Floating Magnets as Two-Dimensional Atomic Models

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The equilibrium configurations (ECs) of any mechanical system made up of interacting parts (when the interactions are due to conservative forces) always correspond to configurations for which the potential energy of the system reaches an extremum (usually a minimum). This idea, although very simple, is extremely useful in understanding a variety of processes that are physically important. The formation of small (and not so small) clusters of atoms is predicted and explained by means of sophisticated algorithms designed to calculate very rapidly the minimum energy configuration of the system.[1] In crystal growth, the early stages of nucleation are understood by minimum energy considerations. The structure of molecules is also calculated with programs that minimize the potential energy.[2] The crystalline structure of solids is also shown to correspond to minimum energy configurations.

In the examples mentioned above, we only included solid-state systems. To describe the properties of gases, the kinetic theory of gases is usually employed where the atoms are considered (in a first approximation) as hard balls, without any long-range interaction force between them. The only forces considered are the ones due to collisions, which are considered to be elastic. If this was true, no gas would change to the solid or the liquid states. To explain the formation of condensed matter, long-range attractive forces among the atoms are needed. On the other hand, to justify the low compressibility of solids (and liquids), a short-range repulsive force is needed. These two kinds of forces must compensate at a certain distance, thus producing the different densities that we observe in the materials.

As we shall see in the next section, a very simple system that permits us to observe the combined effects of a long-range attractive force and a short-range repulsive one can be easily constructed with magnets floating on water. Magnets, and the forces between them, have been discussed previously.[3] Lehman[4] placed magnets floating on an air table, simulating a crystalline structure. He observed the effects of interstitial atoms and their vibrational modes. However, he did not apply a central force, and therefore, he did not observe the configurations we describe. Our setup allows the students to observe directly the attainment of ECs, to see that a particular system may have different ECs, and to appreciate that these different ECs are separated by potential barriers (the activation energies) that must be overcome to lead the system from one EC to another. Also, for more inquisitive observers this system constitutes a challenge to determine if the formulas encountered in the first courses of electromagnetism explain (quantitatively) the observed behavior of the floating magnets.

Physical System and Related Theory

To have a system of floating magnets where short-range repulsive forces, as well as a long-range attractive one could be present, we used as a container a transparent circular pie pan (270 mm diameter, 30 mm deep). The dish was filled with a 10-mm layer of water to permit the magnets to float freely. This transparent container enables us to use an overhead projector to show the system to large audiences.

For the floating magnets, we used small cylindrical ceramic magnets (10 mm diameter, 4 mm thick) with their magnetic axes parallel to the axes of the cylinders. These magnets were glued to the center of circular disks of cork cut from the corks of wine bottles (23 mm diameter, 4 mm thick). With these dimensions the magnets float in a stable way if we place the magnet under the cork, because the center of gravity lies below the buoyancy center.

In order to simulate an attractive force directed to the center of the container, we placed 36 fixed magnets in contact with the container's external wall. Then, if an additional magnet is placed in the interior of the container, the circular arrangement of the 36 fixed magnets will push the intruder to the center, as if a central attractive force existed. If more floating magnets are placed in the water, the magnets will arrange themselves in equilibrium configurations, as we shall see in the following section.

The potential energy of the system (for any number of floating magnets) can be easily calculated, because all the magnets are placed in the same plane and all of their magnetic dipoles are parallel. It is easy to calculate the magnetic field due to a magnetic dipole *M* at any point in the plane perpendicular to the dipole movement. The field at a distance *r* from the dipole (of moment *M*) is given by:[5]

$$B = \frac{\mu_0 M}{4\pi r^3}\ , \qquad (1)$$

and the potential energy U_p of a magnetic dipole *M* placed in the magnetic field *B* produced by another dipole with the same orientation and magnitude is:

$$U_\mathrm{p} = \frac{\mu_0 M^2}{4\pi r^3}, \qquad (2)$$

where *r* represents the distance between the magnets. The corresponding force acting over any of the two magnets is:

$$F = \frac{-dU_\mathrm{p}}{dr} = \frac{3\mu_0 M^2}{4\pi r^4}. \qquad (3)$$

Equation (2) allows us to calculate the potential energy of any configuration of floating magnets. For the interested reader who wishes to obtain a precise numerical value, it is necessary to know that the dipole moment of the employed magnets is $M = (8.37 \pm 0.08) \times 10^{-2}\ \mathrm{C\,cm^2/s}$. This value was obtained from the measured repulsive force between two of the magnets.

Results and Discussion

We used our setup for doing the following simple experiments. We began with the container of water and only the 36 external magnets. Then we started adding the internal magnets one by one, and we observed the following:

1. A single magnet placed in any position will move to the center of the dish. In this position the magnet is at the bottom of a potential well, in a minimum of the potential energy of the system. To move the magnet to any side we would need to do some work, thus increasing the potential energy, which is typical of a stable equilibrium. If we changed the central magnet for one with the opposite polarity, the equilibrium would be unstable and the potential energy would be at a maximum, so if the central magnet is slightly displaced from its position, it will move away from the center until the wall of the container is reached.
2. Two magnets are pushed to the center until the repulsive force between them compensates the force exerted by the external magnets.
3. Three magnets arrange themselves in an equilateral triangle.
4. If we place a fourth magnet exactly in the center of the equilateral triangle mentioned above, in principle it could remain there at an equilibrium position, but in practice this is very difficult to achieve because real magnets are not identical. Therefore, four magnets arrange themselves forming a square.
5. A fifth magnet rapidly placed in the center of the square will remain there. If we place the fifth magnet at any other position it is likely that the five magnets will form a pentagon. Therefore, we clearly have two different configurations, both of them corresponding to minima of the potential energy.
6. In a similar way, six magnets also have two stable configurations: the centered pentagon and the equilateral hexagon. It can be observed that it requires very little energy to go from the hexagon to the pentagon, and a greater amount to go in the

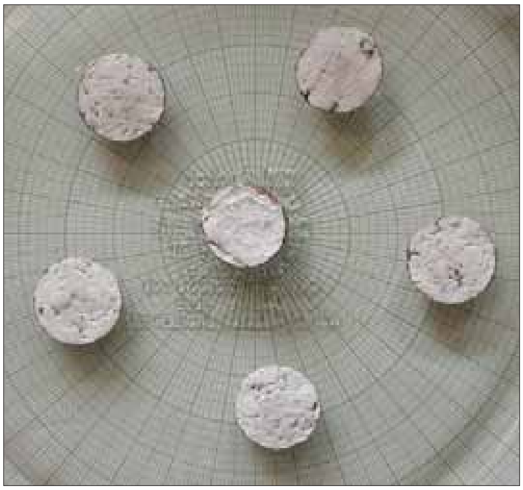

Fig. 1. Centered pentagon: equilibrium configuration of lowest energy for six floating magnets.

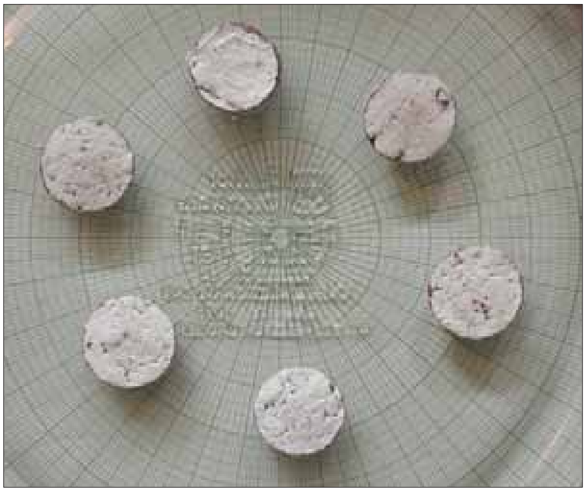

Fig. 2. Hexagon: one of the two equilibrium configurations of six floating magnets. Minimum activation energy needed to transform to the centered pentagon.

opposite direction (i.e., the pentagon has a lower activation energy). In Fig. 1 we can see the centered pentagon formed by the six floating magnets, and Fig. 2 shows the hexagon that is formed when the central magnet is pushed away from the center.

7. With more magnets we have more equilibrium configurations, but always the one with more symmetry corresponds to the more stable, with the minimum configuration energy. With seven magnets the most stable configuration is the centered hexagon, with eight it is the centered heptagon, and with nine magnets the most stable configuration is a heptagon with two magnets inside.
8. Because of the radial symmetry of the external field, the inner magnets have the tendency to arrange themselves in concentric rings, each of them occupied by a different number of magnets. Some of the arrangements are more symmetrical than others. Some of the possible arrangements are indicated in Table 1.
9. When we have many magnets, the average distance between them is approximately the same. If we substitute one of the magnets by another of different magnitude (greater or smaller), we can see the change in the distances to its neighbors (greater or smaller). This simulates the substitution of an impurity atom of a different size. Adding many more, we can simulate an alloy formed by atoms of different sizes.

The stability of a configuration can be judged according to two criteria. According to the first, the greater the activation energy necessary to take the magnets out from the potential wells in which they are placed, the more stable the configuration will be. According to the second, a configuration will be more stable the greater the distances from which we can displace the magnets, without provoking the system to change to another equilibrium configuration. With our system of floating magnets it is possible to appreciate the stability of a configuration either by blowing on the magnets (i.e., giving them energy), or displacing some of them out of their equilibrium positions.

Instead of the circular array of fixed magnets, we can use a coil carrying a direct electric current in order to produce a magnetic field that maintains the floating magnets separated from the wall of the container. The depth of the potential well is proportional to the

Table 1. Number of magnets per layer.

1st layer	2nd layer	3rd layer	4th layer	Total
1	6			7
3	9			12
5	10			15
1	5	10		16
1	6	12		19
1	7	14		22
1	6	12	18	37

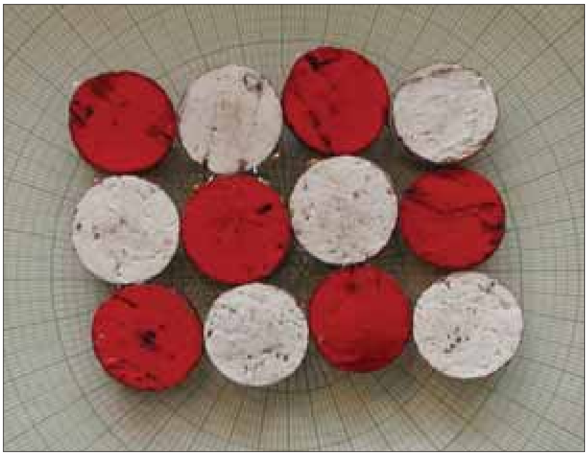

Fig. 3. Ionic model: (a) square lattice formed with six magnets with their dipole moment pointing upwards; (b) six with the opposite orientation.

electric current in the coil. Modulating the field with a superimposed AC current (of the appropriate frequency), the vibration modes of a highly damped system can be observed.

The system of floating magnets is also useful to simulate ionic interactions, because the attractive and repulsive properties of magnets are very similar to those of negative and positive charges, except for the fact that the exponent in the distance's dependence is greater by one. If we place an even number of dipoles, one half with one orientation and the other half oppositely oriented, we obviously observe that the magnets with opposite orientations attract each other until they are in contact (the contact providing the repulsive force required for the balance). The existence of frictional forces may prevent the symmetric equilibrium configuration from being attained. In this case, the configuration is similar to that of an amorphous material. To obtain an ordered structure (crystalline), it is just necessary to provide some thermal energy by stirring the water surface. It can be observed that only magnets with opposite orientations are in contact. In this case, the repulsive force is provided by the contact between them, and therefore, to attain the equilibrium it is necessary for this force to be normal to the contact surface. Figure 3 shows six magnets with the north pole up and six with the opposite orientation. They formed a square lattice after some stirring.

Conclusions

A system of magnets floating on water is shown to be a useful model to appreciate how the combined effects of long-range (attractive) and short-range (repulsive) forces lead to the formation of different equilibrium configurations (ECs). This system allows students to observe that *N* particles (magnets) can arrange themselves in different ECs that may not be equally stable. The activation energy required to change the system from one EC to another can be appreciated directly (by pushing or blowing on the magnets). By using magnets with different polarities, the ionic interaction can also be simulated, and the formation of ionic crystals can be easily understood. This system of floating magnets can also be proposed as a challenge for theoretically oriented students, who should be able to explain and predict theoretically (with the aid of a PC) the ECs observed experimentally.

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