

## **Atomic bremsstrahlung of Al, Ag and Au targets bombarded with 1.5 MeV protons**

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**Abstract.** Continuous X rays produced in light-ion atom collisions have been experimentally and theoretically studied. The experimental results for aluminum target bombarded with ~1.5 MeV protons can be mainly explained by atomic bremsstrahlung of K-shell electrons. We measured the continuous x-ray spectra from a silver target and gold target bombarded with 1.5 MeV protons to investigate atomic bremsstrahlung of L- and M- shell electrons and found that the experimental production cross sections of continuous x-rays are much smaller than the predictions based on the PWBA theory of AB. The discrepancy can be reduced by taking account of a screening effect previously introduced, but this does not resolve the problem completely. We applied the theory of AB based on the Binary Encounter Approximation (BEA) and compared this with the experiment. It is shown by the BEA theory that, without consideration of the screening effect, the agreement between the theory and the experimental cross sections of Al target, Ag target and Au target can be systematically explained by the atomic bremsstrahlung of K-, L- and M- electrons, respectively.

**Keywords:** Continuous X rays, Bremsstrahlung, SEB, AB, PIXE, BEA, PWBA

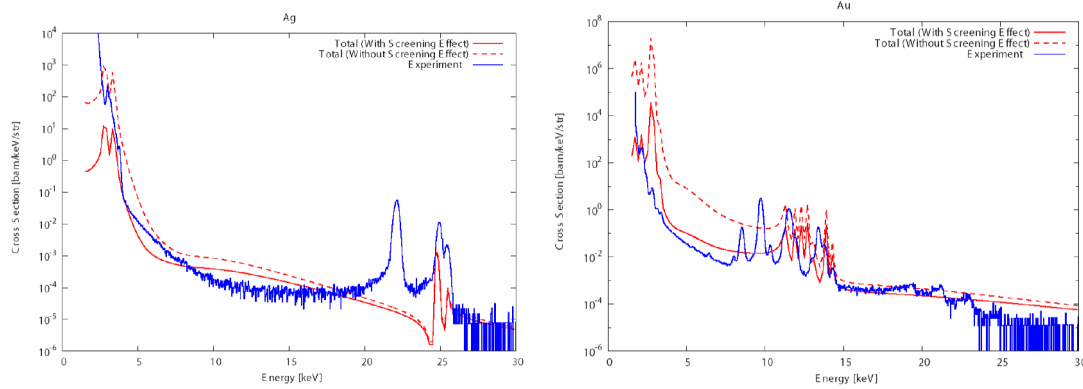
### **INTRODUCTION**

By bombarding a solid target with charged particles such as protons or  $\alpha$  particles, characteristic X-rays of atoms as well as continuous X-rays are produced. It has been experimentally shown that, in the case of light-ion-atom collisions, the production cross sections of continuous X-rays depend on the velocity of incident particles and are proportional to the square of projectile charge. This result suggests that the continuous X-rays are produced through collision processes between target atom and projectile, as well as through inner shell ionizations by heavy charged particles. Up to the present, the following radiative processes have been considered concerning the origin of continuous X-ray production in light-ion atom collisions: Secondary electron bremsstrahlung (SEB)<sup>1,2</sup>, Quasi-free electron bremsstrahlung (QFEB)<sup>3</sup>, Radiative ionization (RI)<sup>4</sup>, Atomic bremsstrahlung (AB)<sup>5,6</sup> (also called polarization bremsstrahlung<sup>7</sup>) and Nuclear bremsstrahlung (NB)<sup>8</sup>. These radiation processes are summarized in Ref. 9 and Ref. 10.

AB is the radiation produced by inner shell electrons of a target atom excited by the projectile to continuum states sequentially returning to the original states. It has been confirmed that, in the case of an Al target bombarded with protons of less than 1.5 MeV, the continuous X-rays in the energy region of a few keV are mainly AB from K-shell electrons<sup>6</sup>. According to the previous works, the main contributions of continuous X-rays above the X-ray energy of K-X-ray, L-X-ray and M-X-ray are the atomic bremsstrahlung of K-, L- and M-shell electrons, respectively. In this work, we discuss these contributions with the theories of Plane Wave Born Approximation (PWBA) and the Binary Encounter Approximation (BEA) which is introduced here.

## COMPARISON WITH PWBA THEORY

We measured recently<sup>11)</sup> the continuous X-rays produced in a gold target bombarded with 1.5 MeV protons and compared the production cross sections with the predictions of the PWBA theory of AB<sup>6)</sup>. The theory overestimated the experimental measurements considerably. To reduce this discrepancy, we have previously introduced the idea of screening the projectile charge with target electrons and could thereby improve the agreement between theory and experiment<sup>11)</sup>. As seen on the right hand side of Fig. 1, however, the experimental spectrum cannot be completely explained by the PWBA theory. Since the continuous yields above 15 keV are explained by the piling-up events of Au-L-X-rays, the theory greatly overestimates the experiment in this X-ray energy region.



**FIGURE 1.** Continuous X-ray cross sections of silver target and gold target bombarded with 1.5 MeV protons and measured at 90 degrees with respect to the beam direction. The dashed line and solid lines show the calculations of PWBA theory without and with projectile charge screening effect, respectively.

To investigate the contribution of L-shell electrons, we measured continuous X-rays from a thin Ag target bombarded with 1.5 MeV protons from the 4.5 MV Dynamitron accelerator of Tohoku University at 90 degrees with respect to the beam direction. At such a low projectile energy, SEB can be neglected and AB is predominant. A comparison between the PWBA calculations of AB and experiment is shown in the left hand side of Fig. 1. The theoretical shape of the continuous X-ray spectrum is very different from the experimental one. The resonance terms in the T-matrix strongly affect the spectrum shape of AB and do not reproduce the experimental results. Therefore, a calculation not including the resonance terms is desired. With such considerations, we introduced a method to calculate the cross sections of AB on the basis of the binary encounter approximation (BEA).

## BINARY ENCOUNTER APPROXIMATION OF ATOMIC BREMSSTRAHLUNG

In the BEA theory, inner shell electrons in an atom are assumed to be free electrons, and the ionization cross section  $\sigma^i$  is derived from the Rutherford cross section  $\sigma^R$  between projectile and inner shell electrons<sup>12)</sup>. The inner shell ionization cross sections  $\sigma^i$  can be expressed by,

$$\sigma^i = \int_0^\infty dv f(v) \int_{\hbar\omega}^\infty dw \int dq \sigma^R \quad (1),$$

where  $f(v)$  is the velocity distribution of inner shell electrons,  $w$  is the energy transferred from the projectile to an inner shell electron and  $q$  is the momentum transfer between the projectile and the inner shell electron. On the other hand, the non-relativistic formula of the production cross section of continuous X-rays in collisions between inner shell electrons and a projectile can be expressed by,

$$\frac{d^2 \sigma^{\text{electron-brems}}}{d\hbar\omega d\Omega} = \sigma^R \frac{1}{4\pi^2 (m_e c)^2} \frac{e^2}{\hbar c} \frac{1}{\hbar\omega} (\vec{q} \times \vec{e}_\omega)^2 \quad (2),$$

where  $\vec{e}_\omega$  is the unity vector of direction. From Eqs. (1) and (2), the cross section of continuous X-rays produced by collisions between inner shell electrons and the projectile is given by<sup>13)</sup>,

$$\frac{d^2 \sigma^{\text{Brems}}}{d\hbar\omega d\Omega} = \int_0^\infty dv f(v) \int_{\hbar\omega}^\infty dw \int dq \sigma^R \frac{1}{4\pi^2 (m_e c)^2} \frac{e^2}{\hbar c} \frac{1}{\hbar\omega} (\vec{q} \times \vec{e}_\omega)^2 \quad (3).$$

In Eq.(3), the integration of transfer energy is taken from  $\hbar\omega$  to  $\infty$ , therefore the cross section given by Eq.(3) contains the contributions of AB and also radiative ionization (RI). However, it is seen in Ref. 6 that the contribution of RI can be neglected compared to that of AB.

Equation (3) can be expressed by,

$$\frac{d^2\sigma^{Brems}}{d\hbar\omega d\Omega} = C_1 - C_2 + \frac{1}{2}(3C_2 - C_1)\sin^2\theta_\omega \quad (4),$$

where

$$C_1 = \frac{1}{4\pi^2(m_e c)^2} \frac{e^2}{\hbar c} \frac{1}{\hbar\omega} \sum_k N_k \int_0^\infty dv f_k(v) \int_{\hbar\omega}^\infty dw \int q^2 dq \sigma^R,$$

and,

$$C_2 = \frac{1}{4\pi^2(m_e c)^2} \frac{e^2}{\hbar c} \frac{1}{\hbar\omega} \sum_k N_k \int_0^\infty dv f_k(v) \int_{\hbar\omega}^\infty dw \int \left(\frac{w}{v_P}\right)^2 dq \sigma^R.$$

$N_k$  and  $f_k(v)$  are the number of electrons of  $k$ -shell state and their velocity distribution, respectively. The velocity distribution  $f_k(v)$  of inner shell electrons is classically obtained by<sup>12)</sup>

$$f_k(v) = \frac{32v_k^5 v^2}{\pi(v_k^2 + v^2)^4} \quad \text{with} \quad U_k = \frac{1}{2} m_e v_k^2 \quad (5),$$

where  $U_k$  is the binding energy of the  $k$ -th shell electron.

According to the method introduced by Bonsen and Vriens<sup>14)</sup>, we can calculate  $C_1$  and  $C_2$  as follows,

$$C_1 = \frac{1}{\pi(m_e c)^2} \frac{e^6}{\hbar c} \frac{1}{\hbar\omega} \frac{z_P^2}{v_P^2} \sum_k N_k \left\{ \int_0^{v_1} dv f_k(v) \left[ \ell n \left( \frac{2m_e v_P (v_P - v)}{\hbar\omega} \right) + \frac{v_P}{v} \left( 1 + \frac{v}{2v_P} \right) \ell n \left( \frac{v_P + v}{v_P - v} \right) - \frac{1}{2} \left( 4 - \ell n \left( 1 - \left( \frac{v}{v_P} \right)^2 \right) \right) \right] \right. \\ \left. + \int_{v_2}^\infty dv f_k(v) \left[ \frac{1}{2} \left( 1 + 2 \frac{v_P}{v} \right) \ell n \left( \frac{2m_e v_P (v_P + v)}{\hbar\omega} \right) - \left( 1 + 2 \frac{v_P}{v} \right) + \sqrt{1 + \frac{2\hbar\omega}{m_e v^2}} - \frac{1}{2} \ell n \left( \frac{\sqrt{1 + \frac{2\hbar\omega}{m_e v^2}} + 1}{\left( 1 + \frac{v}{v_P} \right) \left( \sqrt{1 + \frac{2\hbar\omega}{m_e v^2}} - 1 \right)} \right) \right] \right\}$$

and

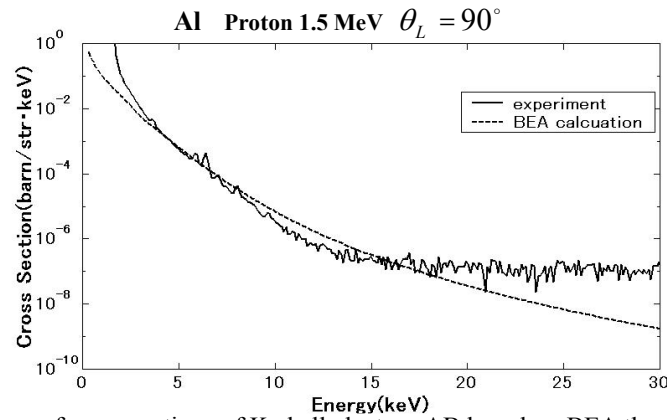
$$C_2 = \frac{1}{\pi(m_e c)^2} \frac{e^6}{\hbar c} \frac{1}{\hbar\omega} \frac{z_P^2}{v_P^2} \sum_k N_k \left\{ \int_0^{v_1} dv f_k(v) \left[ 1 - \frac{v}{v_P} - \frac{\hbar\omega}{2m_e v_P^2} + \frac{1}{3} \left( \frac{v}{v_P} \right)^2 \ell n \left( \frac{2m_e v_P (v_P - v)}{\hbar\omega} \right) \right. \right. \\ \left. \left. + \frac{v_P}{3v} \left( 1 + \frac{1}{2} \left( \frac{v}{v_P} \right)^3 \right) \ell n \left( \frac{v_P + v}{v_P - v} \right) + \frac{v_P}{v} - \frac{1}{18} \left( 12 + \left( \frac{v}{v_P} \right)^2 \right) - \frac{1}{6} \left( \frac{v}{v_P} \right)^2 \left( 4 \frac{v}{v_P} - \ell n \left( 1 - \left( \frac{v}{v_P} \right)^2 \right) \right) \right] \right. \\ \left. + \int_{v_2}^\infty dv f_k(v) \left[ \frac{1}{3} \frac{v_P}{v} \left( 1 + \frac{1}{2} \left( \frac{v}{v_P} \right)^3 \right) \ell n \left( \frac{2m_e v_P (v_P + v)}{\hbar\omega} \right) + \frac{1}{2} \left( \left( 1 + \frac{v}{v_P} \right) - \frac{\hbar\omega}{2m_e v_P^2} \right) \right. \right. \\ \left. \left. - \frac{v_P}{36v} \left( \left( \frac{v}{v_P} + 2 \right)^3 - \left( \sqrt{\left( \frac{v}{v_P} \right)^2 + \frac{2\hbar\omega}{m_e v_P^2}} \right)^3 \right) - \frac{v}{3v_P} \left( \frac{v}{v_P} + 2 - \sqrt{\left( \frac{v}{v_P} \right)^2 + \frac{2\hbar\omega}{m_e v_P^2}} \right) \right] \right\}$$

$$-\frac{1}{6}\left(\frac{v}{v_p}\right)^2 \ell n \left[ \frac{\sqrt{1 + \frac{2\hbar\omega}{m_e v^2}} + 1}{\left(1 + \frac{v}{v_p}\right)\left(\sqrt{1 + \frac{2\hbar\omega}{m_e v^2}} - 1\right)} \right] \right]$$

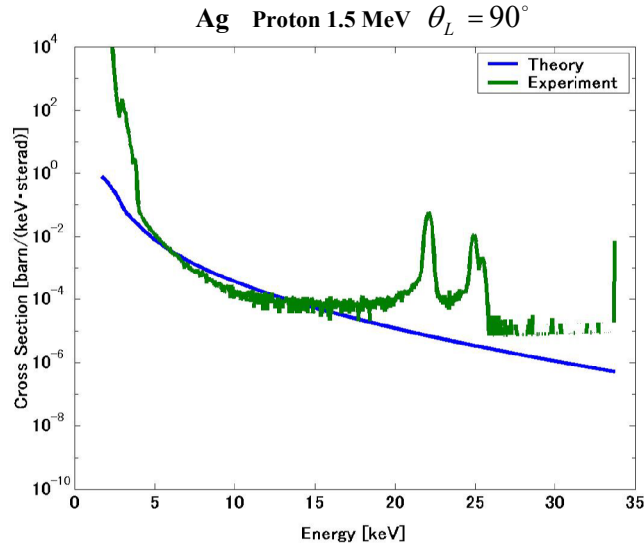
where  $v_1 = v_2 = v_p \left(1 - \frac{\hbar\omega}{2mv_p^2}\right)$  for  $\hbar\omega < 2m_e v_p^2$  and  $v_1 = 0, v_2 = v_p \left(\frac{\hbar\omega}{2mv_p^2} - 1\right)$  for  $\hbar\omega > 2m_e v_p^2$ .

## RESULT AND DISCUSSION

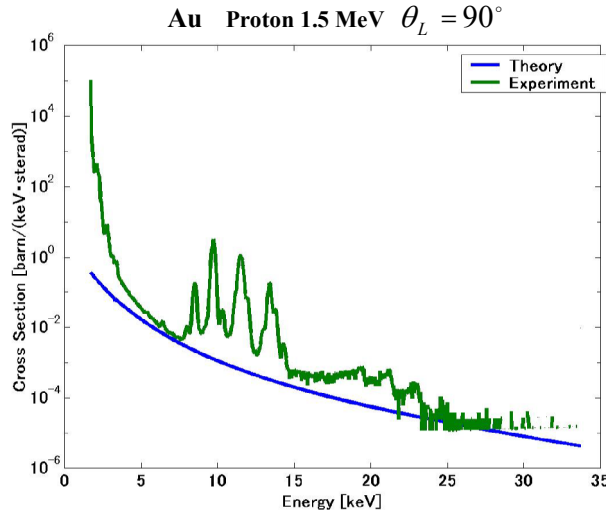
Figures 2, 3 and 4 show the comparisons between the production cross sections of continuous X-rays calculated by Eq.(4) and the experimental ones in the cases of aluminum, silver and gold targets bombarded with 1.5 MeV protons. The agreement between the BEA theory and the experiment is more satisfying than the results of the PWBA theory.



**FIGURE 2.** Comparison of cross sections of K-shell electron AB based on BEA theory with experimental results of aluminum target bombarded with 1.5 MeV protons.



**FIGURE 3.** Comparison of cross sections of L-shell electron AB based on BEA theory with experimental results of silver target bombarded with 1.5 MeV protons.



**FIGURE 4.** Comparison of cross sections of M-shell electrons AB based on BEA theory with experimental results of gold target bombarded with 1.5 MeV protons.

The yields of continuous X-rays in the energy region of 5-10 keV for each target can be explained by the BEA theory. The behavior of BEA predictions in the high X-ray energy region is quite reasonable while the PWBA predictions overestimate the experimental data. This result shows that it is not necessary to introduce the idea of projectile charge screening effect in the process of atomic bremsstrahlung and necessitates improvements of the calculation of the resonance terms in the T-matrix of AB.

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