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# A unified description of s-, p- and d-wave superconductivity

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## Abstract

We report a comparative study of the s-, p- and d-symmetry superconducting states in square lattices within a Hubbard model, in which correlated-hopping interactions are considered in addition to the repulsive Coulomb interactions. This study is carried out by means of the BCS formalism. The results show that the first and second neighbors correlated hoppings favour s- and d-symmetry pairings, respectively. Moreover, an infinitesimal distortion of the right angles in the square lattice induces the appearance of a p-wave (spin-triplet) superconducting state.

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The observation of d-symmetry pairing in the cuprate superconductors has triggered the research of models beyond the standard BCS theory to include anisotropic superconducting gap symmetries [1]. The recent discovery of the p-wave spin-triplet superconducting state in  $\text{Sr}_2\text{RuO}_4$  [2] has highly enhanced this research. The two-dimensional behavior present in both systems could be essential for understanding their peculiarities. During the last years, the Hubbard model has

been extensively studied due to its simplicity and emphasis on the local electron–electron correlation [3]. Three-band Hubbard models have been proposed to describe the dynamics of the carriers on the planes and the electronic states close to the Fermi energy can be reasonably well described by a single-band tight-binding model with next-nearest-neighbor hoppings [4,5]. Recently, we have found that the second-neighbor correlated-hopping interaction ( $\Delta t_3$ ) is essential in the  $d_{x^2-y^2}$  wave superconductivity [6]. In this work, we start from a single-band Hubbard model, in which nearest ( $\Delta t$ ) and next-nearest ( $\Delta t_3$ ) neighbor correlated-hopping interactions are considered in addition of the

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on-site ( $U$ ) and nearest-neighbor ( $V$ ) Coulomb interactions, which can be written as

$$\begin{aligned} \hat{H} = & t_0 \sum_{\langle i,j \rangle \sigma} c_{i\sigma}^+ c_{j\sigma} + t'_0 \sum_{\langle\langle i,j \rangle\rangle \sigma} c_{i\sigma}^+ c_{j\sigma} \\ & + U \sum_i n_{i\uparrow} n_{i\downarrow} + \frac{V}{2} \sum_{\langle i,j \rangle} n_i n_j \\ & + \Delta t \sum_{\langle i,j \rangle \sigma} c_{i\sigma}^+ c_{j\sigma} (n_{i-\sigma} + n_{j-\sigma}) \\ & + \Delta t_3 \sum_{\langle\langle i,j \rangle\rangle \sigma, (i,l), (j,l)} c_{i\sigma}^+ c_{j\sigma} n_l, \end{aligned} \quad (1)$$

where  $c_{i\sigma}^+$  ( $c_{i\sigma}$ ) is the creation (annihilation) operator with spin  $\sigma = \downarrow$  or  $\uparrow$  at site  $i$ ,  $n_{i,\sigma} = c_{i\sigma}^+ c_{i\sigma}$ ,  $n_i = n_{i,\uparrow} + n_{i,\downarrow}$ ,  $\langle i,j \rangle$  and  $\langle\langle i,j \rangle\rangle$  denote respectively nearest-neighbor and next-nearest-neighbor sites,  $t_0$  and  $t'_0$  are the first- and second-neighbor hopping parameters. Let us consider a square lattice with lattice parameter  $a$ . In order to break the degeneracy of p-wave pairing states, we will further consider a small distortion of the right angles in the square lattice, which leads to changes in the second-neighbor interactions and their new values are  $t'_\pm \equiv t'_0 \pm \delta'$  and  $\Delta t_3^\pm \equiv \Delta t_3 \pm \delta_3$ , where  $\pm$  refers to the  $\hat{x} \pm \hat{y}$  direction.

By rewriting Eq. (1) in the momentum space [6], the equations for determining the  $\alpha$ -channel superconducting gaps [ $\Delta_\alpha(\mathbf{k})$ ] and the chemical potential ( $\mu_\alpha$ ) are, within the BCS formalism [6],

$$1 = \frac{(4\Lambda_\alpha - V)}{N_s} \sum_{\mathbf{k}} \frac{g_\alpha(k_x)[g_\alpha(k_x) + \eta_\alpha g_\alpha(k_y)]}{E_\alpha(\mathbf{k})} \times \tanh\left(\frac{E_\alpha(\mathbf{k})}{2k_B T}\right), \quad (2)$$

$$n - 1 = -\frac{1}{N_s} \sum_{\mathbf{k}} \frac{\varepsilon(\mathbf{k}) - \mu_\alpha}{E_\alpha(\mathbf{k})} \tanh\left(\frac{E_\alpha(\mathbf{k})}{2k_B T}\right), \quad (3)$$

where  $\alpha = p$  or  $d$ ,  $n$  is the density of electrons,  $N_s$  is the total number of sites,

$$E_\alpha(\mathbf{k}) = \sqrt{[\varepsilon(\mathbf{k}) - \mu_\alpha]^2 + \Delta_\alpha^2(\mathbf{k})}$$

and

$$\begin{aligned} \varepsilon(\mathbf{k}) = & \left(\frac{U}{2} + 4V\right)n \\ & + 2(t_0 + n\Delta t)[\cos(k_x a) + \cos(k_y a)] \\ & + 2(t'_+ + 2n\Delta t_3^+) \cos[(k_x + k_y)a] \\ & + 2(t'_- + 2n\Delta t_3^-) \cos[(k_x - k_y)a]. \end{aligned}$$

For the p-channel case,  $\Delta_p(\mathbf{k}) = \Delta_p[\sin(k_x a) \pm \sin(k_y a)]$  then  $\eta_p = \pm 1$ ,  $\Lambda_p = \delta_3$ , and  $g_p(k) = \sin(ka)$ , being  $\delta_3 = (\Delta t_3^+ - \Delta t_3^-)/2$ ; whereas for the d-channel  $\Delta_d(\mathbf{k}) = \Delta_d[\cos(k_x a) - \cos(k_y a)]$ ,  $\eta_d = -1$ ,  $\Lambda_d = \Delta t_3$ , and  $g_d(k) = \cos(ka)$ . Finally, the s-channel case can be addressed in a similar manner as discussed in Ref. [6], where  $\Delta_s(\mathbf{k}) = \Delta_s + \Delta_s^*[\cos(k_x a) + \cos(k_y a)]$  leads to three simultaneous equations instead of Eqs. (2) and (3). The critical temperature ( $T_c$ ) of the  $\alpha$  channel is determined by  $\Delta_\alpha(T_c) = 0$ .

In Fig. 1, the dependences of  $T_c$  with  $n$  are respectively shown for the s-, p- and d-channel superconducting states, for a system with  $t'_0 = -0.45 |t_0|$ ,  $U = 8 |t_0|$ ,  $V = 0$ ,  $\Delta t = 0.5 |t_0|$ ,  $\Delta t_3 = 0.05 |t_0|$ , and  $\delta_3 = 0.05 |t_0|$ . Notice that the d-wave is the ground state when  $n$  is close to 2 and for  $n$  around 1 the ground state would be p symmetric if  $U$  tends to infinity, since the p- and d-wave superconducting states do not depend on  $U$  but the  $T_c$  of s-wave superconductors is drastically reduced when  $U$  increases [6]. Moreover, it can be observed that the maximum of

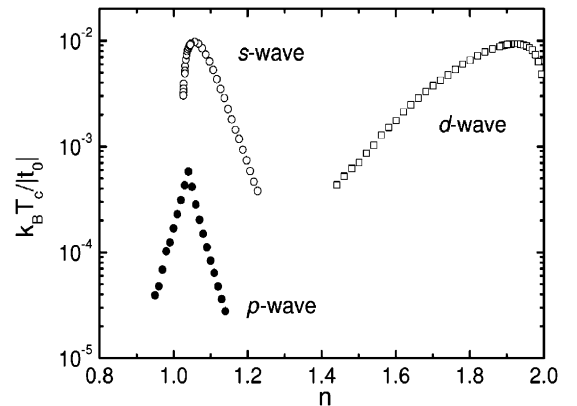


Fig. 1. s- (open circles), p- (solid circles) and d- (open squares) channel superconducting critical temperatures ( $T_c$ ) as functions of the electron density ( $n$ ).

$T_c(n)$  for each channel is located at different optimal doping ( $n_{op}$ ), close to the experimental results for high- $T_c$  superconductors and  $\text{Sr}_2\text{RuO}_4$  [7,8].

In summary, we have studied the symmetry of the superconducting ground state in a square lattice within the Hubbard model, which allows a unified description of the s-, p- and d-wave superconductivities by means of the first- and second-neighbor correlated hoppings. Both correlated hoppings could induce s-channel superconducting states which is destroyed when  $U$  increases allowing the p- and the d-channel superconducting states to be the ground ones for electron density around one and two, respectively.

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