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Axelrod models of social influence with cultural repulsion

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Since both attractive and repulsive effects among agents are important in social systems, we present simulations of two models based on Axelrod's homogenization mechanism that includes repulsion. These models are the *repulsive model*, where all individuals can repel, and the *partially repulsive model* where only a fraction of repelling agents are considered. In these two models, attractive dynamics is implemented for agents with the ability to repel each other only if the number of features shared by them is greater than a threshold parameter. Otherwise, repelling dynamics is used. In the repulsive model, the transition from a monocultural state to a fragmented one often occurs abruptly from one cultural-variability value to the next one and a second transition emerges. For the partially repulsive model, there are also two different transitions present: the initial one being as abrupt as the one found for the repulsive model, whereas the second one follows a less abrupt behavior and resembles that of the original Axelrod model. However, the second transition for this model occurs from a partially fragmented state and not from a monocultural one.

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I. INTRODUCTION

Axelrod devised a model to emulate the way in which culture is exchanged among agents in society [1]. Interaction among agents (or individuals) occurs under the following trends: (1) culturally similar agents have a greater probability of interaction than culturally dissimilar ones; (2) interaction will increase the cultural similarity of the agents. This model yielded interesting results, such as the appearance of a phase transition from a monocultural state to a fragmented one as the cultural variability increases.

There have been variations on Axelrod's model [2]. For example, the agents can also modify their features through an external field [3]. In this case, the external field can be considered as yet another agent which interacts with all the agents in the lattice. The role of the field is analogous to that of mass media, and its effect is to promote order (homogenous state in which the majority of the agents share their features) or disorder (heterogeneous or fragmented state) in the lattice depending on the intensity with which the agents interact with it. Moreover, the role of noise rate on ordering (or disordering) the system has also been widely studied [4–6]. For initially disordered systems, the presence of an appropriately chosen noise rate surprisingly aids in the development of a homogenous state. Nevertheless, above a certain threshold, it favors the transition to a heterogeneous state [5,6].

On the other hand, repulsive behavior in social systems is an important ingredient in many real situations, such as crowd turbulence of pedestrian flows [7,8] and dynamics of attitude change [9], although it has been barely modeled together with attractive behavior in socio cultural dynamics. In other scenarios such as networks of spinlike neurons with excitatory and inhibitory couplings, this last behavior has been represented by repulsive links [10]. In simple opinion spreading models, the attractive and repulsive links can represent friendly and *adversarial relations* among agents, respectively [10,11]. Moreover, the role of agents that tend to adopt the opposite opinion to the majority one (*contrarians*) in the dynamics of majority rule models, has been addressed previously [12–14].

In this work we consider two models based on the Axelrod's homogenization mechanism but including cultural repulsion between agents. These models, described in the following section, are the *repulsive* one, where all individuals can repel others, and the *partially repulsive model* where only a fraction of repelling agents are considered. These models aim to consider the rebellious nature of individuals in a society who try to differentiate themselves from others.

II. MODELS

In the original Axelrod model [1], each agent is represented by a node in a lattice (typically square and with periodic boundary conditions). Additionally, each agent has an F-length vector where each entry corresponds to a cultural attribute or feature (e.g., language, religion, etc.). Each entry starts with randomly chosen values (taken from a uniform discrete distribution) between 1 and an integer q. Thus, q can be considered a measure of cultural variability. A randomly chosen pair of neighbors will interact with a probability equal to the fraction of overlapping cultural features (i.e., the proportion of features they share). Interaction consists on an agent copying a randomly chosen feature from his neighbor (a feature they did not previously share). This procedure is implemented until the lattice reaches a *frozen state*. In this state, all the agents in the lattice have a zero probability of modifying their features, either because all their features are shared with their neighbors, or because they share no feature

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whatsoever with them. The transition from a monocultural state to a fragmented one occurs at a critical value for $q(q_c)$, which increases as F grows. On the other hand, as mentioned before, both attractive and repulsive effects among agents are important in social systems. However, in all the previous models, the basic dynamics is purely attractive in the sense that agents tend to "approach" each other by copying and sharing more features. Additionally, agents in society may not only feel an urge to copy their neighbors but also to be different or rebellious. This fact can be implemented by including a repulsion mechanism to share fewer features when agents interact. Therefore, the dynamics of the Axelrod model can be generalized, preserving its main features in the following way. Interactions between nearest neighbors are of two kinds: (a) *attractive*, forcing one agent to copy another agent increasing their common overlap, as in the original Axelrod's model; or (b) *repulsive*, forcing one agent to change a shared feature with another agent and decreasing their overlap.

Both dynamics are combined in the following simple way. Attractive dynamics will be used only if the proportion of features shared by the agents is greater than a certain parameter $\gamma \in [0,1]$ and, otherwise, repelling dynamics will be used. Thus, γ may be considered a measure of the intolerance of the system. That is, if agents are too similar (above the threshold γ) then there is sympathy or social affinity and they perform a homogenization process, but if agents are too different (below threshold γ) then there is social antipathy, and they perform a repulsive process. This process consists on randomly choosing a feature they share from one of the neighboring agents (randomly chosen as well) and reassigning its value randomly within the considered cultural-variability range, so as to ensure it no longer shares that feature with its neighbor. This is the *repulsive model*, in which all agents follow the same rules. Finally, the *partially or heterogeneous repulsive model* consists on a variation of the *repulsive model*, in that only a fraction ϕ of randomly chosen agents are capable of repelling, and in the interaction of dissimilar agents, we choose to keep a repulsive interaction, that is, the repulsive process will *always* occur between two agents if at least one of them is capable of repelling.

The dynamic process for the repulsive model can be summarized by the following algorithm:

(1) the lattice is swept in an orderly fashion. For each of the agents considered, one of its four nearest neighbors is randomly chosen. These two individuals make up the interacting pair.

(2) Once the interacting pair has been chosen, their similarity is determined by calculating their overlap w (proportion of features shared, i.e., A/F, where A is the number of features shared, and F is the total number of features);

(3) the overlap w is then compared with the degree of repulsion γ. The case $w < \gamma$ will correspond to repulsive behavior between the agents, while $w \geq \gamma$ will correspond to attractive behavior.

(4) In the case of repulsive behavior ($w < \gamma$), one of the previously shared features will be randomly chosen, for one of the two agents (randomly chosen as well). The value of this feature will be randomly reassigned but ensuring that: (a) it will be different from its previous value, and (b) it will lie within the range from 1 to q. If $w=0$, then there is no change (nothing happens).

(5) In the case of attractive behavior ($w \geq \gamma$), one of the features not shared by the individuals is randomly chosen for one of the interacting agents (randomly chosen as well). This agent will then copy the value for this feature from his interacting neighbor.

Finally, the dynamic process for the partially repulsive model can be summarized by the following algorithm:

(1) a fraction ϕL^2 of agents are randomly chosen and these agents are endowed with repulsive behavior;

(2) the lattice is swept in an orderly fashion to choose the interacting pairs of agents, just as in the repulsive model;

(3) once an interacting pair has been chosen, both agents are examined to see if any of them has the ability to repel (i.e., to see if any of them belongs to the fraction ϕL^2 of potentially repelling agents);

(4) in case at least one of them belongs to the fraction ϕL^2 of potentially repelling agents, repulsive dynamics will take place (steps 3 to 5 in the repulsive model algorithm);

(5) in the other case, in which none of the interacting agents belongs to the fraction ϕL^2 of potentially repelling agents, attractive dynamics will occur, i.e., one of the features not shared by the individuals is randomly chosen for one of the interacting agents (randomly chosen as well). This agent will then copy the value for this feature from his interacting neighbor.

III. RESULTS

In order to characterize the frozen state we study the normalized average size of the largest region or cluster $\langle S_{\max}\rangle/L^2$, where L^2 is the total number of agents; the entropy of the system (defined below) and the behavior of the cumulated size distribution as a function of cluster size at the transition. The corresponding averages are taken from at least 30 realizations for each case. All the lattices considered also have periodic boundary conditions.

The criterion to determine if a system has reached the *frozen state* (i.e., a zero probability of change for the values of the entire lattice) was, in certain cases, never reached for the models studied. Due to their repulsive nature, a few agents who had the values of their features previously repelled, always have a nonzero probability of returning them to the value of a nearby cluster. The lattice would, therefore, never reach the frozen state. Hence, a second criterion has to be implemented in order to consider that the system has reached a frozen state. If for a given realization the normalized value of $S_{\max}$, $S_{\max}/L^2$, changes less than a certain percentage in $\sim 10L^2$ iterations (checked at regular intervals of L^2 iterations), the system will be considered to have reached a frozen state. The maximum change in $S_{\max}$ we considered for this criterion was $0.05L^2$. It is worth mentioning that the first criterion failed particularly for values near the transition.

Figure 1 shows $\langle S_{\max}\rangle$ versus q within the *repulsive model* for systems with $F=10$, $L=10$, 20, 30, 40, and 50; and $\gamma = 0.2$, 0.4, and 0.5. Notice that the transitions are considerably sharp, generally occurring from one integer value of q to the next one. Higher values of γ correspond to less toler-

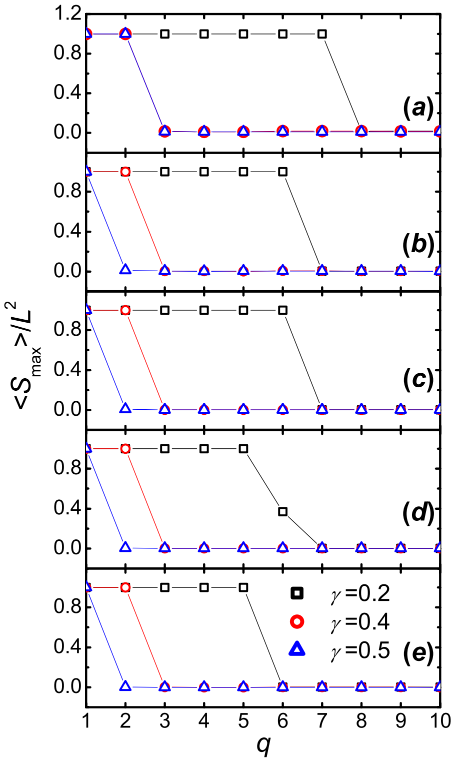

FIG. 1. (Color online) Normalized $\langle S_{\max}\rangle$ versus q for systems with $F=10$, (a) $L=10$, (b) $L=20$, (c) $L=30$, (d) $L=40$, and (e) $L=50$. The values used for γ were 0.2, 0.4, and 0.5.

ant (more repulsive) societies. As expected, the trend is that more tolerant societies present a higher or equal value of q_c than less tolerant ones. Additionally, for more tolerant systems, the transition is not so sharp, and intermediate values of $\langle S_{\max}\rangle/L^2$ between 0 and 1 were found, as can be seen for systems with $\gamma=0.1$ (Fig. 2).

In order to analyze the behavior of systems with smaller values of γ, a greater value of F is required. Note that the overlap fraction w must, by construction, vary in steps of $1/F$ and thus if two neighbors are to interact (have $w>0$) we know $w\geq 1/F$. Repulsive interactions only occur

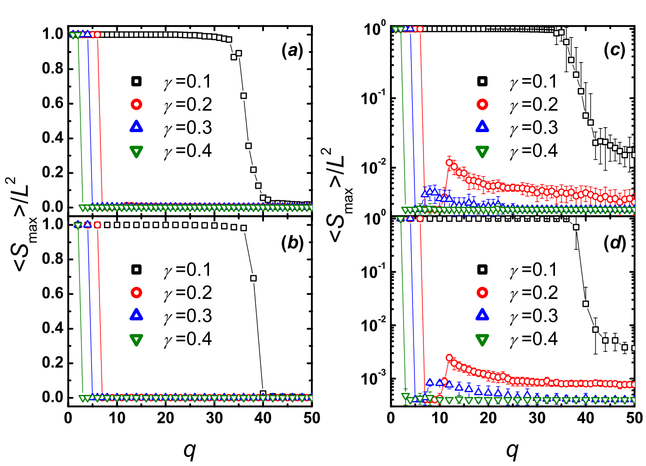

FIG. 2. (Color online) Normalized $\langle S_{\max}\rangle$ versus q for a system with $F=20$ and (a) $L=20$, (b) $L=50$. (c) and (d), respectively, correspond to (a) and (b) but with the vertical axis in logarithmic scale.

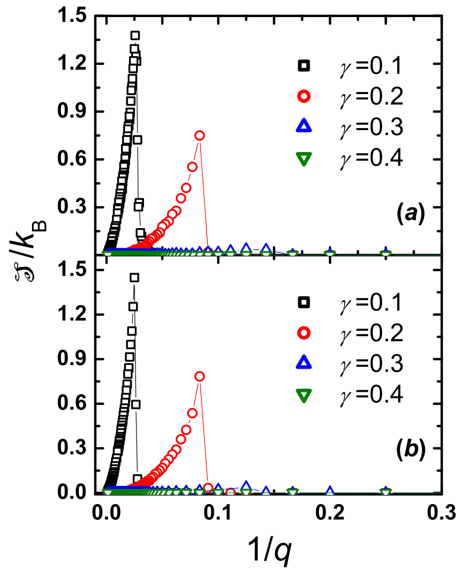

FIG. 3. (Color online) The associated entropy $\mathcal{S}$ versus $1/q$ for systems with $F=20$, (a) $L=20$ and (b) $L=50$.

when $w<\gamma$ and so are impossible if $\gamma\leq 1/F$. We present in Fig. 2 the behavior of $\langle S_{\max}\rangle$ versus q for systems with $F=20$, $L=20$ and 50, and $\gamma=0.1$, 0.2, 0.3, and 0.4.

Figures 2(c) and 2(d) show a second slightly sharp phase transition, almost imperceptible under vertical linear scaling, which occurs after the abrupt one. Typically, fluctuations diverge only inside a transition where intermediate values for the normalized $\langle S_{\max}\rangle$ between 0 and 1 are present. As an example of their behavior, these fluctuations are better shown in Figs. 2(c) and 2(d) in logarithmic scale. Notice that the least repulsive case considered ($\gamma=0.1$) shows only one phase transition.

A useful quantity that characterizes a given system is the entropy *of the distribution of cluster sizes*, which can be defined as: $\mathcal{S}=-k_{\mathrm{B}}\sum_{n=1}^{L^2}\mathcal{P}_n \ln \mathcal{P}_n$, where $\mathcal{P}_n$ is the probability of an agent of belonging to a cluster of n individuals and k_{B} is the Boltzmann constant [15]. This entropy, as a function of $1/q$ (the probability of adopting a particular value for a feature), is shown in Fig. 3 for systems with $F=20$, $L=20$ and 50, and the same values for γ as in Fig. 2.

Interestingly, the sharp peaks in the entropy correspond to the values of q where the second transition takes place, except for $\gamma=0.1$ where the peak corresponds to the only phase transition present in this case. Moreover, those systems with the highest studied values of γ (the most intolerant ones) present very small peaks, an effect that can be associated to the fact that the repulsive dynamics allows only for highly homogenous or completely fragmented states.

The presence of two different phase transitions is due to the fact that, for the first phase transition, the cultural variability q is small. This reduces the interaction between agents and is responsible for the original fragmentation. Upon increasing q, the attractive nature of the agents becomes more prominent, thus favoring the appearance of small clusters. As a matter of fact, the first phase transition may only be considered a subterfuge in the case when it occurs from $q=1$ to $q=2$. It is important to stress that for

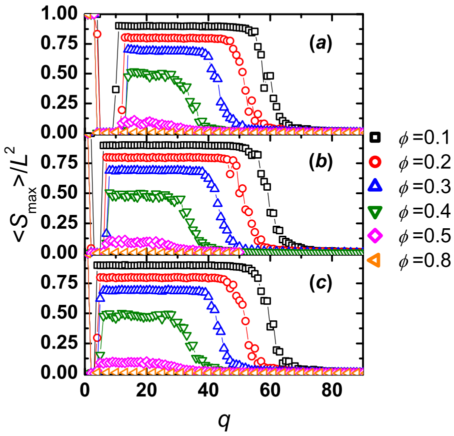

FIG. 4. (Color online) Normalized $\langle S_{\max}\rangle$ versus q for systems with $L=40$, $F=10$ and (a) $\gamma=0.25$, (b) $\gamma=0.5$, and (c) $\gamma=0.75$.

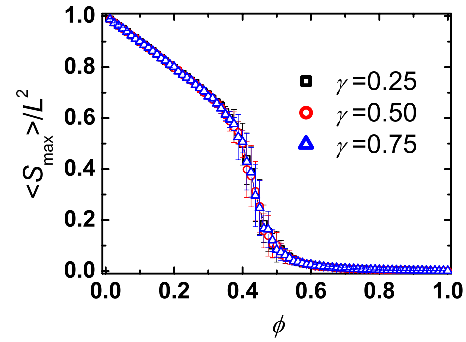

FIG. 6. (Color online) Normalized $\langle S_{\max}\rangle$ versus ϕ for a system with $L=40$, $F=10$ and $q=20$.

$q=1$, the system does not evolve since the features of all agents start and end with the same value 1. Thus, even though we regarded them as two separate phase transitions in this work, it is completely fair to study the system starting from $q=2$. The presence of sharp peaks in the entropy for this model responds to the sharpness of its transitions.

On the other hand, for the *partially repulsive model*, the fraction of agents in the system which may act repulsively is given by the parameter $\phi \in [0,1]$. The purely repulsive model discussed previously may thus be considered as a particular case ($\phi=1$) of this second model. In this one, interactions involving at least one agent belonging to the fraction ϕ of potentially repulsive agents will be repulsive (as in the previous model) if $1/F < w < \gamma$, where γ is the degree of repulsion. Thus, a repulsive agent can have repelling interactions with all its neighbors, whether they are repulsive or not. Otherwise, when none of the interacting individuals belong to the fraction ϕ, one of them will copy a randomly chosen feature of the other with probability 1. Figures 4 and 5 show $\langle S_{\max}\rangle$ versus q for systems with $F=10$, $L=40$, 100, and different values of γ.

Figures 4 and 5 present further aspects of the behavior of the second phase transition. The presence of the two different transitions is particularly evident for systems with smaller values of ϕ (the least repulsive systems). Here, the similarity between the second phase transition and that of Axelrod's model is more evident. This second transition is inherited from the repulsive model and it can be made more evident by choosing lower values of ϕ. It can also be seen, in Figs. 4 and 5, that the change between a fragmented state and a homogenous one as q increases is abrupt. The value of q at which this happens seems to be the same for the two larger values of γ presented. Furthermore, if there are intermediate values of $\langle S_{\max}\rangle$ present at the transition, the fluctuations diverge. It can also be seen in Figs. 4 and 5, that the second phase transition starts from a value of the normalized $\langle S_{\max}\rangle$ lower than 1. Additionally, for smaller values of ϕ, the value of the normalized $\langle S_{\max}\rangle$ from which the second transition starts is close to 1-ϕ, but this trend is not present for greater values of ϕ. In order to analyze this behavior we plot in Figs. 6 and 7, $\langle S_{\max}\rangle$ as a function of ϕ for the same systems shown in Figs. 4 and 5, and for a fixed value of q chosen within the plateau observed before the second transition.

Notice that $\langle S_{\max}\rangle/L^2 \approx 1$-$\phi$ for $\phi \leq 0.3$. Moreover, in the limit where $\phi=1$, the repulsive model is recovered and the second transition becomes imperceptible. Another interesting feature observed in Figs. 6 and 7 is the presence of large fluctuations within the sudden drop of the normalized $\langle S_{\max}\rangle$. This indicates that, inside the drop, the system is out of equilibrium, and that the behavior is analogous to a phase transition. The initial 1-ϕ behavior for $\langle S_{\max}\rangle$ is a consequence of the fraction ϕ of repelling agents. For small values of ϕ, they

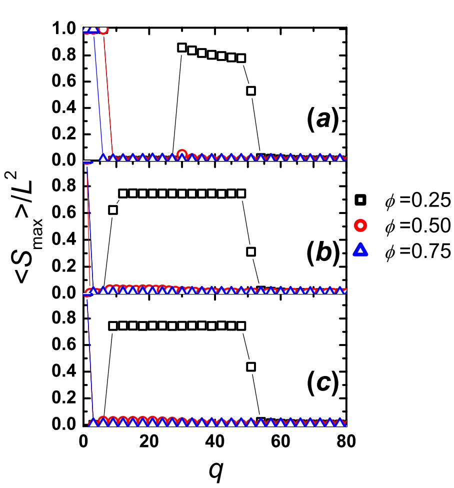

FIG. 5. (Color online) Normalized $\langle S_{\max}\rangle$ versus q for systems with $L=100$, $F=10$ and (a) $\gamma=0.2$, (b) $\gamma=0.5$, and (c) $\gamma=0.8$.

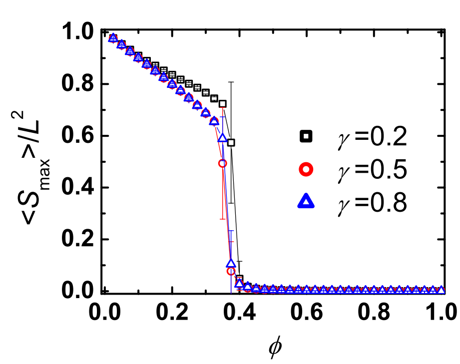

FIG. 7. (Color online) Normalized $\langle S_{\max}\rangle$ versus ϕ for a system with $L=100$, $F=10$ and $q=40$.

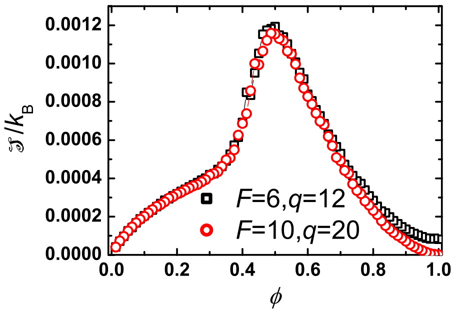

FIG. 8. (Color online) The associated entropy $\mathcal{S}$ of the normalized $\langle S_{\max}\rangle$ versus ϕ for a system with $L=40$ and two different values of F and q. The values of q used for each F correspond to the plateau found before the second transition.

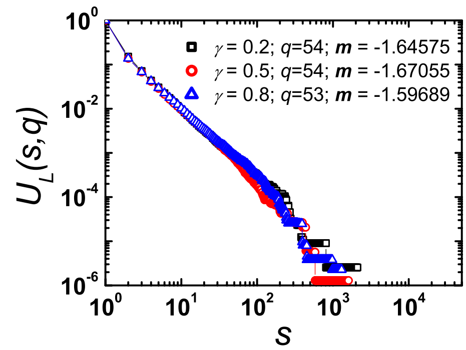

FIG. 9. (Color online) Cumulated distribution $U_L(s,q)$ of region sizes s for $q\approx q_c$, $L=100$, $F=10$, $\phi=0.25$, and $\gamma=0.2$, 0.5, and 0.8 with the corresponding slopes m.

are too few to form a "cluster of repellers." However, as ϕ increases, they can interconnect among themselves and consequently break any surviving large cluster and, in this way, alter the 1-ϕ behavior of $\langle S_{\max}\rangle$. Additionally, in Fig. 8 we show the corresponding entropy as a function of ϕ for a system with $L=40$ and $F=6,10$.

Observe that the entropy behavior is independent of F and it also presents a maximum that corresponds to the region within which the sudden drop occurs, showing again that the behavior is similar to a phase transition. Finally, we study the behavior of the cumulated size distribution as a function of the cluster size [16]. The cumulated size distribution (U_L) is defined as the fraction of regions of size equal or larger than s: $U_L(s,q)=\sum_{s'=s}^{L^2}P_L(s',q)$, where $P_L(s,q)$ is the probability distribution of size s regions in the system for a given q. In Fig. 9 we plot the cumulated size distribution (CSD) as a function of s for systems with $L=100$, $F=10$, $\phi=0.25$ and three different values of γ: 0.2, 0.5, and 0.8. These systems present a second phase transition near $q=54$, 54, and 53, respectively.

Notice that the CSD studied follows a power law whose exponent m slightly depends on the value of γ chosen. It is worth mentioning that only values of q near the second phase transition yield power-law results for the CSD as a function of cluster size. No power-law behavior is found for the first phase transition of the two models studied in this work.

IV. CONCLUSIONS

The repulsive model shows that introducing the possibility of a little rejection in all agents is responsible for the fragmentation of the system into a heterogeneous state even for small cultural variability. For this model, two phase transitions are found and the entropy, as a function of $1/q$, presents sharp peaks corresponding to the value of q where the second phase transition occurs. No peak is observed for the first phase transition, responding to the fact that the repulsive behavior allows for only completely homogenous or fragmented states. On the other hand, the partially repulsive model shows that, indeed, a certain degree of cultural variability is necessary in order to form clusters of a size comparable to that of the lattice. The cultural variability needed for that to happen is typically low. It also tends to be independent of ϕ (at least for the values of γ studied). Actually, the ϕ parameter seems to determine from which value the second transition will occur. Thus, the manipulation of the γ and ϕ parameters turns out to be fundamental in favoring stages in which a homogenous society may exist. This is in agreement with common sense, since both parameters represent the lack of tolerance of agents to engage in attractive behavior, as well as the proportion of the less tolerant individuals in a society. Moreover, the average maximum cluster size found for a given value of q within the plateau before the second phase transition, when considered as a function of ϕ, also presents a phase transition. The entropy related to this phase transition consists of maxima which do not depend on the chosen value of F. The cumulated size distribution as a function of cluster size presents power-law behavior only for values of cultural variability near the second transition. This further indicates that this particular transition is more akin to those in the original Axelrod model and its variants [16], whereas the first phase transition is of different nature. We hope that this work may stimulate further research of combined attractive and repulsive behavior in social dynamics.

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