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SOCIAL HIERARCHIES WITH AN ATTRACTIVE SITE DISTRIBUTION

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We reinvestigate the model of Bonabeau *et al.*[1] of self-organizing social hierarchies by including a distribution of attractive sites. Agents move randomly except in the case where an attractive site is located in its neighborhood. We find that the transition between an egalitarian society at low population density and a hierarchical one at high population density strongly depends on the distribution and percolation of the valuable sites. We also show how agent diffusivity is closely related to social hierarchy.

Keywords: Social and economic systems; hierarchies; randomness; diffusivity.

1. Introduction

An example of how hierarchical inequalities are created is given by the model of Bonabeau *et al.*[1] For example, the transition thousands of years ago from more egalitarian hunting and gathering societies to more hierarchical agricultural and city life could be described by a phase transition.[2] In the original Bonabeau model, agents diffuse on the square lattice to the four nearest neighbors. Agents (animals, individuals, communities, countries, etc) are initially equal and then they diffuse randomly.[1] When an agent moves onto an occupied site, a fight breaks out between the two and a memory function of the outcome is stored. The basic feature of the model is the introduction of an agent fitness based in its memory that evolves with the following rule. Whenever an agent wins, its probability to win again increases and when it loses, its probability to win again decreases.

In the same spirit of the above model, there are some previous works.[3–8] The phase transition between an egalitarian and hierarchical regime (or society) observed at a critical density was very weak[1] and it was reinforced[5,6] by introducing a feedback mechanism on the probability of an agent's rise or fall in the hierarchy. Since these models produced the same number of weak (low fitness) and powerful (high fitness) agents in the hierarchy, a slight modification was made to reproduce the more realistic case of more weak agents.[8] Numerical simulations have been reported on the Bonabeau model on a fully connected graph, where a "forgetting" control parameter is crucial and spatial degrees of freedom are absent.[9] Later on, the model was applied to agents moving in scale-free networks[4] exhibiting a sharp transition from an egalitarian to an hierarchical society, with a very low population density threshold which depends on the network size. Finally, an analytical model within a mean-field theory is developed with a distribution obeying a nonlinear master equation which also exhibits a phase transition.[10]

In all the previous models, it is assumed that the sites in the lattice are all equivalent and the agents decide the direction of motion at random. In many environments not all directions or sites are equivalent. Usually, some places are more attractive than others due to several factors, like for example the availability of natural resources or the strategic value. To improve upon the Bonabeau model, in this work we assign to each site a value determined by a statistical distribution. Then we consider agents move in such a way that they try to reach high value sites.

2. The Model

In the Bonabeau model,[1] agent diffusion occurs in a Margittay Neumann neighborhood of four neighbors. They diffuse randomly and when agent i wants to move into the site of agent k, a fight breaks out between the two. If i wins, i and k exchange sites on the lattice; if k wins none of them move. Therefore these rules imply that the population density is kept constant through the whole evolution. Following Stauffer,[8] the probability q for i to win is given by

$$q = \frac{1}{1 + \exp(\sigma[h(k) - h(i)])}, \tag{1}$$

where $h(i)$ counts the weighted number of victories minus the weighted number of loses of person i, and the amount of inequality is measured by

$$\sigma = \sqrt{\langle q^2 \rangle - \langle q \rangle^2}, \tag{2}$$

and the average $\langle\,\rangle$ is performed over all the agents. Initially the probability of losing is 50% since all h are zero.

To improve over the Bonabeau model, we propose to keep the fighting rules of the Bonabeau model but changing the way in which the agents move. By considering that each site in the lattice can be valuable or non-valuable, the new rules of agent movement are the following:

(1) If all the neighbor's site values are less than the value of the site occupied originally by the agent, then the agent remains in the site, since we assume that it is not worthwhile to take a less valuable site.

(2) When condition (1) is not fulfilled, the agent moves to the neighboring site with the highest value. However, it can happen that more than one of the neighboring sites have the same high value. In such a case, one site is chosen at random among the most valuable. Finally, it can also happen that the original site has the same value as the maximum value of the neighbors, then the agent always moves to one of the randomly chosen most valuable neighbors. When all sites are equally valuable, our rules are similar to the Bonabeau case.

We use a random distribution of sites, with probability x of having a valuable site, and $1 - x$ for non-valuable sites. Thus, the resulting lattice is akin to a two-dimensional percolation problem or random binary alloy.[11] It is worthwhile to remark that the percolation transition occurs for $x = 0.59$, i.e., for $0.59 \leq x \leq 1.0$ there is an infinite spanning cluster of valuable sites.

3. Results

Our simulations were made on 250×250 square lattices. Figures 1(a) and 2(a) shows the results of inequality as a function of the density p of agents. For $x = 1.0$, we recover the results of Stauffer, where a phase transition is observed around $p = 0.35$, with an inequality of $\sigma = 0.45$ for high populations. As the concentration is diminished, two important features are observed. The first is a gradual shift of the phase transition toward low density populations. When $x = 0.60$ the phase transition almost occurs at $p = 0$. In that sense, the introduction of valuable sites tends to enhance the inequality for low p. However, at high populations, there is a decrease in the inequality as valuable sites are introduced. Also, in Fig. 1(b) we plot the agents diffusion constant, defined as,

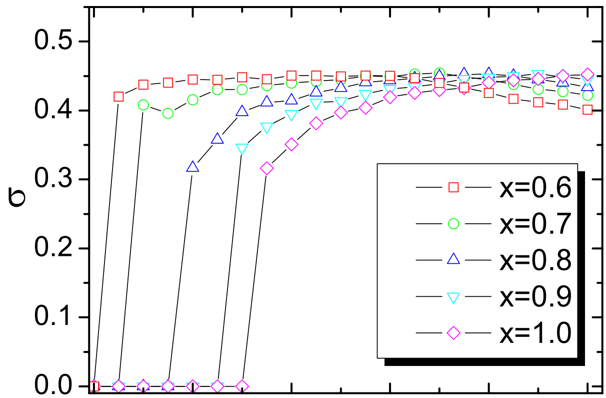
$$D = \frac{1}{N} \sum_{i=1}^{N} \left(\lim_{t \to \infty} \frac{\langle x_i^2 \rangle_t}{t} \right) ,$$

where $\langle x_i^2 \rangle_t$ is the mean square displacement of agent i at time t and N is the total number of agents. The distance between sites is taken as 1 and the time is measured in units of steps of the simulation.

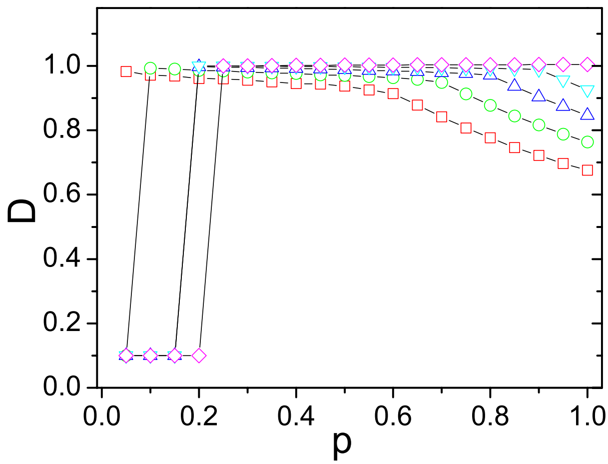
The diffusivity plot Figs. 1(b) and 2(b) reveals an important fact, even for the Bonabeau model: the phase transition in the inequality is precluded by a transition in the diffusivity. In this case, the transition to inequality occurs around the critical concentration $p_c^\sigma = 0.35$ while diffusivity jumps at $p_c^D = 0.25$. A second observation from Fig. 1, is that p_c can be tuned by the concentration of valuable sites. This happens because an important aspect of the present model is that agents try to stay or move toward valuable sites. A rough estimation of this change is to suppose a lattice of attractive sites with $(1 - x)$ impurities that decrease diffusion, in this case the critical concentration is given by $p_c^D \approx x/4$.

Another interesting feature observed in Fig. 1(a), is the decreasing of σ with x at high populations. The reason is that when valuable sites do not percolate, they decrease the diffusivity since agents try to avoid those sites. In this limit, it is useful to remember the picture of electrons or phonons in a crystal, which are scattered by impurities that decrease the diffusivity. Then, as D decreases, fights are less common and σ diminishes. Notice that the place where D diminishes at high population, corresponds nearly to $p = x$. This effect is a consequence of the tendency of agents to reach attractive sites.

Between $x = 0$ and $x = 0.5$, see Fig. 2, we observe the absence of a phase transition for almost all x, i.e., hierarchies almost always exist, except when $x \to 0$,



(a)

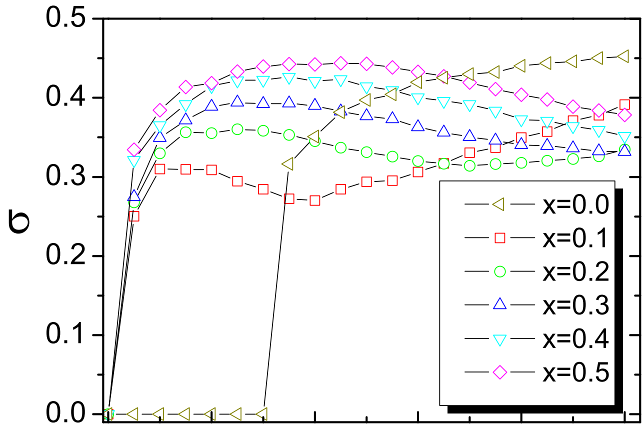


(b)

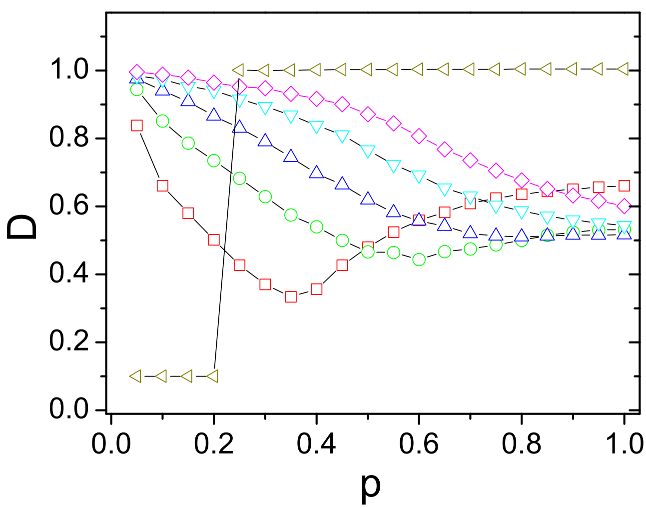
Fig. 1. (a) Inequality σ as a function of the population density p, for different values of the rich site concentration x. The symbol code for each value of x is in the inset of (a). (b) Diffusion constant as a function of p, for the same values of x as in (a). The symbol code is the same as the inset of (a).

where the Bonabeau model is recovered. However, the transition to the Bonabeau model is not as smooth as the case $x \to 1$. We have observed that the transition occurs when the concentration of sites is very small, of order 10^{-3}.

Another feature is that the inequality σ tends to decrease as a function of x for a given p. This tendency is also reflected in D, and is a consequence of the asymmetry between the probability of establishing agents around valuable sites. Agents in nonvaluable sites are always trying to jump into valuable sites, while the other agents just stay in their places. As a consequence, diffusivity is decreased.



(a)



(b)

Fig. 2. (a) σ versus p and (b) D versus p for concentrations of strategic sites between $x = 0$ and 0.5.

4. Conclusions

We have studied the effects of including sites with different values in the Bonabeau model. The results show that in general, the inequality is enhanced and the critical concentration for a phase transition observed by Stauffer is moved towards lower values of the density. The transition to the homogeneous case is very sharp in the case of few attractive sites. Thus, the introduction of an inhomogeneous distribution of valuable or attractive sites has very important effects upon the artificial society.

Acknowledgments

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