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AXELROD MODEL OF SOCIAL INFLUENCE WITH CULTURAL HYBRIDIZATION

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Since cultural interactions between a pair of social agents involve changes in both individuals, we present simulations of a new model based on Axelrod's homogenization mechanism that includes hybridization or mixture of the agents' features. In this new hybridization model, once a cultural feature of a pair of agents has been chosen for the interaction, the average of the values for this feature is reassigned as the new value for both agents after interaction. Moreover, a parameter representing social tolerance is implemented in order to quantify whether agents are similar enough to engage in interaction, as well as to determine whether they belong to the same cluster of similar agents after the system has reached the frozen state. The transitions from a homogeneous state to a fragmented one decrease in abruptness as tolerance is increased. Additionally, the entropy associated to the system presents a maximum within the transition, the width of which increases as tolerance does. Moreover, a plateau was found inside the transition for a low-tolerance system of agents with only two cultural features.

Keywords: Sociophysics; Axelrod model; multivariate models of consensus formation.

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1. Introduction

In Axelrod's original model,[1] interaction among agents (or individuals) follows two main trends: (1) agents who culturally share more features have a greater chance of interaction than those who share less; (2) the purpose of interaction is to increase the cultural similarity of the agents. One of the most interesting results obtained from this model is the appearance of a phase transition from a monocultural state to a fragmented one as the cultural variability (usually represented by an integer q) is increased, which in turn shows that it is possible to have culturally fragmented states

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through local attraction. Within Axelrod's model, a pair of agents who differ by one unit in each feature is regarded as different as those who differ by $q-1$ units. This has been thought to be a disadvantage of the model since many cultural issues could be better described by continuous scales and this is the reason why models, which actually take into consideration the amount of difference between agents, have been developed.[2–5] For example, some models[5,6] assign a "metric" to the values that the agents' features can take, such that the contribution to the overlap of a given feature is given by $[1-(\sigma_{i,f}-\sigma_{j,f})/(1-q)]/F$, where $\sigma_{i,f}$ is the value of the fth feature of ith agent. In this work we consider a model similar to Axelrod's homogenization mechanism but including cultural hybridization between agents. The term hybridization implies that individuals' features change simultaneously upon interaction as opposed to only one individual changing one of its features. It also implies that change can occur through continuous variations of a feature, instead of discrete ones. This model takes into consideration these two behaviors present in real social interactions in order to see how they affect the behavior of these systems.

2. The Hybridization Model

In the original Axelrod model,[1] the system consists on a square lattice with periodic boundary conditions in which each node represents an agent. Associated to each agent there is an F-length vector where each entry corresponds to a cultural attribute or feature, such as language, religion, etc. The values for each entry are chosen randomly from a uniform discrete distribution between one and a positive integer q, where q is also known as the cultural variability of the system. For the hybridization model, the assignment of values for the features is the same as the version used by Castellano *et al.*,[7] in which the values are extracted from a Poisson integer distribution of values with mean q. This allows for the exploration of the behavior in systems with very low cultural variability. In the original model proposed by Axelrod, once the lattice is set, a pair of neighbors will interact with a probability equal to the fraction of overlapping cultural features (i.e. the proportion of features they share). When agents interact, one of the two (randomly selected) copies a randomly chosen feature from its neighbor (also randomly chosen from all the features they did not previously share). This procedure is carried on until the lattice reaches a frozen state, in which all the agents in the lattice have a zero probability of modifying their features, either because all their features are shared with their neighbors and no subsequent interaction will change either of them, or because they share no feature whatsoever with them. The transition from a monocultural state to a fragmented one occurs at a critical value for q (q_c), which tends to increase as F grows. As we stated before, in most of the previous models, the agents' features are regarded as a set of independent symbols. Agents can only be completely different or completely equal, with no intermediate state. Additionally, interaction consists of one agent becoming equal in one feature to its neighbor, within a discrete set of values for the features. There is, therefore, no metric defined which may allow to determine whether an agent is more or less similar to a neighbor. This metric can be implemented by means of a new parameter, τ, that represents the maximum absolute difference which two agents must have for a given feature in order to be considered different. In our model, interaction will take place under the same conditions as proposed by Castellano *et al.*[7] Agents will interact if they are similar, as described above, within a value for the stated parameter, for a randomly chosen feature. The larger the value of the parameter, the higher the likelihood that two agents will be considered similar. Additionally, after the dynamics have concluded, two neighboring agents are considered as belonging to a same cluster if the absolute value difference of each of the features is less than the value of τ. We shall therefore name this parameter (τ) tolerance. Interaction will, additionally, increase the similarity of the agents by assigning a randomly chosen feature (which can be the same feature used for the initial comparison) the average value of the ones the chosen feature previously had for both agents. The dynamic process for the hybridization model can be summarized by the following algorithm:

(1) The features of the agents are originally assigned with integer values $0, 1, 2, 3 \ldots$ taken from a Poisson distribution with mean q, so that the probability of extracting the value k is $P(k) = q^k e^{-q}/k!$
(2) The lattice is swept in an orderly fashion. For each of the agents considered, one of its four nearest neighbors is randomly chosen. These two individuals make up the interacting pair.
(3) Once the interacting pair has been chosen (say, agents i and j), their similarity is measured by calculating the absolute value difference for a randomly chosen feature (f).
(4) If the absolute value difference is less or equal than the tolerance τ ($|\sigma_{i,f} - \sigma_{j,f}| \leq \tau$), a feature is randomly chosen (let us denote this new feature as f') and the new value of this feature for both individuals will be the average of the values prior to interaction. In this way, in case of interaction, the new feature's value shared by both interacting agents is given by $\sigma^*_{i,f'} = (\sigma_{i,f'} + \sigma_{j,f'})/2 = \sigma^*_{j,f'}$, where $*$ indicates the value posterior to interaction. On the other hand, if $|\sigma_{i,f} - \sigma_{j,f}| > \tau$, nothing happens.
(5) The algorithm checks if the system has reached the *frozen state* every $n = 10\,000$ sweeps of the lattice. If so, the procedure stops. The *frozen state* is reached when, for each first-neighbor pair of agents i and j, $|\sigma_{i,f} - \sigma_{j,f}| > \tau$; for all its features f; or $|\sigma_{i,f} - \sigma_{j,f}| \leq \tau$ for all its features f.

Previously, Jacobmeier[2] has addressed a model along similar lines to the one presented here, where N agents, forming a directed Barabási–Albert network, have discrete opinions on several subjects. The interaction between two of them starts when the absolute value of the distance of all opinions to each other is less than a given value and they can get closer after each repeated interaction.

3. Results

In order to characterize the frozen state, we study the normalized average size of the largest region or cluster $\langle S_{\max}\rangle/N$, where $N = L^2$ is the total number of agents, and the entropy of the system as a function of q. The corresponding averages of $\langle S_{\max}\rangle$ are taken from at least 30 realizations for each value of q. All the considered lattices have periodic boundary conditions. A point worth stressing is that the frozen state is usually reached for a number of sweeps of the lattice considerably lower than n, i.e. checking for the frozen state every n sweeps usually involves doing many more sweeps after the frozen state has been reached. However, even after having reached the frozen state, the agents could still undergo finer changes through the averaging process described above. Therefore, implementing an algorithm which checks for the frozen state every n sweeps carries the advantage of allowing the system to relax to what we call an *asymptotical state*, which constitutes a stronger version of the frozen state.

For a system with $F = 2$, $\tau = 0.25$ and $L = 40$, Fig. 1(a) shows the behavior of the normalized $\langle S_{\max}\rangle$ as a function of q for the simple frozen state and the asymptotical state. Logarithmic scale is used for q in order to better appreciate the nature of transitions in systems with low tolerance. Even though the values of $\langle S_{\max}\rangle$ for a

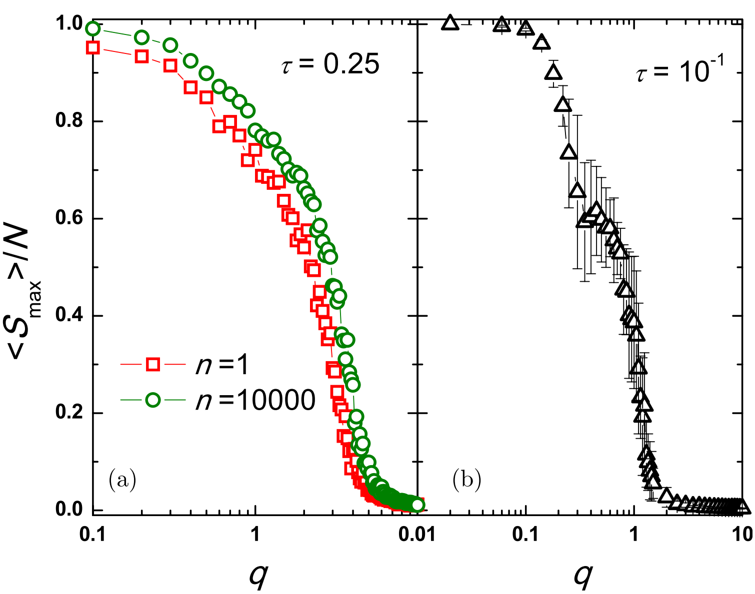


Fig. 1. (Color online) (a) Normalized $\langle S_{\max}\rangle$ vs q for a system with $F = 2$, $\tau = 0.25$ and $L = 40$. We checked at n, $2n$, $3n$, ... sweeps of the lattice whether the frozen state has been reached. Red squares and green circles correspond to $n = 1$ (simple frozen state) and $n = 10\,000$ (asymptotic state), respectively. (b) Normalized $\langle S_{\max}\rangle$ vs q for a system with $F = 2, \tau = 0.1$ and $L = 50$ showing the fluctuations (standard deviations) as error bars. Throughout this work, fluctuations diverged only within the transition from a monocultural state to a fragmented one. They are not presented in every graph in order not to crowd the image.

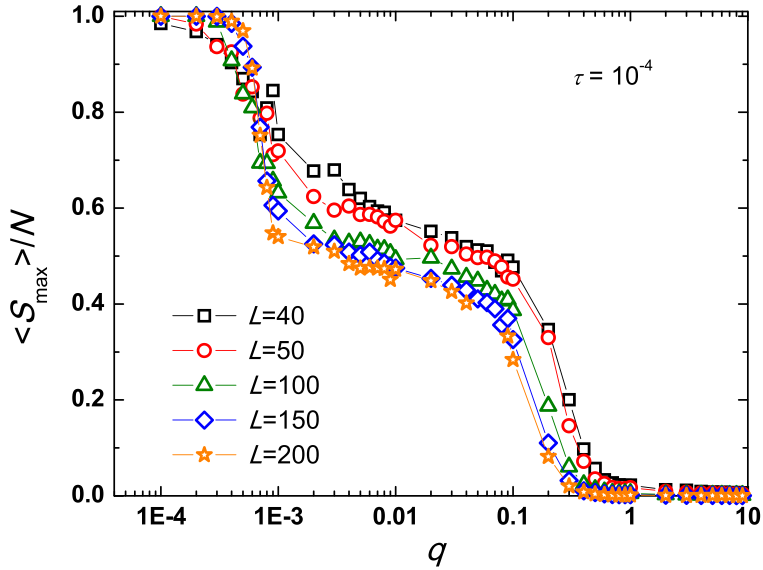


Fig. 2. (Color online) Normalized $\langle S_{\rm max}\rangle$ vs q for a system with $F = 2$, $\tau = 0.0001$ and $L = 40$, 50, 100, 150 and 200.

particular q in the asymptotical state are slightly larger than those in the simple frozen state, q_c remains the same for both cases. The slightly larger values found in the asymptotical state respond to the fact that the system, when using the asymptotical algorithm, is generally allowed to relax much more after the frozen state has been found, allowing the size of the largest cluster to grow further. All the following figures presented in this paper were obtained by using the asymptotic state criterion instead of the simple frozen state. Figure 1(b) is presented as an example that shows the typical behavior of the fluctuations of the values of $\langle S_{\rm max}\rangle$, defined as their standard deviations. Figure 2 shows $\langle S_{\rm max}\rangle$ versus q for systems with $F = 2, \tau = 0.0001$ and $L = 40, 50, 100, 150$ and 200; whereas Fig. 3 shows $\langle S_{\rm max}\rangle$ versus q for systems with $F = 2, L = 100$ and nine different values of τ. An interesting and unexpected result found for systems with $F = 2$ and $\tau \to 0$ is that the transition from a monocultural state to a fragmented one is given by a sharp jump of the size of the largest cluster from close to one to about $1/2$ for increasing q, followed by an interval of reduced slope or "*plateau*" and then another jump of $\langle S_{\rm max}\rangle/N$ to nearly zero. Notice in Fig. 2 that by augmenting the system size L, the sharpness of these jumps increases as expected for phase transitions.

Moreover, when τ increases for a fixed system size (Fig. 3), the passage from a monocultural state to a fragmented one becomes smoother and the "*plateau*" disappears. As expected, more tolerant societies present a higher or equal value of q_c than less tolerant ones, which in turn suggests that systems with higher tolerance present higher resilience to fragmentation. The generation of individual clusters in these systems requires the use of very high values for q. Additionally, more tolerant

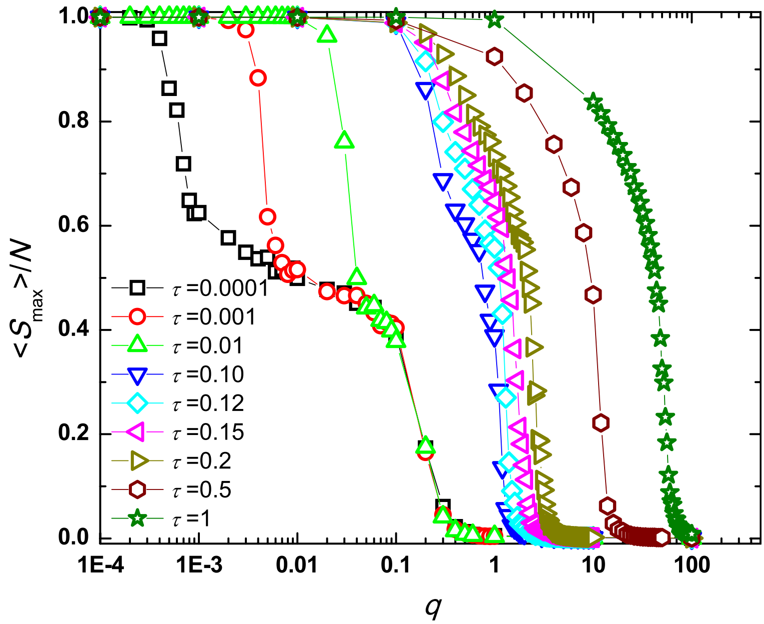


Fig. 3. (Color online) Normalized $\langle S_{\max}\rangle$ vs q for a system with $L = 100$, $F = 2$ and various values of tolerance (τ).

systems allow for a larger number of intermediate values of the normalized $\langle S_{\max}\rangle$ within the transition. Also, the q range for which intermediate values of the normalized $\langle S_{\max}\rangle$ are found is also larger, showing that various degrees of cultural variability may allow for a diversity of cultural regions when the tolerance is large enough. Moreover, the appearance of the "*plateau*" for systems with $F = 2$ and $\tau \to 0$, suggests a certain resistance of the system to undergo fragmentation despite increasing q. To better understand this "*plateau*," we present, in Fig. 4, a contour plot of one of the agents' features in the system with $\tau = 0.01$ and $q = 0.06$ (Fig. 4(a)), $q = 0.08$ (Fig. 4(b)) at the asymptotical state.

Figure 4 presents a visual guide of the values of the agents for the system described and the distance in values from one color to the next corresponds to twice the tolerance considered in the system. The clusters with a pattern inside represent agents whose values for the features are far away from those present in the majority of agents. However, from the size of these clusters it is obvious that they are not part of the largest cluster. Notice in Fig. 4(a) a large cluster, formed by those agents whose analyzed feature has a value between 0.10 and 0.12, which seems to be interconnected by thin strands. It can be seen in Fig. 4(b) that, whereas the blocks constituting the large cluster reduce in size as q increases, the strands connecting the blocks in the large cluster first become thinner at a lower rate, rather than breaking as q increases and thus allowing for the large cluster not to fragment itself yet, thus originating the plateau. However, further increments of q will force more variability

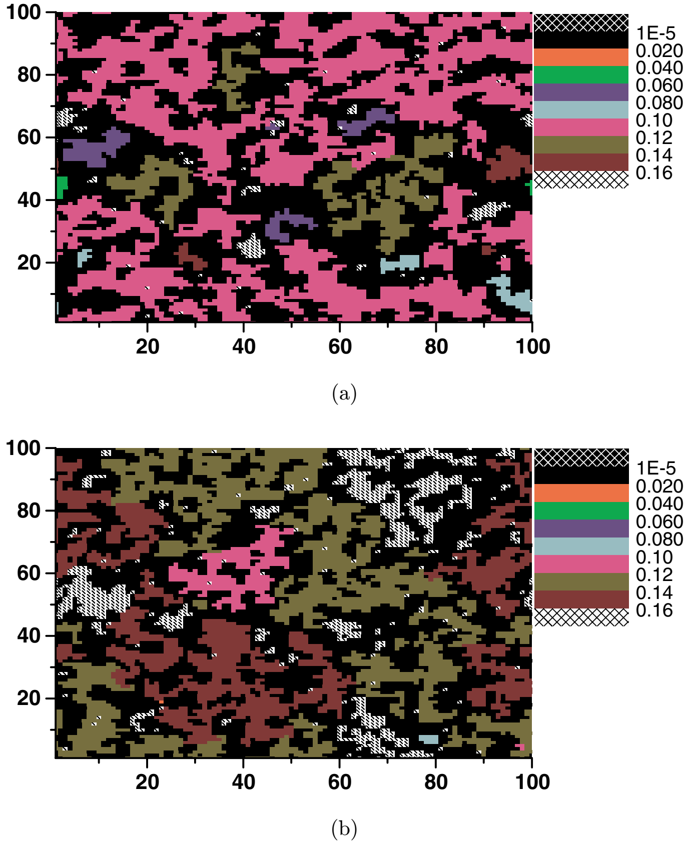


Fig. 4. (Color online) Contour plot for one of the features of the agents in systems with $L = 100$, $F = 2$, $\tau = 0.01$, (a) $q = 0.06$ and (b) $q = 0.08$. The values chosen for q place the system right at the beginning of the plateau (a) and right at the end (b).

upon these agents, breaking the large cluster into smaller ones and leading the transition to a fragmented frozen state. For systems with $F = 2$, the presence of the plateau was a constant feature regardless of the value of L. However, such feature is absent in systems with $F > 2$. The probability of interaction between agents given by $P = (k/F)(1 - k/F)$ where k is the number of features in common from the total F, has a value of $1/4$ if $F = 2$ and k is not equal to zero or to F. Therefore, the probability of interacting for individuals with $F = 2$ can only be 0 (if they already share all features or if they share none) or $1/4$ (if they only share one feature). The interaction probability values that systems with $F > 2$ may adopt is typically lower than this maximum value. As in previous models, the transitions from monocultural states to fragmented ones are reached for higher values of q as F increases.[1,7] However, in contrast with them, the transitions for large values of F are less sharp, even for large values of L. This is in agreement with the idea that tolerant societies allow for the coexistence of various considerably large communities for large spans of

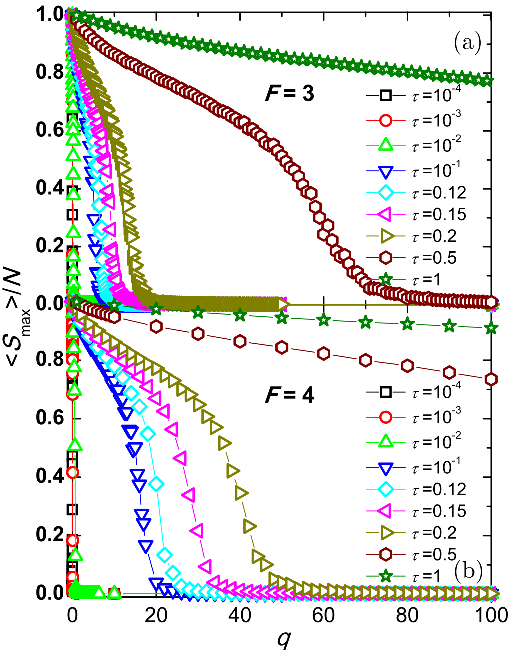


Fig. 5. (Color online) Normalized $\langle S_{\max}\rangle$ vs q for a system with $L = 100$ and (a) $F = 3$, (b) $F = 4$ and various values of τ.

cultural variability, instead of just allowing for monocultural and highly fragmented systems. This can be seen in Fig. 5 where we show the normalized $\langle S_{\max}\rangle$ as a function of q for systems with $L = 100$ and $F = 3, 4$. Figure 6 shows the same for a system with $L = 100$ and $F = 7, 10$.

A useful quantity that characterizes a given system is the entropy, which can be defined as: $\mathcal{S} = -k_B \sum_{m=1}^{N} P_m \ln P_m$, where P_m is the probability of an agent of belonging to a cluster of m individuals and k_B is the Boltzmann constant.[8] This entropy was originally used as function of $1/q$, since $1/q$ represents the probability of adopting a particular value for a feature if q is used as the cultural variability in the original Axelrod model.[8] However, since in this work q represents the mean of a Poisson distribution of values, it already stands for a probability and the entropy is plotted as a function of q. Since $P \ln P = 0$ for both $P = 0$ and $P = 1$, the entropy adopts values near zero in two cases: (a) when the largest cluster tends to span the whole lattice and (b) when the lattice is completely fragmented. Nonzero values for the entropy can thus be expected when there is coexistence of several cluster sizes within the same system (a “disordered state”). The entropy as a function of q is shown in Fig. 7 for systems with $L = 100$, $F = 2, 3, 4$; and two values of τ.

The behavior of the entropy consists in the presence of maxima, the width of which depends on the value of τ. For smaller tolerance values, the peaks are more narrow. Their width grows as τ increases. As can be seen in Fig. 7(b), the peaks for

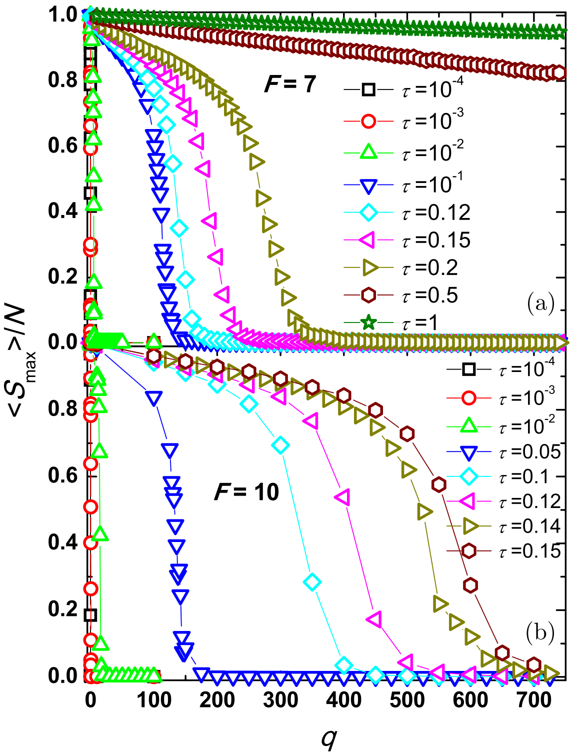
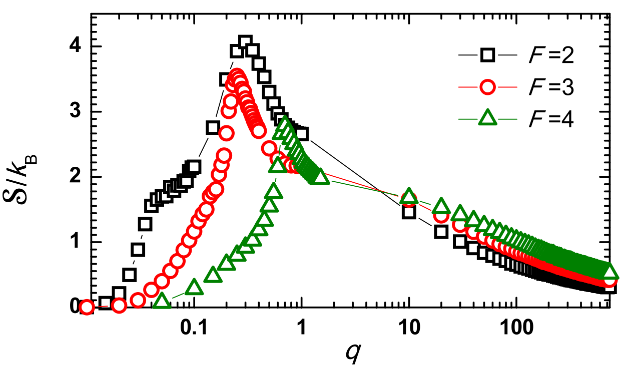
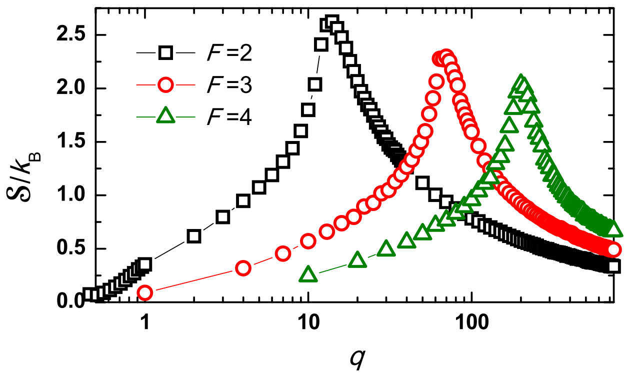


Fig. 6. (Color online) Normalized $\langle S_{\max} \rangle$ vs q for a system with $L = 100$ and (a) $F = 7$, (b) $F = 10$ and various values of τ.



(a)

Fig. 7. (Color online) The associated entropy $\mathcal{S}$ vs q for systems with $L = 100$, $F = 2$, 3, 4 and (a) $\tau = 0.01$, (b) $\tau = 0.5$.



(b)

Fig. 7. (*Continued*)

$\tau = 0.5$ are considerably wide, consistent with the large span of the values of q involved in the passage from a monocultural state to a fragmented one. Additionally, for the smallest values of τ included, the entropy for $F = 2$ shows a plateau consistent with the plateau discussed for Fig. 3.

4. Conclusions

The smooth transition from a monocultural state to a fragmented one present in this model shows that coexistence of various different communities is possible when dealing with a highly tolerant society, even when considering a large span of values representing the cultural variability. The presence of the "*plateau*" for systems with low tolerances and the lowest number of features ($F = 2$) responds to the fact that $F = 2$ presents a higher probability of interaction among agents, which in turn allows for the preservation of a large cluster despite slight increases of the cultural variability q. The entropy as a function of q presents peaks corresponding to the value range of q for the phase transition. These peaks decay very slowly due to the smooth nature of the transitions from a monocultural towards a fragmented state. As expected, both the span of values of q in which the transition occurs, as well as the width of the peaks in the entropy, increase as the tolerance and the number of features (F) increase. Also, the larger offset of q_c present in systems with large tolerances suggests the possibility of supporting partially homogenous societies even when the cultural variability is high.

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