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Optimizing strategic blocks with asymmetric bilateral propensities with symmetric propensities

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Resumen

Hay modelos tomados de la física que se han utilizado para explicar la formación de coaliciones o bloques de agentes. Estos modelos son útiles para entender cómo las alianzas (en guerras, partidos políticos, etc.) tienden a agrupar amigos en mismos bloques y a enemigos en bloques separados minimizando la frustración total. Todos estos modelos suponen interacciones recíprocas entre los agentes, ya que este es el caso más común en la física. Sin embargo, se destaca el importante hecho de que las interacciones humanas no son, en general, de reciprocidad, es decir, no existe una "tercera ley de Newton social". Aquí mostramos que este defecto fundamental de los modelos de la coalición puede ser resuelto mediante la construcción de interacciones simétricas efectivas (en las que los modelos físicos bien conocidos funcionan) a partir de interacciones no recíprocas. En los casos de varios modelos con afinidades asimétricas se proponen varias estrategias cualitativas para lograr interacciones simétricas efectivas. En varias de estas estrategias empleamos el valor medio simétrico en el que las afinidades compiten entre sí para mantener cierta información parcial de las tendencias asimétricas.

Palabras clave: Dinámica de sistemas sociales, estructuras y organización de sistemas complejos, vidrios de espín y otros magnetos aleatorios

Abstract

There are models taken from physics that have been used to explain the formation of coalitions or blocks of agents. Such models are useful to understand how alliances (in wars, political parties, etc.) tend to cluster friends in same blocks and enemies in separate blocks by minimizing the total frustration. All of these models assume reciprocal interactions between agents, since this is the most common case in physics. However, we point out the important fact that human interactions are, in general, not reciprocal, *i.e.*, there is no "social Newton´s third law". Here we show that this fundamental flaw of coalition models can be solved by constructing effective symmetric interactions (in which well-known physical models work) from non-reciprocal interactions. For various model cases with asymmetrical propensities we propose several qualitative strategies to achieve effective symmetric interactions. In many of these strategies we employ the symmetric average value in which propensities compete in strength to keep some partial information of the asymmetrical propensities.

Keywords: Dynamics of social systems, Structures and organization in complex systems, Spin glasses and other random magnets.

Introducción

Modeling conflicts and cooperation among social agents is an important research subject due to its relevance for understanding key aspects of human organization[3] . In the last years there has been a renewed interest in this field due to three factors; i) the recognition of human societies as complex systems, ii) new models employing physical concepts (sociophysics and econophysics), and iii) powerful computers to simulate large systems.

Since there exist complex structures in familiar, cultural, monetary, commercial, political, military systems, etc. that clearly exhibit the existence of blocks, here we address the formation of coalitions or blocks of agents at a general level. The main ideas behind these models are two-fold; the existence of bilateral affinities or propensities among agents or actors and some kind of algorithms that leaders (chiefs or leaders of states, political parties,

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3 See for example, different related topics in Axelrod (2006), Weisbuch (1991).

or any other set of people) employ in order to take decisions. From the point of view of complexity theory, the processes of taking decisions or assuming responsibilities are in fact emerging properties of complex networks.

Generally, in social sciences, it is desirable to avoid conflicts, dissatisfactions and frustrations. For instance, frustration can be defined as the prevention of the progress, success, or fulfillment of something. "Frustration" in our systems has a very precise meaning arising in two cases; when friends are in different blocks or alliances, and when two enemies are clustered together.

So first we will employ bilateral "propensities" or "affinities" that are defined to measure or estimate friendship and enmity between agents. Then our goal is to find the optimum partition of the set of agents trying to make clusters (as much as possible) of friends in same subsets or blocks and to set enemies in separate blocks. The key point is to minimize the total frustration by optimizing the partition of the whole set of agents in blocks.

We will employ bilateral "propensities" or "affinities" that are defined to measure or estimate friendship and enmity between agents, and then we look for guidance in physical systems to minimize frustration, since in physics many laws can be written as a minimization of some variables. Geometrical frustration is an important feature in magnetism. It stems from the topological arrangement of magnetic entities or spins with competing magnetic coupling parameters. For example, ferromagnetic interactions between two magnetic entities or spins tend to align magnetic moments parallel to each other, and antiferromagnetic interactions tend to align magnetic moments antiparallel to each other. Thus, it is possible that a magnet experiences opposite competing influences from neighbouring magnets. If there is ferromagnetic interaction between magnets A and B and they are *antiparallel* to each other (due to stronger interactions of A and B with other magnets), then one says that there exists magnetic frustration. That is, the presence of conflicting interactions forbids simultaneous minimization of the interaction energies.

In particular, spin glasses (Nishimori, 2001) are disordered materials that exhibit a high magnetic frustration due to competing interactions. Coalition forming in social sciences has been studied using concepts from the theory of spin glasses (Florian & Galam, 2000; García *et al.*, 2007; Samaniego-Steta *et al.*, 2008). The analog of the magnetic coupling parameter in social systems is the propensity or affinity between agents. The tendency of two social agents either to be in conflict or to be in different blocks (antiferromagnetic) or to cooperate or to be in the same block (ferromagnetic) is simulated similarly to magnetic systems. These models have been applied to the dismembering of Yugoslavia (Florian & Galam, 2000) and to the Iraq invasion (García *et al.*, 2007; Samaniego-Steta *et al.,* 2008) in the year 2003, where it is assumed that different political opinions or regimes, religions, ethnic groups, etc. tend to split agents in different blocks. In García *et al.* (2007) and Samaniego-Steta *et al.* (2008) it is also shown that in some cases the theory has to go beyond bilateral propensities in order to reproduce the main features of the required interactions, and thus it must include three-body interactions.

In physics almost all models assume a reciprocal interaction, *i.e.*, a kind of Newton´s third law. However, human relations are not reciprocal. In general, the affinity or propensities (empathy, affection, friendship, hatred, love, amount of money owed to other person, etc.) are not symmetric. For instance, negative relationships may have more importance than positive relationships to understand outcomes in the context of social networks in work organizations (Labianca & Brass, 2006). In decision theory or in artificial intelligence (Bayesian or belief networks), asymmetric networks are employed (Hertz *et al.*, 1991).

On the other hand, as mentioned in Galam & Vinogradova (2002), there exist instabilities in alliances when three agents are involved (conflicting triangular network), as in the historical examples such as the triangle of England, Spain and France, the countries of the whole European Union, the Soviet and the Western camps (Galam & Vinogradova, 2002). These cases illustrate the existence of instability in the formation of coalitions during a significant period of their history.

Usually, bilateral propensities have been modeled as symmetrical (Florian & Galam, 2000; García *et al.*, 2007; Samaniego-Steta *et al.*, 2008), that is, propensity P_{ab} experienced from agent *a* towards agent *b* is equal to P_{ba} , but in real social systems, propensities are generally asymmetrical ($P_{ab} \neq P_{ba}$). Here we propose that asymmetrical propensities can be substituted by effective symmetrical propensities in order to employ well-known physical symmetrical-propensity models.

The layout of this work is the following. In Sect. II we present the theory of optimizing strategic blocks with symmetric bilateral propensities. In Sect. III we discuss several social scenarios or strategies with asymmetrical propensities that can be converted into simpler systems with symmetrical propensities, and finally Sect. IV is devoted to concluding remarks.

I. Optimizing partitions to minimize social frustrations in the symmetric model

We use two variables; one is the "bilateral affinity" or "propensity" P_{ij} between agents *i* and *j* and the other is their possible "distance" d_{ij} between them. In this section bilateral we assume that propensities are symmetrical, that is, propensity P_{ij} experienced from agent *i* towards agent *j* is equal to P_{ji} ($P_{ij} = P_{ji}$). The values of the affinity are defined for convenience in the interval [-1,1]. If the affinity is positive, then the individuals *i*, *j* have a friendly relationship and if it is negative it is unfriendly. It must be noticed that some of these affinities may not be measured very objectively or accurately, so these variables belong to the realm of fuzzy mathematics (Ragin, 2008).

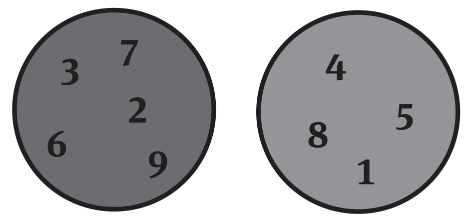
In a given set our goal is to find the optimum partition trying to cluster friends in same blocks or subsets and enemies in separate blocks or subsets. Ideally, all friends will be together, and enemies will always be in separate blocks, but in general, some frustration is unavoidable (see Fig. 1).

We now present the formalism to minimize a variable called total frustration. The family of subsets A_i of a set A is a partition if and only if:

$$
\begin{aligned}
&1.\ A_i \neq \emptyset\ \forall\, i \in I.\\
&2.\ \bigcup_{i\in I} A_i = A.\\
&3.\ A_i \cap A_j \neq \emptyset \Rightarrow A_i = A_j.
\end{aligned}
\qquad (1)
$$

The set of agents will be divided in a partition according to their relative affinities or propensities. We use two variables; bilateral affinity or propensity P_{ij} between agents *i* and *j*, and their possible "relative distance" d_{ij} between them. The values of the affinity can be scaled for convenience in the interval [-1,1]. If the total affinity is positive; $P_{ij}>0$, then the individuals *i*, *j* have a friendly relationship and if it is negative; $P_{ij}<0$, then it is unfriendly. The absolute value of a bilateral propensity represents the intensity, being neutral if it is zero. In the usual cases studied before, the propensities are symmetric, that is, $P_{ij} = P_{ji}$. It must be noticed that these affinities may not be very precise, so fuzzy mathematics could be used for these situations (Ragin, 2008). When two agents are in the same block, their relative distance will be zero, and will be nonzero – the particular value it is not important – if they belong to different subsets or blocks. Then relative distances d_{ij} are defined to take only two values; $d_{ij} = 0$ if *i* and *j* are in the same block, and $d_{ij} = 1$ if *i* and *j* are in different blocks.

Figure 1. Example of agents partitioned into two blocks



Let us suppose that propensities are such that agents 3 and 5 are friendly, and agent 5 is enemy of agent 1. Agent 5 suffers two sources of frustration; friend in another block and sharing block with unfriendly agent 1.

Source: own elaboration.

We define the frustration between these elements *i* and *j* as

$$E_{ij} = P_{ij}\, d_{ij} \qquad (2)$$

Now, let us analyze the values of E_{ij} for two different combinations of distances (*i. e.* belonging or not to the same block) and propensities:

A. If $P_{ij} > 0$, $P_{ij} = |P_{ij}|$. Then, $E_{ij} = \begin{cases} 0 \cdot |P_{ij}| = 0, ij\ \textit{friends} \\ 1 \cdot |P_{ij}| = P_{ij}, ij\ \textit{enemies} \end{cases}$,

B. If $P_{ij} < 0$, then $P_{ij} = -|P_{ij}|$. Then, $E_{ij} = \begin{cases} 0 \cdot \left(-|P_{ij}|\right) = 0, ij\ \textit{friends} \\ 1 \cdot \left(-|P_{ij}|\right) = -|P_{ij}|, ij\ \textit{enemies} \end{cases}$

Notice that, by construction, when $P_{ij} > 0$, the *minimum* value of E_{ij} occurs (and it is 0) when friendly elements *i*, *j* are in the same block, and when $P_{ij} < 0$ the *minimum* value of E_{ij} occurs (and it is -|P_{ij}|) when the elements *i,j* are enemies. In both cases, A and B, the minimum value of E_{ij} is given by the value of d_{ij} that yields the correct behavior; namely friends in the same block and, on the contrary, in different blocks, if they are enemies. Thus, minimizing the frustration E_{ij} maximizes the agent "satisfaction" yielding the desired result.

To generalize Eq. 2 for more than two elements, we can define the total frustration E_T as:

$$E_T = \sum_{i \neq j} E_{ij} = \sum_{i \neq j} P_{ij} d_{ij} \,, \qquad (3)$$

where we recall that in this section the propensities are symmetric, that is, $P_{ij} = P_{ji}$.

For a given set of propensities, each partition is uniquely defined by the set $\{d_{ij}\}$ yielding a particular value of E_T. As in the simplest case of only two agents, the set $\{d_{ij}\}$ (the ouput) that minimizes the total frustration E_T in Eq. 3 is the optimal one for a given set of propensities $\{P_{ij}\}$ (the input). Now we can understand why calling *E* a measure of frustration is really appropriate. The *strategy* of minimizing total frustration always yields the expected optimal results; states with less frustration mean more friends sharing blocks and more enemies in different blocks. Of course, it is possible to obtain partitions or states with the same energy (degenerate states), including those states with minimum energy.

II. Modeling asymmetric propensities to make them symmetrical

Now we deal with systems involving more realistic asymmetric propensities. We already mentioned that in the past the study of block formation has been modeled assuming all propensities are symmetrical, and secondly, in real social systems propensities are generally asymmetrical, in contrast to most cases of interactions in physical sciences. Newton´s action-reaction third law states that the mutual forces of action and reaction between two bodies are equal in magnitude and opposite in direction. However, human interactions are not reciprocal, *i.e.*, there is no "social Newton´s third law". In these cases Eq. 3 cannot be used since there are two unequal propensities linking two given agents. However, one can try to minimize the ***local*** frustration of a given agent by analyzing all his/her propensities with other agents, but the total frustration E_T cannot be defined in the case of asymmetric propensities. Then we have to look for possible solutions for a problem that formally does not have a simple solution. These facts are well known in the analog case of magnetic interactions, such as in spin glass systems (Florian & Galam, 2000).

In order to model something symmetrically that in principle is not symmetric, we can say that, first, there is no simple solution to this problem, so we tried to symmetrize propensities, in such a way that the new symmetric variables do contain information on the original asymmetry problem. Furthermore, depending on the particular problem, we propose different ways to pass that asymmetry information to the new symmetric propensities or affinities. Using lower-case or smaller minuscule letters (*p*) for asymmetric propensities and upper-case or majuscule letters (*P*) for asymmetric propensities, let us now discuss various cases:

Average propensities. For example, if David loves or is attracted to María with a strength of 0.8 (p_{ij} = 0.8) but María hates him or dislikes him with a strength of 0.2 (p_{ji} = - 0.2), then David wants to share the same block with her, but she does not want to share a block with him. Then, how can we satisfy both of them? Setting them in either the same or different blocks will not satisfy them simultaneously. However, their propensities imply that in this case love overcomes hate, so the average (0.8 - 0.2)/2 = 0.3 represents the total feeling. Therefore, the first choice is to take the average

$$P_{ab} = \frac{p_{ab} + p_{ba}}{2}$$

, yielding finally $P_{ab} = P_{ba}$

This means that largest propensity wins but it loses some strength by subtracting the related propensity (opposite sign). In case that p_{ab} and p_{ba} share the same sign, then the average keeps the same sign. This is the simplest and more logical choice to map asymmetric propensities systems into symmetrical ones in order to employ Eq. 3.

This case is the most natural way to implement symmetric propensities, because *i)* two positive propensities reinforce each other (if David loves María and she loves him too, then their symmetric love is stronger); *ii)* two negative propensities reinforce each other (if David hates María and she hates him too, then their symmetric hate is stronger), and *iii)* propensities of different sign compete against each other (if David loves María, but she hates him, then the *stronger* feeling will prevail in a symmetric fashion). Notice how in this way we are able to keep the most important features of the asymmetric interaction.

Now we discuss other possibilities of possible internal dynamics that yield interesting results. In some of them some parameters could be introduced to go beyond bilateral propensities in order to model "global" environments.

Copying the highest propensity. If we suppose that the system dynamics reinforces the stronger affinity, then we could imagine that the weaker affinity copies the stronger affinity:

$$P_{ab} = \begin{cases} p_{ab}, & \text{if } p_{ab} \geq p_{ba} \\ p_{ba}, & \text{if } p_{ba} \geq p_{ab} \end{cases},$$

where P_{ab} is the symmetric affinity, and $P_{ab} = P_{ba}$.

Exaggeration. Sometimes agents can be carried away by emotional excesses or tend to exaggerate their propensities. For such situations, we can define a constant *c*, above which propensities lead to an effective propensity P_{ab} larger than the average, with the sign of the largest component:

$$P_{ab} = \begin{cases} \operatorname{sgn}(p_{ab} + p_{ba}) \dfrac{|p_{ab}| + |p_{ba}|}{2}, & \text{if } p_{ab} \text{ or } p_{ba} \in [-1,-c] \cup [c,1] \\ \dfrac{p_{ab} + p_{ba}}{2}, & \text{in other case} \end{cases},$$

where P_{ab} is the symmetric affinity, and $P_{ab} = P_{ba}$.

Preference. Some propensities can be altered depending on an added parameter *k*:

$$P_{ab} = \begin{cases} \dfrac{p_{ab} + p_{ba}}{2} + k, & \text{if } p_{ab} \text{ or } p_{ba} \geq k \\ \dfrac{p_{ab} + p_{ba}}{2}, & \text{in other case} \end{cases},$$

and if P_{ab} exceeds the value 1, then it is set equal to 1. Here P_{ab} is the symmetric affinity, and $P_{ab} = P_{ba}$. Also the case of *k* with opposite sign is a possibility as an added parameter.

Average with bound. It is possible to impose a bound to increase or decrease propensities. For example, workers could be required to have positive affinities, in order to relate among all amicably. Values are averaged and a minimum bound *c* is defined, so if the average cannot be less than *c*:

$$P_{ab} = \begin{cases} c, & \text{if } \dfrac{p_{ab} + p_{ba}}{2} < c \\ \dfrac{p_{ab} + p_{ba}}{2}, & \text{in other case} \end{cases}$$

where P_{ab} is the symmetric affinity, and $P_{ab} = P_{ba}$ ..

III. Concluding remarks

We employ the general strategy of minimizing the dissatisfaction or frustration of agents, who prefer to be in blocks or groups with agents who have positive affinity avoiding those with whom they have negative propensities.

The most important contribution of this paper is to simplify the formation of blocks of complex realistic systems with asymmetrical propensities. For this purpose, we presented several qualitative models that illustrate some possible features in social systems. These features can be mixed or combined to better model specific situations.

However, there exists a rich spectrum of social situations that cannot be fully described here, and that could be very interesting to study more systematically.

Let us notice, for instance, that the average symmetric propensity between two agents vanishes (first case in Sect. II above). This result could arise from the following two very different situations. First, if element *a* loves *b* with the same strength that *b* hates *a*, then "feelings" cancel each other, thus finally producing a lot of internal "tension". Secondly, if there are no feelings between both elements *a* and *b* then the relation is of total "indifference". In both cases $P_{ab} = P_{ba} = 0$, despite being totally different! Obviously, in this case, one could count all the vanishing (or very small) average symmetric propensities in the propensities matrix, and then perform further analysis to evaluate both the total tension and indifference in the system.

Given the richness of real social situations, this work is a contribution that gives very *general guidelines* for a realistic description of the formation of strategic alliances or blocks. The procedure is simple; in order to find the optimal partition, we map a given set of asymmetrical propensities describing several social scenarios or strategies into a new set with symmetrical propensities and then minimize the corresponding total frustration given by Eq. 3. In this way, the strategy is to minimize the dissatisfaction or frustration of agents, who prefer to be in blocks or groups with agents who have positive affinity avoiding those with whom they have negative propensities.

Although in the case of asymmetric propensities the total frustration E_T cannot be calculated, however, one can calculate the *local* frustration of a given agent by analyzing all their propensities with other agents. For example we can add up all the positive propensities experienced by an agent and also all the positive propensities experienced by that agent. The sums will give important information on how much that agent is *locally* "loved or hated" and can lead to novel future investigations.

We hope that this work can help to understand, model and resolve many situations that arise in various social fields, such as schools, working places, political organizations, etc.

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