

Conditions for *superluminal* transmission of evanescent light pulses through optically opaque barriers

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The *superluminal* transmission of evanescent light pulses through optically opaque barriers, is analyzed using simple considerations of causal electrodynamics. By exact numerical calculations, as well as with analytical arguments within the stationary-phase approximation, we show that *superluminal* transmission occurs whenever the main frequency components of the pulse are confined to frequency regions where the presence of the barrier decreases the density of states of the electromagnetic modes of the system. We also show that these frequency regions correspond to the transmission gaps of wide enough barriers. We discuss a very simple theory for the density of states of the barrier system and compare the results of such a theory with numerical calculations. The results are illustrated with two different models for the barrier, and we find the limits of validity and of occurrence of the phenomenon. We argue that causality is not violated in this type of situations.

Keywords: Superluminal transmission; electromagnetic pulses; tunneling times; Hartman effect; causality

Se analiza la transmisión *superlumínica* de pulsos de luz evanescentes a través de barreras ópticas opacas, usando consideraciones sencillas de la electrodinámica causal. Por medio de cálculos numéricos exactos, así como con argumentos analíticos dentro de la aproximación de la fase estacionaria, mostramos que la transmisión *superlumínica* ocurre siempre que las componentes principales de las frecuencias del pulso estén confinadas a regiones de frecuencias donde la presencia de la barrera disminuye la densidad de estados de los modos electromagnéticos del sistema. Mostramos también que estas regiones de frecuencia corresponden a las brechas de transmisión de barreras suficientemente anchas. Discutimos una teoría sencilla para la densidad de estados de la barrera del sistema y comparamos los resultados de tal teoría con cálculos numéricos. Ejemplificamos nuestros resultados con dos modelos diferentes para las barreras y hallamos los límites de validez y de ocurrencia del fenómeno. Argumentamos que la causalidad no se viola en este tipo de situaciones.

Descriptores: Transmisión *superlumínica*; pulsos electromagnéticos; tiempos de tunelaje; efecto Hartman; causalidad

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1. Introduction

In most of the textbooks of classical electrodynamics [1] it is seemingly explained that there can be no propagation of electromagnetic signals faster than the speed of light in vacuum. Nonetheless, many experimental reports on *superluminal* propagation of electromagnetic radiation have recently appeared. These experiments analyze the behavior of either single photons in wavepacket states or of electromagnetic pulses traveling through dielectric barriers [2], or through microwave cavities [3-5], or across media with anomalous dispersion near an absorption or gain line [6], or between two gain lines [7,8], or in frustrated total internal reflection [10,11]. The salient common features of these experiments are

- i) That either the electromagnetic pulses or the media are chosen in such a way that most of the frequency components of the radiation lie in frequency regions where the propagation is evanescent, namely, where the barrier is optically *opaque*.
- ii) Typically, the experiments measure the *coincidental* arrival of a pair of pulses, one that crossed the opaque barrier, and one traveling unimpeded.

Such a measurement yields the apparent result that the transmitted pulse crossed the barrier at speeds greater than the speed of light in vacuum. This is what we call here *superluminal* transmission. The explanation of this result, as in

any of the different types of experiments, may be cast in the same way: the detection of the coincidental arrival refers to the coincidence in arrival of the *peaks* of the pulses, and one can show, as we do below, that there is no causal connection between the arrival of the peaks [12]. Although this simple statement should be enough to rule out the possibility of *superluminal* transmission, we find it valuable to address and review these issues within the framework of classical electrodynamics. Our objective is to establish the conditions for the occurrence of *superluminal* transmission. We believe that we can gain a better understanding of the propagation of light pulses through matter, and we can also verify, at least within the present framework, that neither *superluminal* transmission can occur nor violations of causality.

To be more precise, in this article we analyze, using *causal* classical electrodynamics, the tunneling of a classical light pulse through an optical opaque barrier. By an optical opaque barrier we mean any arrangement of optical components which produce, in the frequency domain, gaps in their transmission amplitudes. We restrict ourselves to the case of 1D tunneling by considering plane-wave pulses traveling perpendicular to the interface of barriers consisting of slabs of a single material with a transmission gap, as in the experiments by Wang *et al.* [8], or an alternate array of layered materials, as in the experimental arrangement of Spielmann *et al.* [2]. In these arrangements, a gap can be defined as the frequency region in which the normal modes of the corres-

ponding boundless system become evanescent in the dissipationless limit. For example, in the case of a slab made of a single material, this happens at the frequencies in which the index of refraction becomes purely imaginary. An alternative and probably more precise definition of a gap refers to it as the frequency region in which the density of states of the electromagnetic modes in the corresponding boundless system, vanishes. According to both of these definitions, a gap can be found only in lossless materials, since for dissipative materials the propagation is never truly evanescent [9] and the density of states cannot be properly defined. In real materials, however, one could still identify frequency regions that will become gaps through a proper analysis of the dissipationless limit.

There have appeared in the literature several studies that analyze this same transmission or tunneling phenomenon along similar [13–16] and other lines [17–19]. Within the stationary-phase approximation, in which it is assumed that the shape of the pulse suffers only slight distortions while traveling through the medium, it has been already established that the time delay between the peak of the tunneling pulse and the unimpeded one is given by the so-called phase time. The phase time is defined as the frequency derivative of the phase of the transmission amplitude. As we shall illustrate using two different models for the barrier, it turns out that the phase time is always negative for thick enough barriers and for frequencies within the transmission gaps. It follows that *superluminal* transmission occurs whenever the phase times become negative and the main frequency components of the incident pulse lie within the gap [2, 3, 6, 13, 16]. We shall also see that the phase time is always positive for frequencies outside the gaps and we shall argue that this is a requirement of causality.

Thus, our goal in this study is to establish the conditions under which these phase times turn out to be negative. Our main result is that *superluminal* transmission will be possible whenever the presence of a barrier causes a *decrease* in the density of states of the electromagnetic modes of the system in comparison with the density of states in the absence of the barrier. We illustrate this by proposing a simple approximation to evaluate this change in the density of states. Furthermore, we show that this decrease in the density of states *always* occurs for thick enough lossless barriers and that the corresponding phase times, besides being negative, are proportional to their width, whenever this width is not too large. We have already given a brief account of these arguments, see Ref. 20. Moreover, we also show here that as the width of the barriers increases the pulse transmission becomes *subluminal* again, but the shape of the transmitted pulse is too distorted to be still considered a "pulse". We then display a direct numerical test for the limits of validity of the stationary-phase approximation in relation with the conditions found here for *superluminal* tunneling.

As mentioned above, we proceed by concentrating our study in the analysis of two particular models, first, a barrier made of a dissipative material with a single Lorentzian reso-

nance and, second, a barrier made of alternating layers of non-dissipative, dispersionless materials with high and low indexes of refraction. Both of these models are treated in a unified manner through the use of a transfer-matrix formalism. The first model will serve to obtain illustrative analytic results, while the second will serve to show explicitly the relationship between our calculations and the experiments of Spielmann *et al.* [2]. We will also show that in the frequency regions *outside* the gaps the density of states is always increased by the presence of the barrier. We argue that this is a consequence of causal propagation which, in our analysis, is guaranteed at the onset by construction. In this regard, we will also verify that causality, understood as the fact that effects cannot precede the causes, is indeed satisfied. We recall, nevertheless, that Heitmann and Nitz [21] have argued that Einstein causality, namely, that no signal can travel faster than light in vacuum, cannot be verified in the type of experiments discussed here. Although we proceed by analyzing the two particular models of barriers discussed above, we shall conclude that our results are quite "universal", and they hold for any slab with transmission gaps. It is also of interest to mention that the present problem is very similar to the non-relativistic quantum situation of a particle tunneling through a barrier [22]. In this case it is found that, as the barrier becomes thicker, the tunneling probability becomes smaller, as expected, but the tunneling time becomes shorter. This is known as the Hartman effect [23], and the electromagnetic situation we consider may also be seen as an extension of such a result.

The present article is organized as follows. In the next section we write down the *causal* analytic expressions for the transmitted and reflected fields for a normally incident plane-wave pulse in terms of the transfer matrix of the barrier. In Sec. 3, we analyze the phase of the transmission amplitude for the particular cases mentioned above and introduce the relationship between its frequency derivative (phase time) and the change in the density of states due to the presence of the barrier. Then, it is explicitly shown that in the frequency domain corresponding to transmission gaps the phase time can become negative, while it is always positive outside these gaps. In Sec. 4, we use a simple analytic argument to demonstrate that the *superluminal* effect appears for an incident pulse whose frequency bandwidth is inside the gap; as a follow up one can immediately show that the time duration of the transmitted pulse is shorter than the incident one, in agreement with previous analysis [19] and recent experimental results [2]. We corroborate the analytic results derived in this section, through a numerical analysis of the transmission of pulses across the barriers discussed in Sec. 2. This section ends with a brief discussion about the consequences of causality. Section 5 is left for our conclusions, a brief analysis of the effects of dissipation and some final remarks.

2. Transmitted and reflected fields

The purpose of this section is to introduce our notation, our main definitions and the transfer matrix formalism. The phy-

sical system consists of a linearly polarized electromagnetic pulse traveling in vacuum and impinging normally on a slab of thickness d . For definiteness, we place the slab between $x = 0$ and $x = d$, and assume that the pulses are polarized along the y -direction. The dielectric properties of the slab vary only along the x -direction, and we consider plane-wave pulses whose electric and magnetic fields vary only along the x -direction. Thus we end up effectively with a 1D-problem. We call $E_i(x, t)$, $E_r(x, t)$ and $E_t(x, t)$ the amplitudes at time t , of the incident, reflected and transmitted electric fields, respectively. The amplitudes of the reflected and transmitted electric fields are related to the incident one by the causal relationships,

$$E_r(x, t) = \int_{-\infty}^t dt' \hat{r}(t-t') E_i(x, t'), \quad (1)$$

for $x \leq 0$, and

$$E_t(x, t) = \int_{-\infty}^t dt' \hat{t}(t-t') E_i(x, t'), \quad (2)$$

for $x \geq d$. The field $E_i(x, t)$ is defined for all x and t and the coefficients $\hat{r}(\tau)$ and $\hat{t}(\tau)$ are the reflection and transmission amplitudes of the barrier, assuming that in the remote past ($t \rightarrow -\infty$) there are no reflected and transmitted fields. The relationship, in the frequency ω domain, between the time Fourier transforms of the electric field $E_y(x = 0^+; \omega)$ and the magnetic field $B_z(x = 0^+; \omega)$ with $E_y(x = d^-; \omega)$ and $B_z(x = d^-; \omega)$, can be written as

$$\begin{bmatrix} E_y(x = d^-, \omega) \\ B_z(x = d^-, \omega) \end{bmatrix} = \tilde{M}_d(\omega) \begin{bmatrix} E_y(x = 0^+, \omega) \\ B_z(x = 0^+, \omega) \end{bmatrix}, \quad (3)$$

where $\tilde{M}_d(\omega)$ is a 2×2 matrix, called the transfer matrix. It is a simple exercise to show that the time Fourier transform of the reflection and transmission amplitudes $r(\omega)$ and $t(\omega)$ are given by [25]

$$r = \frac{M_{11} + M_{12} - M_{21} - M_{22}}{M_{12} + M_{21} - M_{11} - M_{22}}, \quad (4)$$

and

$$t = -\frac{2}{M_{12} + M_{21} - M_{11} - M_{22}} e^{-i\omega d/c}, \quad (5)$$

where M_{ij} ($i, j = 1, 2$) are the elements of \tilde{M}_d , and we have suppressed their explicit ω dependence.

The transfer matrix of a system composed by N slabs, each of them of thickness d_j and index of refraction n_j , is given by the matrix multiplication of the transfer matrices of each slab [16, 25], that is,

$$\tilde{M}_d = \tilde{M}_{d_1} \cdot \tilde{M}_{d_2} \cdot \dots \cdot \tilde{M}_{d_N}, \quad (6)$$

where $d = \sum_j d_j$. The transfer matrix of a single slab of thickness d_j and index of refraction $n_j(\omega)$ can be readily

written in closed form [25], while for a multilayer system the matrix multiplication given in Eq. (6) has to be performed prior to the substitution into Eqs. (4) and (5). In this case, it is not possible to obtain a simple closed-form-expression for $r(\omega)$ and $t(\omega)$, thus their calculation has to be done numerically.

Taking into account the causal property of the coefficients $\hat{r}(\tau)$ and $\hat{t}(\tau)$ (e.g. $\hat{t}(\tau) = 0$ for $\tau < 0$), one can write general analogous expressions for the reflected and the transmitted pulses as integrals over frequency, that is,

$$E_r(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega R(\omega) e^{i\phi_r(\omega)} E_0(\omega) e^{-i\omega(x+ct)/c} \quad (7)$$

and

$$E_t(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega T(\omega) e^{i\phi_t(\omega)} E_0(\omega) e^{i\omega(x-ct)/c}, \quad (8)$$

where we have defined

$$r(\omega) = R(\omega) e^{i\phi_r(\omega)} \quad (9)$$

and

$$t(\omega) = T(\omega) e^{i\phi_t(\omega)}. \quad (10)$$

Here $R(\omega)$ and $T(\omega)$ are the moduli of the reflection and transmission amplitudes, while $\phi_r(\omega)$ and $\phi_t(\omega)$ are their corresponding phases. Here $E_0(\omega) e^{i\omega x/c}$ is the plane-wave component of the incident field with frequency ω .

The expressions for the fields Eqs. (7) and (8) are the basis for our analysis. We stress the well-known fact that in order to get the shape of the reflected and transmitted pulses, one just needs to know the frequency dependence of the reflection and transmission amplitudes. The transmission and reflection amplitudes are functions of the width d of the barrier, and this dependence plays an important role in the *superluminal* effect.

3. Models, phase time, and density of states

In this section, we define the two models for the barriers that shall be analyzed in this work and we shall find the frequency regions that correspond to the transmission gaps. Our analysis will be concentrated on the study of the modulus and *phase* of the transmission amplitude inside and outside the frequency regions of the transmission gaps. These results shall then be used to calculate the density of states of the electromagnetic modes. We shall show that inside the gaps, and for sufficiently wide barriers, the density of states is decreased due to the presence of the slab. This will serve, in the next section, to show that *superluminal* transmission is related to this decrease in the density of states and that it is a quite generic property of materials with transmission gaps. Since most of the content of this section has already been analyzed by us in Ref. 20, we shall present here only some definitions and the main results to make this paper self-contained. The models are:

- a) A slab of length d made of an absorbing material with an index of refraction $n(\omega)$ having a single lorentzian

resonance [1], that is,

$$n(\omega) = \sqrt{1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\gamma\omega}}, \quad (11)$$

where ω_p is a model parameter with units of frequency, ω_0 is the resonance frequency, and γ is the damping parameter related to energy dissipation. The real and imaginary parts of $n(\omega)$ will be denoted as $n(\omega) = n'(\omega) + in''(\omega)$.

- b) A multilayer of alternating media with (real) high and small indexes of refraction, n_1 and n_2 and with equal widths $d/2$. This is the case experimentally analyzed by Spielmann *et al.* [2].

According to one of our definitions, the gaps are frequency regions in which the normal modes of the corresponding boundless system become evanescent in the dissipationless limit. In model (a), the wave vector q of a plane wave inside the corresponding boundless system is given by

$$q = \frac{\omega}{c} n(\omega), \quad (12)$$

where $n(\omega)$ is the index of refraction given by Eq. (11) and we are using cgs units. In the dissipationless limit ($\gamma \rightarrow 0$) the plane waves become evanescent when the wavevector q is purely imaginary, and this occurs in the frequency region $\omega_0 \leq \omega \leq \sqrt{\omega_0^2 + \omega_p^2}$, which defines the gap. This model has been thoroughly discussed [13, 15, 3, 14, 19], and its similarity with non-relativistic quantum tunneling through a barrier [22] has been also analyzed.

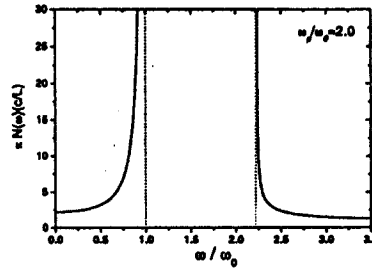
The corresponding boundless system for model (b) is a periodic superlattice with the period given by the total length d of the two layers of index of refraction n_1 and n_2 . In this case, the dispersion relation of the normal modes of the system, that is, the relation between their Bloch wavevector κ and the frequency ω , is given by [25]

$$\cos \kappa d_0 = \frac{1}{2} [M_{11}(\omega) + M_{22}(\omega)], \quad (13)$$

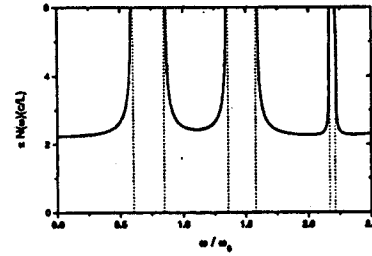
where d_0 is the length of the period of the superlattice and $M_{ij}(\omega)$ are the ij -th elements of the transfer matrix corresponding to d_0 . Direct substitution of the closed-form expressions of $M_{ij}(\omega)$, for a single slab, in the matrix elements of Eq. (6), yields [16, 25]

$$\cos \kappa d_0 = \cos\left(\frac{k_1 d_0}{2}\right) \cos\left(\frac{k_2 d_0}{2}\right) - \frac{1}{2} \left(\frac{n_1}{n_2} + \frac{n_2}{n_1}\right) \sin\left(\frac{k_1 d_0}{2}\right) \sin\left(\frac{k_2 d_0}{2}\right), \quad (14)$$

where $k_1 = n_1 \omega / c$, $k_2 = n_2 \omega / c$, and $d_0/2$ is the width of each layer. Bloch evanescent modes appear when the frequency is such that κ becomes purely imaginary. These frequency regions are also known as photonic band gaps where there is no energy transport.



(a)



(b)

FIGURE 1. Density of electromagnetic states, Eq. (15), a) model (a), Eq. (11). Time units are rescaled with ω_0 and b) model (b), Eq. (14). Time units are rescaled with $d_0/2c$ where d_0 is the width of a single bilayer.

One can easily show that in the case of a boundless system the density of states $N(\omega)$ of the electromagnetic modes is given by

$$\pi N(\omega) = L \left| \frac{d\omega}{d\kappa} \right|^{-1}, \quad (15)$$

where $L(\rightarrow \infty)$ denotes the size of the system and the wavevector κ is real. When κ is purely imaginary, $N(\omega) = 0$, and this defines a gap. For model (a), $\kappa = q$ and $N(\omega)$ for the boundless system becomes $\pi N(\omega) = L(n' + \omega dn'/d\omega)/c$, which is plotted in Fig. 1a for a specific choice of parameters. For the infinite superlattice, one combines Eqs. (14) and (15) to determine the density of states. This is shown in Fig. 1b for a set of parameters corresponding to the experiment of Spielmann *et al.* [2]

When the barriers are of finite width the calculation of the density of states is not as simple as described above. However, Avishai and Band [26], using an S -matrix formalism previously developed by Dashen, Ma and Bernstein [27], derived a relationship between the phase time τ_* , defined as,

$$\tau_*(\omega) \equiv \frac{d\phi_s(\omega)}{d\omega}, \quad (16)$$

and the density of states $N(\omega)$. They found

$$\tau_*(\omega) = \pi [N(\omega) - N_0(\omega)], \quad (17)$$

where ϕ_t is the phase of the transmission amplitude of the barrier, $N(\omega)$ is the density of states of the system in the presence of the barrier and $N_0(\omega)$ is the density of states of the system in the absence of the barrier. In other words, τ_p is proportional to the *change* in the density of states due to the presence of the barrier.

As we shall see in the next section, the *superluminal* effect can be traced back to the fact that the phase time becomes negative within the gaps. Moreover, with the transfer matrix formalism we can numerically calculate the phase time for both models and quantify the extent and limitations of the effect. However, in order to gain physical insight into the question of why the phase time is negative for frequencies within the gaps, in Ref. 20 we have introduced a simple superposition approximation for the density of states, in the presence of the barrier, that yields a simple explanation to the *superluminal* effect.

First we note from Eq. (15) that in vacuum $\pi N_0(\omega) = L/c$. Therefore, the sign of τ_p is determined by the difference between $N(\omega)$ and L/c . It is clear that $N(\omega)$ should also scale with L so that the difference $N(\omega) - L/c$ should depend only on the width d of the barrier, remaining finite in the limit $L \rightarrow \infty$. Thus, the approximation consists in assuming that the density of states, for an infinite system ($L \rightarrow \infty$) in the presence of a barrier of finite width d , is also given by Eq. (15) with L replaced by d . In this way, one obtains that $\pi N(\omega) = (L - d)/c + d/(d\omega/d\kappa)$ for frequencies outside the gaps, and $\pi N(\omega) = (L - d)/c$ for frequencies within the gaps. Therefore, the phase time τ_p can be expressed as

$$\tau_p(d, \omega) = -\frac{d}{c} + d \left| \frac{d\omega}{d\kappa} \right|^{-1}, \quad (18)$$

for frequencies outside the gaps, and

$$\tau_p(d, \omega) = -\frac{d}{c}, \quad (19)$$

for frequencies within the gaps. In the regions of normal dispersion outside the gaps, $d\omega/d\kappa < c$. Therefore, this approximation leads to the interesting conclusion that τ_p should *always* be positive outside the gaps and *always* negative inside the gaps.

Now, using Eqs. (5) and (10) we can write the phase ϕ_t of the transmission amplitude as

$$\phi_t(d, \omega) = -\omega d/c + \alpha(d, \omega), \quad (20)$$

where $\alpha(d, \omega)$ is the phase of $1/(M_{11} + M_{22} - M_{12} - M_{21})$, and M_{ij} are the elements of the transfer matrix $\overline{M}_d(\omega)$. Then the phase time $\tau_p = d\phi_t/d\omega$ can be written as

$$\tau_p(d, \omega) = -\frac{d}{c} + \frac{d\alpha(d, \omega)}{d\omega}. \quad (21)$$

One can see that this expression is similar to Eqs. (18) and (19). Hence, the superposition procedure described above amounts to take

$$\frac{d\alpha(d, \omega)}{d\omega} \approx d \left| \frac{d\omega}{d\kappa} \right|^{-1}, \quad (22)$$

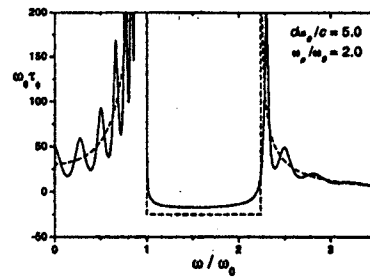


FIGURE 2. Transmission phase time, $\tau_p = d\phi/d\omega$ as a function of frequency, for model (a), using $\omega_p/\omega_0 = 2$ and $\gamma/\omega_0 = 0$, for a barrier of width $d = 5.0c/\omega_0$. The dotted lines correspond to the superposition approximation, Eqs. (18) and (19), and the solid lines correspond to the exact calculation of the transmission amplitude, Eq. (5).

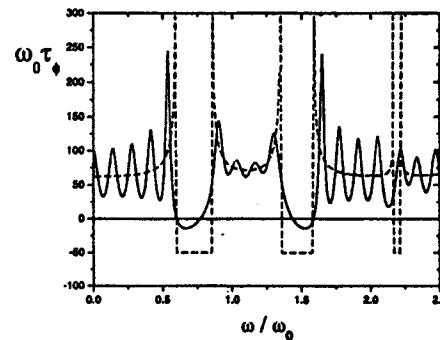


FIGURE 3. Transmission phase time, $\tau_p = d\phi/d\omega$ as a function of frequency, for model (b), for a barrier of width $d = 5d_0$. The values of the indices of refraction are $n_1 = 1.5$ and $n_2 = 2.8$. The dotted lines correspond to the superposition approximation, Eqs. (18) and (19), and the solid lines correspond to the exact calculation of the transmission amplitude, Eq. (5).

for frequencies outside the gaps, and

$$\frac{d\alpha(d, \omega)}{d\omega} \approx 0, \quad (23)$$

for frequencies within the gaps.

In Ref. 20, we compared the exact and the approximated expressions for the phase time and we have found that the latter holds better as the barrier becomes wider. Here, in Figs. 2 and 3, we show the comparison between these expressions for a case with intermediate values of the barrier width. From these Figures we see that the agreement is excellent as far as obtaining the negative values of the phase time for frequencies within the gap. The oscillations of the phase time for frequencies outside the gap in the exact case, of course, cannot be described by the superposition approximation since those oscillations are Fabry-Perot-like interferences due to the finiteness of the barrier.

The general conclusion, therefore, is that as the width of the barrier is increased, the results of the superposition

procedure and the exact ones approach each other, yielding $\tau_s \approx -d/c$ in the gaps (the analog of the Hartman effect [23]). As mentioned, in the next section we shall show that a negative τ_s for frequencies in the gap essentially accounts for *superluminal* transmission, especially for wide barriers. However, we shall also show that when the barrier becomes *too* wide, the transmission turns back to be subluminal again.

4. Superluminal transmission of electromagnetic pulses

In this section we show that that within the stationary-phase approximation the phase time τ_s is equal to the time delay between the peak of the tunneling pulse and the peak of the unimpeded one. Then, we shall see that *superluminal* transmission occurs for $\tau_s < 0$, and according to last section this condition is satisfied for frequencies within the gap of thick enough barriers. Therefore, *superluminal* tunneling will be allowed under the following conditions:

- i) The main frequency contributions of the incident pulse must lie within the frequency gap of the barrier.
- ii) The barrier must be *sufficiently* wide.

We end this section by comparing the results of the stationary-phase approximation with exact (numerical) calculations using the two models described before; we shall see that if the barrier becomes too wide, the transmission becomes subluminal again.

Let us start by going back to Eq. (8), the equation for the transmitted field. In this equation our basic assumption is that the main frequency contributions of the incident field, $E_0(\omega)$, lie within the frequency range of a transmission gap. This will be possible for pulses which are frequency-limited [3]. Nevertheless, any field with a well-defined front edge, as well as the most common idealizations such as a gaussian wave packet, have "long tails" of frequency components lying outside the gap. To be specific, let us consider a gaussian pulse. That is, the incident field is a pulse of duration τ_0 centered at $x = ct$ and with central frequency ω_c , namely,

$$E_i(x, t) = E_i^0 e^{-(x-ct)^2/2(c\tau_0)^2} e^{i\omega_c(x-ct)/c}. \quad (24)$$

In the frequency domain we can write

$$E_i(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega E_0(\omega) e^{i\omega(x-ct)/c}, \quad (25)$$

with the function $E_0(\omega)$ given by

$$E_0(\omega) = \frac{\tau_0 E_i^0}{\sqrt{2\pi}} e^{-(\omega-\omega_c)^2 \tau^2/2}. \quad (26)$$

From Eq. (8) we find that we can safely assume that condition (i) above is satisfied whenever the product $T(\omega) E_0(\omega)$ is much larger for frequencies inside the gap than for frequencies outside of it. However, one must keep in mind that this cannot be true for *arbitrarily* wide barrier because then $T(\omega)$

becomes exponentially smaller (inside the gap) as the gap becomes wider.

In the stationary-phase approximation one keeps, in Eq. (8), only the lowest-order terms in a Taylor expansion of the phase $\phi_t(\omega)$ [cf. Eqs. (20) and (23)] and up to the second-order ones in the amplitude $T(\omega)$ of the transmission amplitude. The Taylor expansion is made around the central frequency ω_c of the pulse and one assumes that $\omega_c \pm \tau_0^{-1}$ lies well within the gap. Hence, the transmitted pulse can be approximated as,

$$E_t(x, t) \approx E_i^0 T(\omega_c) e^{-(x-ct-d')^2/2(c\tau_t)^2} \times e^{i\omega_c(x-ct-d')/c}. \quad (27)$$

That is, the transmitted pulse is also a gaussian packet of duration $\tau_t = \sqrt{\tau_0^2 - \Delta\tau_t^2}$, and centered at $x = ct + d'$. Here

$$d' = -c \left. \frac{d\phi_t}{d\omega} \right|_{\omega_c} = -c\tau_s(\omega_c), \quad (28)$$

and

$$\Delta\tau_t^2 = \frac{1}{T(\omega_c)} \left. \frac{d^2 T}{d\omega^2} \right|_{\omega_c}. \quad (29)$$

We have further assumed that the central frequency ω_c of the pulse is chosen at the minimum of the transmission gap, that is, $dT/d\omega|_{\omega_c} = 0$. Note that the transmitted pulse is shorter in duration than the incident one; this is in agreement with experimental results [2] and earlier theoretical estimates [19].

Comparing Eq. (27) with the expression of the incident pulse, Eq. (24), one finds that the transmitted pulse has also a gaussian shape, traveling at speed c , with its peak going at a distance d' from the peak of the unimpeded incident gaussian packet. Thus for $\tau_s(\omega_c) > 0$, one has $d' < 0$, and the peak of the transmitted pulse goes behind the peak of the unimpeded one (subluminal), while for $\tau_s(\omega_c) < 0$, one has $d' > 0$, and the peak of the transmitted pulse goes ahead (*superluminal*). Because of the factor $T(\omega_c)$ in Eq. (27), the intensity of the transmitted pulse is much smaller than the intensity of the incident one. This observation, in addition to the fact that the pulse becomes shorter, is essential for reconciling the fact that the transmission is causal notwithstanding the peak of the transmitted pulse lies ahead. We return to this point below.

In conclusion, the stationary-phase approximation predicts *superluminal* transmission whenever the phase time $\tau_s(\omega_c) < 0$, and according to the discussion in the last section, $\tau_s(\omega_c) < 0$ whenever the presence of the barrier accounts for a decrease in the density of states for frequencies ω_c within the gap. Nevertheless, this conclusion is based in the validity of Taylor expansions, and these might become questionable for frequencies around the edges of the transmission gaps. Therefore, in order to check the more general validity of the above conclusion, we now proceed to perform a direct numerical determination of d' and $\Delta\tau_t$.

In Figs. 4 and 5 we plot the distance d' between the peaks of the transmitted pulse and a freely traveling one, as a function of d , for the two models (a) and (b), and for a specific choice of the model parameters. As stated above, this distance

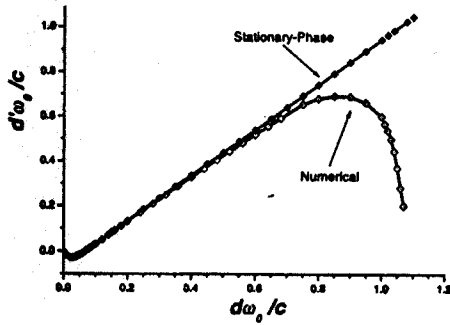


FIGURE 4. Calculated distance between the peak of a transmitted pulse and the peak of a freely traveling one as a function of the width d of the barrier, for model (a) using $\omega_p/\omega_0 = 2$ and $\gamma/\omega_0 = 0.01$. The incident pulse is a gaussian function with central frequency $\omega_c/\omega_0 = 1.5$ and duration $\omega_0\tau = 15$. In the figure "stationary-phase" refers to the stationary phase approximation, and "numerical" refers to direct numerical calculation of the distances between the peaks.

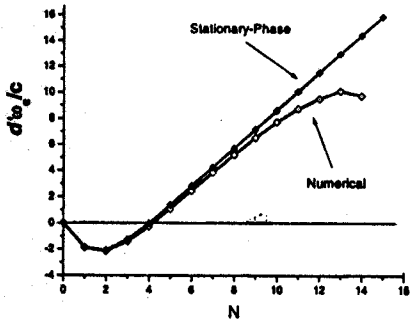


FIGURE 5. Calculated distance between the peak of a transmitted pulse and the peak of a freely traveling one as a function of the number N of bilayers in barrier, for model (b) using $n_1 = 1.5$ and $n_2 = 2.8$. The incident pulse is a gaussian function with central frequency $\omega_c/\omega_0 \approx 0.72443$ and duration $\omega_0\tau = 50$. In the figure "stationary-phase" refers to the stationary phase approximation, and "numerical" refers to direct numerical calculation of the distances between the peaks.

is obtained directly from the *numerical*, otherwise exact, evaluation of the transmitted field given in Eq. (8). In these Figures we also plot d' as a function of d as taken from Eq. (28), for the same choice of model parameters. One can see that the agreement between the two calculations is quite close, within a certain range of values of d , yielding support to the validity of the expression given in Eq. (28), for d' in stationary-phase approximation. A very interesting feature in these Figures is that *superluminal* transmission occurs up to a given value of d and then it tends to become *subluminal* again. That is, as d becomes larger, the position of the peak of the transmitted pulse starts to recede. The reason for this behavior is that for very wide barriers the main frequency com-

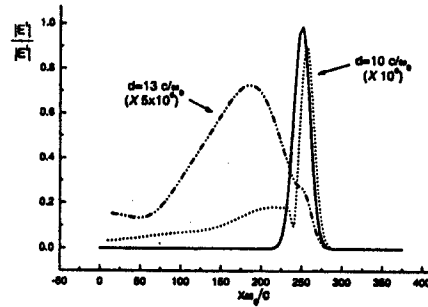


FIGURE 6. Snapshots of a freely traveling pulse and two transmitted pulses for different widths of the barrier. The barrier and the incident pulse correspond to those of Fig. 4. The magnification of each pulse is indicated in the Figure. Note how for $d = 10c/\omega_0$ the transmission is still *superluminal* while for $d = 13c/\omega_0$ it is (again) *subluminal*.

ponents of the incident pulse inside the gaps are so strongly suppressed that the contribution to the transmitted pulse of the frequency tails outside the gaps become as important as the ones inside. Therefore, the failure of Eq. (28) to describe the behavior of the spatial shift d' of the transmitted pulse for such wide barriers, comes from the fact that for those barriers condition (i) is no longer satisfied. That is, the product $T(\omega)E_0(\omega)$ is no longer larger for frequencies inside the gaps than for frequencies outside. In order to illustrate the behavior of these pulses which go through very wide barriers we show, in Fig. 6, two snapshots of the transmitted pulse for two different values of d , one *superluminal* and the other *subluminal*. The calculations are done for the dissipative medium of model (a) with a specific choice of model parameters. In the subluminal case the transmitted field appears so distorted that it can hardly be called a "pulse" since its frequency components outside the gap are now the dominant ones.

We have also numerically verified that the transmitted pulse is shorter than the incident one for both models (a) and (b). In Figs. 7 and 8 we compare the results of these calculations with the formula for $\Delta\tau_t^2$ given by Eq. (29) and with a direct numerical evaluation of the width of the transmitted pulses. The agreement between these results, again, is only valid up to a certain value of d , as can clearly be seen in both Figures. The reason is the same as before, that is, once the frequency components of the incident field outside the gap begin to contribute significantly to the pulse shape, the transmission is no longer *superluminal* and the simple arguments and expressions used above cease to be valid. Nevertheless, besides its connection with *superluminal* transmission, the most important aspect of the narrowing of the transmitted pulse, is that it can be seen as a direct consequence of causality.

About causality, we have shown that it is not violated, simply because the transmitted fields are obtained from expressions that are causal by construction, [cf. Eq. (2)]. However, we find worthwhile to illustrate, in a more intuitive fashion, that causality is preserved regardless of the fact that the

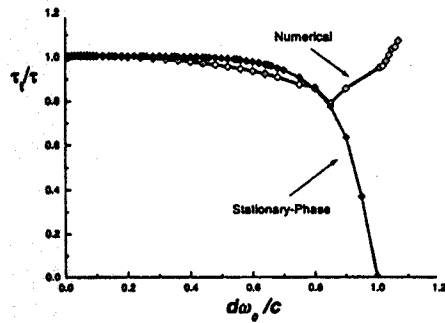


FIGURE 7. Time duration τ_t of the transmitted pulse as a function of the width d of the barrier for model (a), using the parameters of Fig. 4. The values of τ_t are scaled with the value τ of the incident pulse. "Stationary-phase" refers to the calculation with the stationary phase approximation, see Eq. (29), while "numerical" refers to the exact numerical evaluation of the transmitted pulse.

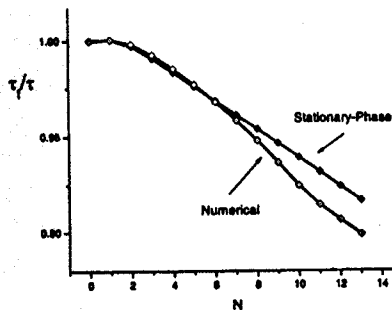


FIGURE 8. Time duration τ_t of the transmitted pulse as a function of the number N of bilayers in the barrier for model (b), using the parameters of Fig. 5. The values of τ_t are scaled with the value τ of the incident pulse. "Stationary-phase" refers to the calculation with the stationary phase approximation, see Eq. (29), while "numerical" refers to the exact numerical evaluation of the transmitted pulse.

peak of the transmitted pulse lies ahead. As a consequence of causal propagation, it must be obeyed that the intensity of the transmitted pulse, at a given position $x \geq d$, integrated from $t' = -\infty$ to the observation time t is smaller than the corresponding integrated intensity of a freely traveling pulse, that is,

$$\int_{-\infty}^t dt' |E_t(x, t')|^2 \leq \int_{-\infty}^t dt' |E_i(x, t')|^2. \quad (30)$$

One can easily check that, at least in model (a), this inequality is automatically satisfied, *independently* of whether the peak of the transmitted pulse is ahead or not of the unimpeded one. But one can also check that if the duration τ_t of the transmitted pulse were equal or larger than the duration τ_0 of the incident pulse, then Eq. (30) would not be satisfied and causality

would be violated. Therefore, the shortening of the transmitted pulses observed in Spielmann *et al.* experiments [2], is a consequence of causality.

It is also interesting to notice that for transmission *outside* the gaps it is not true that, for given x and t , $E_t(x, t)$ is always smaller than $E_i(x, t)$. Therefore, it is crucial that in such a case the peak of the traveling pulse does not move ahead of the freely traveling one, otherwise the causal condition [Eq. (30)] would not be satisfied. This is prevented by the fact that outside the gaps the phase time is always positive, yielding transmitted pulses with peaks lagging the peak of the freely traveling ones.

We recall here the point of view put forward by Heitmann and Nimtz [21] that Einstein causality, namely the statement that no signal can travel faster than the speed of light in vacuum, cannot be verified in the type of experiments discussed here. Their point being that the pulses produced in the laboratory, as well as idealizations such as gaussian pulses, do not have well defined fronts. It is known that Einstein causality is a consequence of causality, in the sense that causes cannot precede effects, *cf.* Eqs. (1) and (2), and Lorentz invariance of Maxwell equations [1]. In this regard, we have certainly only verified the latter statement of causality with Eq. (30).

5. Final Remarks

In this article we have studied the conditions for the occurrence of *superluminal* transmission of plane-wave light pulses through 1D opaque barriers. By *superluminal* we understand the fact that the peak of the transmitted pulse is ahead of the peak of a freely traveling pulse unimpeded by the barrier. We have shown that this behavior is quite generic to any barrier independent of the specific properties of the material or materials it is made of. More important, we have found that the main requirement for *superluminal* propagation is that the frequency bandwidth of the pulse should be composed by frequencies in which the presence of the barrier yields a decrease in the density of states. This leads one to conclude that the best conditions for this to happen is when the density of states of the corresponding boundless material or material system which composes the barrier is already null, like in the frequency regions known as transparency gaps. With this in mind, we propose an extremely crude superposition approximation for the density of states of the system in the presence of the barrier by assuming that the local density of states in the barrier region scales with the width of the barrier in the same way as in the boundless system. Then, to test the fairness of this approximation we worked out in detail the exact calculation of the density of states, as a function of frequency, for two specific model systems with barriers made of:

- i) A dissipationless material with a lorentzian resonance in its dielectric response.
- ii) A finite number of bilayers of loss free materials with a large contrast in their indexes of refraction.

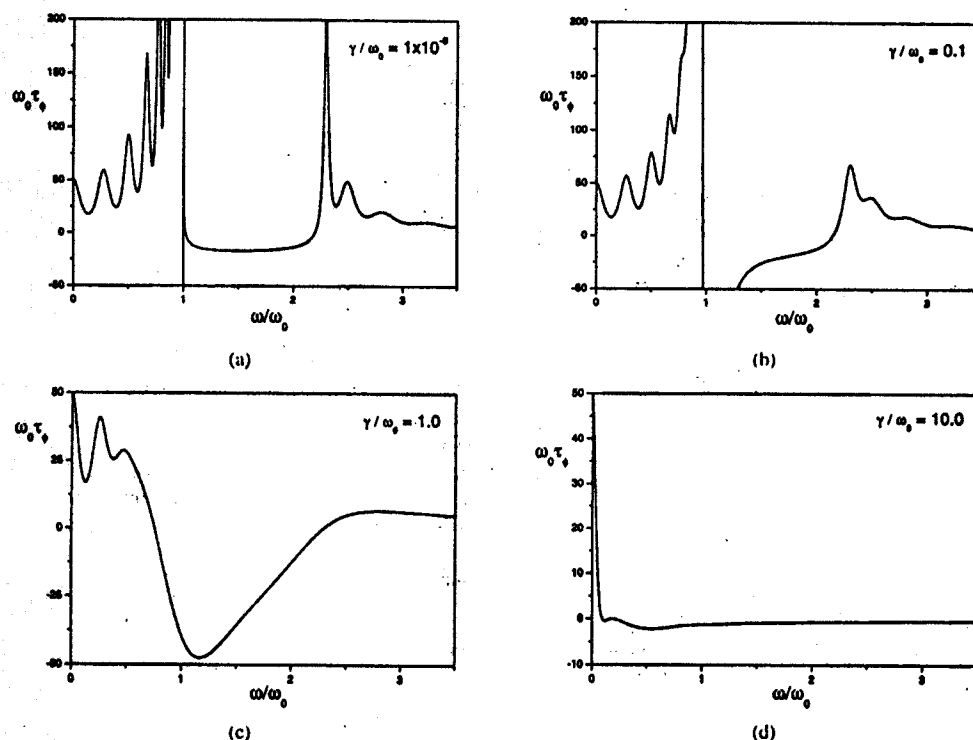


FIGURE 9. Transmission phase time, $\tau_p = d\phi/d\omega$ as a function of frequency, for model (a), using $\omega_p/\omega_0 = 2$, a barrier of width $d = 5.0c/\omega_0$, and a) $\gamma/\omega_0 = 10^{-8}$, b) $\gamma/\omega_0 = 0.1$, c) $\gamma/\omega_0 = 1$, and d) $\gamma/\omega_0 = 10$. Note that as the dissipation is increased the resonance line is wider and this affects the behavior of the phase time in the transmission gap.

This last barrier system was used by Spielmann *et al.* [2] to perform measurements of the tunneling time of light pulses. We found out, that although the fine structure in the change in the density of states due to the presence of the barrier is not well reproduced by the superposition approximation, the main general conclusions derived from it about *superluminal* transmission in wide barriers, hold out correctly.

For thin barriers the superposition approximation obviously fails and the transmission of pulses with a frequency bandwidth comprised within the transmission gaps, instead of keeping being *superluminal*, becomes subluminal again. The critical barrier width between *superluminal* and subluminal transmission is determined by a width such that the density of states in the frequency gap is not altered by the presence of the barrier. We showed all this in the specific models mentioned above using the stationary-phase approximation for the description of the pulse transmission. Then we check the validity of the stationary-phase approximation by performing exact calculations of the time delay between the pulse arrival of the pulse which tunnels through the barrier and the unimpeded one. We conclude that the results for the time delay between pulses derived from the stationary-phase approximation are quite correct when the width of the barrier is not too thick. Nevertheless, we also show that in the case

of very thick barriers

- i) The distortion of the pulse shape is so strong, that it becomes difficult to assess a peak displacement to such a pulse.
- ii) The pulse transmission becomes subluminal again, due to the role played by the frequency "tails" of the incident pulse.

We have also analyzed the narrowing in the duration of the *superluminal* transmitted pulse and have argued that besides previous predictions [19] and the experimentally verified account of this phenomenon, it is also a direct consequence of very general causality requirements.

Finally, we have calculated the effects of dissipation in the *superluminal* effect for the case of model (a), [cf. Eq. (11)]. The first aspect to keep in mind is that as the dissipation becomes different from zero, strictly speaking, the transmission gap is no longer a gap, even for an infinite system; of course, if the dissipation is small there still exists a very low transmission in the gap region and the *superluminal* transmission still occurs. To be precise, we have studied the effect of dissipation by varying the parameter γ/ω_0 from a value of 10^{-8} up to 10. In Fig. 9 we show the transmission phase time τ_p for different values of γ/ω_0 keeping the other

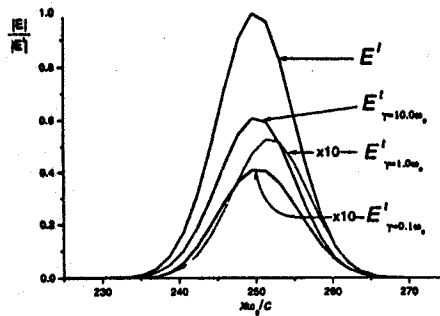


FIGURE 10. Snapshots of a freely traveling pulse and transmitted pulses for different values of the dissipation coefficient γ/ω_0 .

model parameters fixed, $\omega_p/\omega_0 = 2$, $d\omega_0/c = 5$. Note that, as the dissipation is increased, the superposition approximation is less valid because the resonance line becomes wider (i.e. around ω/ω_0). For the same reason, the stationary phase approximation begins to break down as γ/ω_0 increases. We can, however, calculate numerically the transmission of a pulse and assess the robustness or not of the phenomenon. In Fig. 10 we show snapshots of transmitted pulses for different values of the dissipation parameter. We find that up $\gamma/\omega_0 \approx 0.1$ the peak of the transmitted pulse is still ahead of the peak of a freely traveling one, essentially the same distance as in the case of no dissipation. For large values of the dissipation, say $\gamma/\omega_0 \approx 10$, first, the gap disappears and the transmission coefficient is almost independent of the frequency; thus, the pulse passes through a region

where the dielectric function is almost constant yielding a transmitted pulse with no *superluminal* effect but with larger intensity than in the case of no dissipation. The most interesting case is when the dissipation coefficient is in the intermediate regime, namely $\gamma/\omega_0 \approx 1$. In this situation, as we can see from Fig. 9c, neither the superposition approximation nor the stationary-phase approximation can be used. However, one finds from the exact calculation that the transmitted pulse is still *superluminal* and, because of the large value of the dissipation, its intensity is higher than in the dissipationless case; an additional result is that the central frequency of the transmitted pulse is shifted from the corresponding value of the incident frequency. This latter case deserves a more extensive study but it is out of the scope of the present work.

We close this section by remarking that although the above-mentioned tunneling experiments which measure the peak velocity and the pulse duration of the transmitted pulse may be fully interpreted and explained within causal classical electrodynamics, there are still many others questions, such as group, front and energy velocities [24], experimentally accessible and verifiable, whose full understanding and elucidation deserves further attention.

Acknowledgments

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