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## Nodal effects on the electronic specific heat of anisotropic superconductors

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## Abstract

Based on the BCS formalism, the electronic specific heat of d-symmetry superconducting states is studied. This study is performed on a square lattice described by a generalized Hubbard model, in which correlated-hopping interactions are included in addition to the repulsive Coulomb ones. Instead of the exponential temperature dependence for the s-channel, the results show second and higher power-law behaviors for d-wave superconducting specific heat, depending on the angular dependence of the single-excitation energy gap occurred at different carrier concentrations. The results of this study could help to understand a variety of power-law behaviors observed in  $La_{2-x}Sr_xCuO_4$  superconductors.

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Keywords: Superconducting gap symmetry; Hubbard model; Electronic specific heat

The energy spectrum of elementary excitations in solids determines the temperature dependence of their specific heat. In particular, for a superconductor it gives information regarding to the symmetry of its superconducting state. An swave superconductor has an exponentially temperaturedependent electronic specific heat (C), while an anisotropic nodal superconducting gap leads to a power-law C, as occur in the cuprate superconductors [1] and in  $Sr_2RuO_4$  [2]. For these materials, three-band Hubbard models have been proposed to describe the dynamics of the carriers on the CuO<sub>2</sub> and RuO<sub>2</sub> planes, and the electronic states close to the Fermi energy can be reasonably well described by a singleband square-lattice tight-binding model with second neighbor hoppings [3,4]. In this work, we study C of d-wave superconducting states on a square lattice with a lattice parameter a and a single-band generalized Hubbard Hamiltonian [5]:

$$\hat{H} = t \sum_{\langle i,j \rangle,\sigma} c^+_{i\sigma} c_{j\sigma} + t' \sum_{\langle \langle i,j \rangle \rangle,\sigma} c^+_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} + \frac{V}{2} \sum_{\langle i,j \rangle} n_i n_j + \Delta t \sum_{\langle i,j \rangle,\sigma} c^+_{i\sigma} c_{j\sigma} (n_{i-\sigma} + n_{j-\sigma})$$

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$$+ \Delta t_3 \sum_{\substack{\langle \langle i,j \rangle \rangle, \sigma \\ \langle i,j \rangle, \langle j,l \rangle}} c^+_{i\sigma} c_{j\sigma} n_{l-\sigma}, \tag{1}$$

where  $c_{i\sigma}^+$  ( $c_{i\sigma}$ ) is the creation (annihilation) operator with spin  $\sigma = \downarrow$  or  $\uparrow$  at site *i*,  $n_{i,\sigma} = c_{i\sigma}^+ c_{i\sigma}$ ,  $n_i = n_{i,\uparrow} + n_{i,\downarrow}$ ,  $\langle i, j \rangle$  and  $\langle \langle i, j \rangle \rangle$  denote first- and second-neighbor sites,  $\Delta t$  and  $\Delta t_3$  are first- and second-neighbor correlated hopping integrals, respectively. Applying the BCS formalism to Eq. (1), two coupled integral equations are obtained [5]. They determine the superconducting gap ( $\Delta_d$ ) and the chemical potential ( $\mu$ ) for a given temperature (T) and electronic density (n). The critical temperature ( $T_c$ ) is obtained when  $\Delta_d = 0$ . The C of superconducting states is given by [6]

$$C = \frac{2k_{\rm B}\beta^2}{4\pi^2} \iint_{1BZ} d^2k f(E_{\bf k})[1 - f(E_{\bf k})] \times \left[ (E_{\bf k})^2 + \beta E_{\bf k} \frac{dE_{\bf k}}{d\beta} \right],$$

where  $\beta = 1/k_{\rm B}T$ , f(E) is the Fermi–Dirac distribution,  $E_{\bf k} = \sqrt{[\varepsilon({\bf k}) - \mu]^2 + \Delta_d^2({\bf k})}$  is the single-excitation energy,  $\varepsilon({\bf k})$  is the mean-field dispersion relation,  $\mu$  is the chemical potential

and  $\Delta_d(\mathbf{k}) = \Delta_d[\cos(k_x a) - \cos(k_y a)]$  [5]. The electronic specific heat of the normal state can be obtained by taking  $\Delta_d$  equal to zero.

Fig. 1 shows  $T_c$  as a function of n for t' = -0.45|t|,  $\Delta t = 0.5|t|$ ,  $\Delta t_3 = 0.1|t|$ , V = 0 and arbitrary U. Observe that the maximum  $T_c$  is located at the optimal n = 1.73, similar to that observed in cuprate superconductors since the doping concentration x is related to 2-n in this model.

In Fig. 2, two electronic densities (a) n = 1.2 and (b) n = 1.94 are chosen from Fig. 1 to calculate their C and compared with experimental data obtained from  $La_{2-x}Sr_x$  $CuO_4$  for x = 0.22 and 0.1, respectively [7]. Insets of Fig. 2 show the corresponding theoretical angular dependences of the single-excitation energy gap  $(\Delta)$  defined as the minimum value of  $E_{\mathbf{k}}$  in  $\mathbf{k}$  direction [8]. The polar angle is given by  $\theta = \tan^{-1}(k_y/k_x)$ . Notice that for the overdoped regime, n < 1.73,  $\Delta$  has a  $d_{x^2-y^2}$  symmetry and in consequence C is proportional to  $T^2$  as obtained in Ref. [9]. However, for the underdoped regime, n > 1.73,  $C \propto T^{\nu}$  with v > 2 since  $\Delta$  has a d<sub>xv</sub>-like symmetry without real nodes. For low-energy single-particle excitations, the lack of real nodes has a similar effect as in an s-wave superconductor. In consequence, for  $T \ll T_c$ , C has an almost exponential behavior. The residual C/T value at T = 0 in experimental data could be due to the chemical or electronic inhomogeneity of the sample [10], and this fact is not considered in the theory.

In conclusion, the low-temperature behavior of *C* is very sensitive to the deepness of nodes in  $\Delta$ , which symmetry depends on *n*, as found in scanning tunneling experiments [11]. It is worth mentioning that  $d_{xy}$ -like gaps without real nodes have been observed in cuprate superconductors [12]. These results could help to understand the different *C*(*T*) behaviors observed in d-wave superconductors.

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Fig. 1. Critical temperature ( $T_c$ ) as a function of the electron density (n) for t' = -0.45|t|,  $\Delta t = 0.5|t|$ ,  $\Delta t_3 = 0.1|t|$ , V = 0 and arbitrary U.



Fig. 2. Theoretical (open circles) d-wave electronic specific heat (*C*) versus temperature (*T*) for systems with the same parameters as in Fig. 1 in comparison with experimental data (solid circles) obtained from  $La_{2-x}Sr_xCuO_4$  [7]. Insets: corresponding single-excitation energy gaps  $(\Delta/|t|)$  as a function of the polar angle ( $\theta$ ).

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