, one that crossed the medium and one traveling unimpeded, occurred as if the transmitted pulse had crossed the medium at speeds greater than the speed of light in vacuum; and this is what we will address as superluminal transmission. But this apparently paradoxical result may be explained by arguing that the detection of the coincidental arrival refers to the coincidence in arrival of the *peaks* of the pulses, and therefore there is no causal connection between them [3]. Here we reexamine this argument by first pointing out that what is common to all these experiments is the tunneling of a light pulse through an optically opaque barrier, and then we analyze this phenomenon within the framework of *causal* classical electrodynamics. By an optically opaque barrier we mean any arrangement of optical components that produces, in the frequency domain, gaps in the transmission amplitudes. In these arrangements, a gap can be defined as the frequency region in which the normal modes of the corresponding boundless barrier become evanescent in the dissipationless limit, or, more precisely, the frequency region in which the density of states of the electromagnetic modes in the corresponding boundless system vanishes. According to this definition, a gap can be found only in lossless materials, since for dissipative materials the propagation is never truly evanescent and the density of states cannot be properly defined. However, in real materials one could still identify frequency regions that will become gaps through a proper analysis of the dissipationless limit. We illustrate our analysis by considering plane-wave pulses traveling perpendicular to the interface of barriers consisting of slabs of a single material or dielectric barriers made of an alternating array of layered materials.

There are several articles that study this same transmission or tunneling phenomenon along similar [15–18] and other [19–21] lines, and it has already been established that, within the stationary-phase approximation, the time delay between the peak of the tunneling pulse and the unimpeded one is given by the phase time. The phase time is defined as the frequency derivative of the phase of the transmission amplitude and the stationary-phase approximation demands only a slight distortion in the shape of the pulse due to the tunneling process. Superluminal transmission occurs whenever the phase times become *negative* and the main frequency components of the incident pulse lie within the gap [2,7,15,18]. Our objective here is to establish the conditions under which these phase times turn out to be negative. Our analysis is performed within the framework of classical causal electrodynamics and we find that superluminal transmission will be possible whenever the presence of a barrier causes a decrease in the density of states of the electromagnetic modes of the system in comparison with the density of states in the absence of the barrier. Furthermore, we show that this decrease in the density of states always occurs for thick enough lossless barriers and that the corresponding phase times, besides being negative, are proportional to their width, whenever this width is not too large. This last statement is, essentially, the electromagnetic version of the well-known Hartman effect [5].

We start by defining the two models that will be used in our work and locate the frequency regions corresponding to the gaps. (a) The first is a slab of length d made of a medium with an index of refraction $n(\omega)$ having a single Lorentzian resonance [22], that is, $n(\omega)=\sqrt{1+\omega_p^2/(\omega_0^2-\omega^2-i\gamma\omega)}$, where ω_p is a model parameter with units of frequency, ω_0 is the resonance frequency, and γ is the damping parameter related to energy dissipation. (b) The second is a multilayer of alternating media with (real) high and low indices of refraction n_1 and n_2 and with equal widths $d/2$. This is the system experimentally analyzed by Spielmann *et al.* [7].

According to our definition, the gaps are frequency regions in which the normal modes of the corresponding boundless system become evanescent in the disipationless limit. In model (a) this happens in the frequency window $\omega_0 \leqslant \omega \leqslant \sqrt{\omega_p^2+\omega_0^2}$, which defines the gap. This model has been extensively analyzed in the literature [15,17,16,21], and its similarity with nonrelativistic quantum tunneling through a barrier [6] has also been thoroughly discussed. In the case of model (b) the corresponding boundless system is a periodic superlattice with the period given by the total length d of the two layers with indices of refraction n_1 and n_2. For simplicity we will assume n_1 and n_2 to be real. In this case, the dispersion relation of the normal modes of the system, that is, the relation between their Bloch wave vector κ and the frequency ω, is given by [18,23]

$$\cos \kappa d = \cos(k_1 d/2)\cos(k_2 d/2) - \frac{1}{2}\left(\frac{n_1}{n_2}+\frac{n_2}{n_1}\right)\sin(k_1 d/2)\sin(k_2 d/2), \qquad (1)$$

where $k_1 = n_1\omega/c$, $k_2 = n_2\omega/c$, and $d/2$ is the width of each layer. Bloch evanescent modes will appear whenever the frequency is such that κ becomes purely imaginary. In these frequency regions, also known as photonic band gaps, there is no energy transport, and there is, in general, an infinite number of gaps.

According to our definition, the gaps are also defined as the frequency regions where the density of states vanishes. In the case of a boundless system the density of states $N(\omega)$ of the electromagnetic modes is given by

$$\pi N(\omega) = L\left|\frac{d\omega}{d\kappa}\right|^{-1}, \qquad (2)$$

where $L(\to\infty)$ denotes the size of the system and the wave vector κ is real. When κ is purely imaginary one gets $N(\omega)=0$, in agreement with our definition of a gap. One also sees that the density of states scales with the size of the system. For model (a) one has $\kappa = q$ and $N(\omega)$ for the boundless system becomes $\pi N(\omega) = L(n' + \omega\, dn'/d\omega)/c$. Also, for the infinite superlattice, one combines Eqs. (1) and (2) to determine the density of states, which also scales as the size L of the system. One can check that $N(\omega)$ vanishes within the gaps and it has a divergent monotonic increase as both edges of the gap are approached from outside. This divergent behavior is associated with the vanishing of the group velocity $d\omega/d\kappa$.

For one-dimensional systems with barriers of finite width the calculation of the density of states is not as immediate. Nevertheless, Avishai and Band [24], using the S-matrix formalism developed by Dashen, Ma, and Bernstein [25], found a relationship between the phase time τ_ϕ, defined as $\tau_\phi(\omega) \equiv d\phi_t(\omega)/d\omega$, and the density of states $N(\omega)$. They found

$$\tau_\phi(\omega) = \pi[N(\omega) - N_0(\omega)], \qquad (3)$$

where ϕ_t is the phase of the transmission amplitude of the barrier, $N(\omega)$ is the density of states of the system in the presence of the barrier, and $N_0(\omega)$ is the density of states of the system in the absence of the barrier. In other words, τ_ϕ is proportional to the *change* in the density of states due to the presence of the barrier. If the barrier is of finite width and located in vacuum, then according to Eq. (2) $\pi N_0(\omega) = L/c$. Therefore, the sign of τ_ϕ will be determined by the difference between $N(\omega)$ and L/c. It will be negative in frequency regions where $N(\omega) < L/c$, and it will vanish when $N(\omega) = L/c$. Now, one expects $N(\omega)$ to scale also with L in such a way that the difference $N(\omega) - L/c$ will depend only on the width d of the barrier, and will remain finite in the limit $L\to\infty$. For example, in the crudest possible approximation, one might assume that the density of states of a finite system of width d is also given by Eq. (2), where L should be now replaced by d. In this case one can write $N(\omega)$ as a simple superposition of the densities of states in the different regions of space, that is, $\pi N(\omega) = (L-d)/c + d/(d\omega/d\kappa)$ for frequencies outside the gaps, and $\pi N(\omega) = (L-d)/c$ for frequencies within the gaps. Combining now Eq. (2) and $\pi N_0(\omega) = L/c$, one gets that the phase time τ_ϕ can be written, for frequencies *outside* the gaps, as

$$\tau_\phi(d,\omega) = -\frac{d}{c} + d\left|\frac{d\omega}{d\kappa}\right|^{-1}, \qquad (4)$$

and, for frequencies *within* the gaps, as

$$\tau_\phi(d,\omega) = -\frac{d}{c}. \qquad (5)$$

In general, in the regions of normal dispersion outside the gaps, one has $d\omega/d\kappa < c$, and thus this crude approximation leads one to the interesting conclusion that τ_ϕ should always be positive outside the gaps and always negative within. Nevertheless, one might expect this conclusion to hold only for wide barriers, since in this case the superposition procedure should be a better approximation.

To check the validity of these ideas we now proceed to the exact calculation of τ_ϕ, for our model systems, using causal electrodynamics. First, we write the phase ϕ_t of the transmission amplitude as $\phi_t(d,\omega) = -\omega d/c + \alpha(d,\omega)$. Then the phase time $\tau_\phi = d\phi_t/d\omega$ can be written as

$$\tau_\phi(d,\omega) = -\frac{d}{c} + \frac{d\alpha(d,\omega)}{d\omega}. \qquad (6)$$

One can see that this expression is similar to Eqs. (4) and (5); thus the superposition procedure described above amounts to taking

$$\frac{d\alpha(d,\omega)}{d\omega} \approx d\left|\frac{d\omega}{d\kappa}\right|^{-1} \qquad (7)$$

for frequencies outside the gaps, and

$$\frac{d\alpha(d,\omega)}{d\omega} \approx 0 \qquad (8)$$

for frequencies within the gaps. This last equation simply means that for wide barriers $\alpha(d,\omega)$ should be a slowly varying function of ω.

Simple closed-form expressions for $\alpha(d,\omega)$ can be obtained for model (a) in the dissipationless limit. In this limit, the complex index of refraction $n(\omega) = n'(\omega) + in''(\omega)$ becomes purely real outside the gap and purely imaginary within the gap. Thus in the regions outside the gap one can write $n(\omega) = n'(\omega)$ and $\alpha(d,\omega)$ will be given by

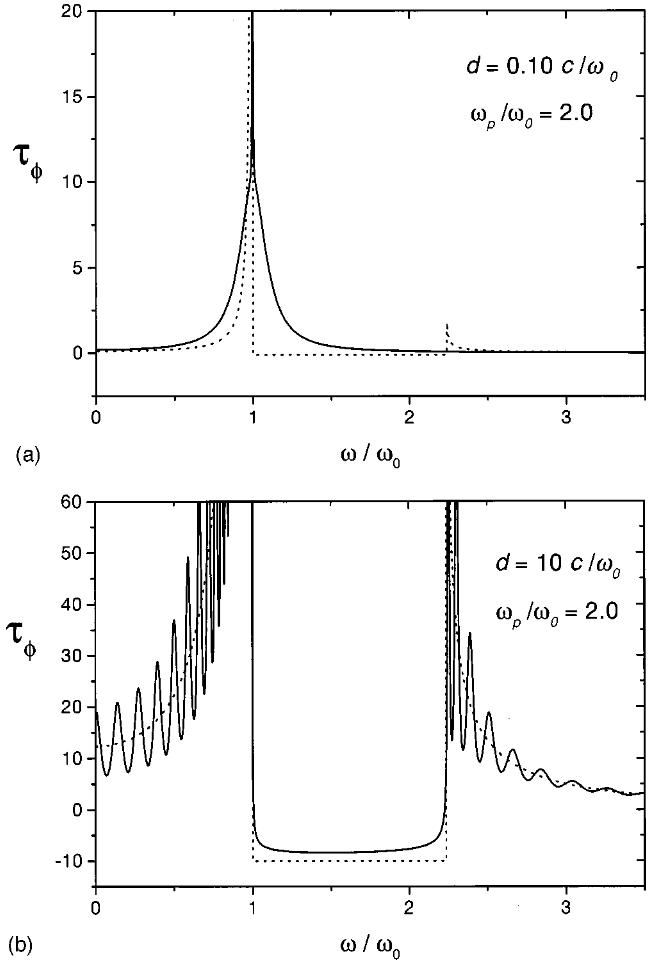

FIG. 1. Transmission phase time $\tau_\phi = d\phi_t/d\omega$ as a function of frequency, for model (a), using $\omega_p/\omega_0 = 2$ and $\gamma/\omega_0 = 0.01$, for barriers of width (a) $d = 0.1c/\omega_0$ and (b) $d = 10.0c/\omega_0$. The dotted lines correspond to the superposition approximation, Eqs. (4) and (5), and the solid lines correspond to the exact calculation of the transmission amplitude.

$$\alpha(d,\omega) = \tan^{-1}\left\{\left(\frac{1+n'(\omega)^2}{2n'(\omega)}\right)\tan[\omega n'(\omega)d/c]\right\}, \quad (9)$$

while in the region within the gap one writes $n(\omega) = in''(\omega)$, and $\alpha(d,\omega)$ will be correspondingly given by

$$\alpha(d,\omega) = \tan^{-1}\left\{\left(\frac{1-n''(\omega)^2}{2n''(\omega)}\right)\tanh[\omega n''(\omega)d/c]\right\}. \quad (10)$$

Equation (9) may be used to check that in the frequency regions outside the gaps τ_ϕ is always positive. Also, one can use Eq. (10) to examine the behavior of τ_ϕ for frequencies within the gap, recalling that causality implies $n''(\omega) \geqslant 0$ [22]. From this equation it is immediately clear that, as d grows, $\alpha(\omega,d)$ becomes a function independent of d, and thus for sufficiently wide barriers τ_ϕ becomes negative. Furthermore, as d is increased, Eq. (9) displays also the fact that $\alpha(\omega,d)$ becomes a slowly varying function of ω, in agreement with Eq. (8). All these results support the general conclusion derived from the superposition procedure mentioned above.

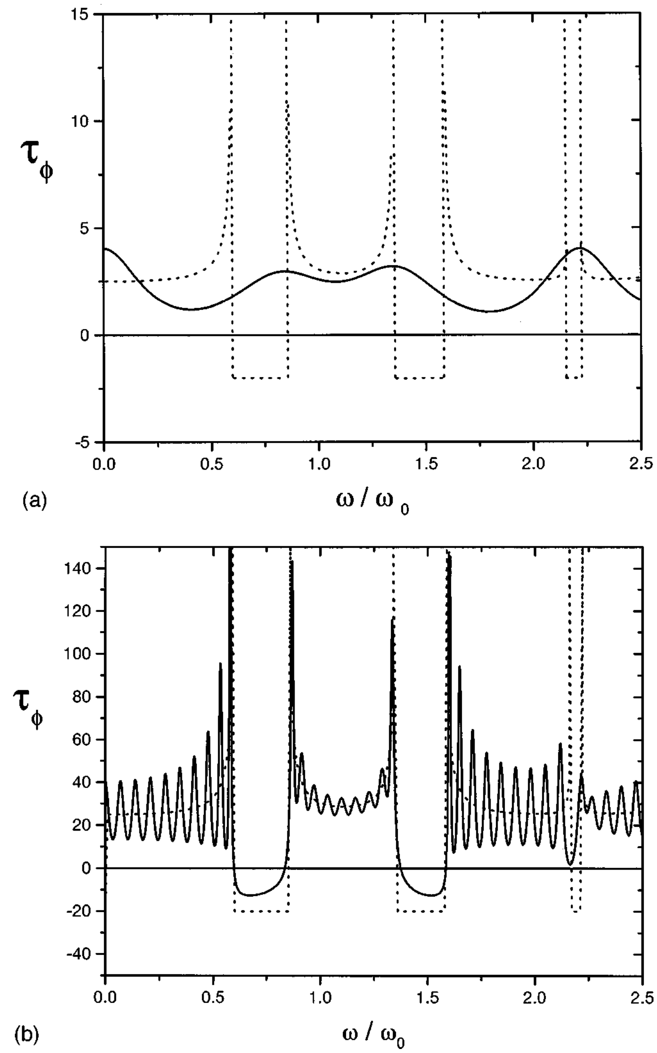

FIG. 2. Transmission phase time $\tau_\phi = d\phi_t/d\omega$ as a function of frequency, for model (b), for barriers of width (a) $d = d_0$ and (b) $d = 10d_0$. The values of the indices of refraction are $n_1 = 1.5$ and $n_2 = 2.8$. The dotted lines correspond to the superposition approximation, Eqs. (4) and (5), and the solid lines correspond to the exact calculation of the transmission amplitude.

We now perform the exact calculation of $\tau_\phi(d,\omega)$ for model (a) in the dissipationless limit, using Eq. (6) and the expressions for $\alpha(\omega,d)$ given by Eqs. (9) and (10). To display a quantitative comparison between the exact results and the results of the superposition procedure, in Fig. 1 we plot both results for $\tau_\phi(d,\omega)$ as a function of frequency, for different values of d. One can see that for frequencies outside the gap $\tau_\phi(\omega)$ is always positive in both calculations, although the exact calculation displays a richer structure with an oscillatory behavior coming from interference effects due to the finite width of the barrier. For frequencies within the gap, the exact value of $\tau_\phi(d,\omega)$ is negative for barriers with $d = 10.0c/\omega_0$, but it is positive for the very thin barrier with $d = 0.1c/\omega_0$. We further explore the behavior of the exact value of $\tau_\phi(d,\omega)$ for barriers of different widths and we find that $\tau_\phi(d,\omega)$ is always negative for thick barriers, but starts to be positive for thin barriers with $d \lesssim 0.3\omega_0/c$. Therefore, for frequencies inside the gap and thin enough barriers ($d \lesssim 0.3\omega_0/c$), there is a disagreement between the exact results and the ones from the superposition procedure, not only in the numerical values but even in the *sign* of τ_ϕ. On the other hand, for these same frequencies and as the width of the barrier is increased, $d\alpha(d,\omega)/d\omega \approx 0$; thus the results of the superposition procedure and the exact ones approach each other ($\tau_\phi \approx -d/c$).

In Fig. 2 we show the corresponding results for model (b) with a set of parameters chosen to coincide with those of the experiment of Spielman *et al.* [7]. In this case there is a collection of gaps, but the behavior of $\tau_\phi(d,\omega)$ within and around each gap, as a function of frequency and width, is very similar to that described above for model (a), in the dissipationless limit. Therefore, one arrives exactly at the same general conclusion about the behavior of $\tau_\phi(d,\omega)$ as the one reached in our above discussion of model (a). Furthermore, this leads us to propose, as a general conclusion, that independent of the model used to describe the transmission barrier superluminal tunneling will be allowed under the following conditions: (i) the main frequency contributions of the incident pulse must lie within the frequency gap of the barrier, and (ii) the barrier must be sufficiently wide.

Nevertheless, this conclusion is based on the validity of the stationary-phase approximation and the assumption that all the frequency components lie within the gap. Therefore, in order to check the more general validity of the above conclusion, we have performed a direct numerical determination of the distance d' between the peaks of the transmitted pulse and a freely traveling one [26]. We obtained the result that the agreement between the the exact and the stationary-phase calculations is quite close, within a certain range of values of *d*, yielding support to the validity of the expression $d' = -c\tau_\phi(\omega_c)$ given by the stationary-phase approximation. A very interesting feature of this comparison is that ''superluminal'' transmission occurs up to a given value of *d* and then it becomes subluminal again. The reason for this behavior is that for very wide barriers the main frequency components of the incident pulse inside the gaps are so strongly suppressed that the contribution to the transmitted pulse of the frequency tails outside the gaps becomes as important as that from inside.

We close this report by remarking that, although the above-mentioned tunneling experiments, which measure the peak velocity and the pulse duration of the transmitted pulse, may be fully interpreted and explained within causal classical electrodynamics, there are still many others questions, such as the group, front, and energy velocities [27], experimentally accessible and verifiable, whose full understanding and elucidation deserves further attention.

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[1] C. G. B. Garret and D. E. McCumber, Phys. Rev. A **1**, 305 (1970).
[2] A. M. Steinberg, P. G. Kwait, and R. Y. Chiao, Phys. Rev. Lett. **71**, 708 (1993).
[3] R. Landauer, Nature (London) **365**, 692 (1993).
[4] W. Heitmann and G. Nimtz, Phys. Lett. A **196**, 154 (1994).
[5] T. H. Hartman, J. Appl. Phys. **33**, 3427 (1962).
[6] See, for example, R. Landauer and Th. Martin, Rev. Mod. Phys. **66**, 217 (1994).
[7] Ch. Spielmann, R. Szipöcs, A. Stingl, and F. Krauz, Phys. Rev. Lett. **73**, 2308 (1994).
[8] D. Mugnai, A. Ranfagni, and L. S. Schulman, Phys. Rev. E **55**, 3593 (1997).
[9] M. Scalora *et al.*, Phys. Rev. A **52**, 726 (1995).
[10] P. Balcou and L. Dutriaux, Phys. Rev. Lett. **78**, 851 (1997).
[11] M. Deutsch and J. E. Golub, Phys. Rev. A **53**, 434 (1996).
[12] D. L. Fisher and T. Tajima, Phys. Rev. Lett. **71**, 4338 (1993).
[13] A. M. Steinberg and R. Chiao, Phys. Rev. A **49**, 2071 (1994).
[14] L. J. Wang, A. Kuzmich, and A. Dogariu, Nature (London) **406**, 271 (2000).
[15] Th. Martin and R. Landauer, Phys. Rev. A **45**, 2611 (1992).
[16] K. Hass and P. Busch, Phys. Lett. A **185**, 9 (1994).
[17] T. Emig, Phys. Rev. E **54**, 54 (1996).
[18] J. Ruiz, M. Ortuño, E. Cuevas, and V. Gasparian, J. Phys. I **7**, 653 (1997).
[19] R. Y. Chiao, Phys. Rev. A **48**, 34 (1993).
[20] Y. P. Wang and D. L. Zhang, Phys. Rev. A **52**, 2597 (1995).
[21] Y. Japha and G. Kurizki, Phys. Rev. A **53**, 586 (1996).
[22] J. D. Jackson, *Classical Electrodynamics*, 2nd. ed. (Wiley, New York, 1975).
[23] W. L. Mochán, M. del Castillo-Mussot, and R. G. Barrera, Phys. Rev. B **35**, 1088 (1987).
[24] Y. Avishai and Y. B. Band, Phys. Rev. B **32**, 2674 (1985).
[25] R. Dashen, S. H. Ma, and H. J. Bernstein, Phys. Rev. **187**, 345 (1969).
[26] V. Romero-Rochín, R. P. Duarte-Zamorano, S. Nilsen-Hofseth, and R. G. Barrera (unpublished).
[27] J. C. Garrison *et al.*, Phys. Lett. A **245**, 19 (1998).