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Diffusion of knowledge and globalization in the web of twentieth century science

Q1   G.G. Naumis [a], J.C. Phillips [b],*

[a] Univ Nacl. Autonoma Mexico, Inst Fis, Mexico City 01000, DF, Mexico
[b] Department of Physics and Astronomy, Rutgers University, Piscataway, NJ, 08854-8019, United States

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ABSTRACT

Scientific communication is an essential part of modern science: whereas Archimedes worked alone, Newton (correspondence with Hooke, 1676) acknowledged that "If I have seen a little further, it is by standing on the shoulders of Giants." How is scientific communication reflected in the patterns of citations in scientific papers? How have these patterns changed in the 20th century, as both means of communication and individual transportation changed rapidly, compared to the earlier post-Newton 18th and 19th centuries? Here we discuss a diffusive model for scientific communications, based on a unique 2009 scientometric study of 25 million papers and 600 million citations that encapsulates the epistemology of modern science. The diffusive model predicts and explains, using no adjustable parameters, a surprisingly universal internal structure in the development of scientific research, which is essentially constant across the natural sciences, but which because of globalization changed qualitatively around 1960. Globalization corresponds physically to anomalous diffusion, which has been observed near the molecular glass transition, and can enhance molecular diffusion by factors as large as 100.

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1. Introduction

Stretched exponential "fat tails" have been observed in the distributions of radio and light emissions from galaxies; of US GOM OCS oilfield reserve sizes; of World, US and French agglomeration sizes; of country population sizes; of daily Forex US–Mark and Franc–Mark price variations; of Vostok (near the South Pole) temperature variations over the last 400,000 years; of Raup–Sepkoski's kill curve; and of earthquake sizes and fault displacements [1]. Almost all of these databases are sparse for $\sim 10^2$–10^3 entries or fewer. Early informetric studies of citation patterns of "only a few" million citations identified power law patterns, which were interpreted in terms of a cumulative advantage model (analogous to properties of equilibrium materials near a critical point; in everyday terms, the rich get richer). Power-law patterns have been interpreted in many interesting ways [1–3]. The exponents represent adjustable parameters that cannot be determined without a specific (preferably objective physical) model, which should describe systems in equilibrium.

Scientometric studies of citation patterns have shown a mixed distribution, with power laws prevailing at larger numbers of citations and stretched exponentials (SE) giving better fits for smaller numbers [3]. Power law functions are simple and obey simple relations (a power of a power is itself a power, which means that systems described by power laws are "scale free"), but stretched exponentials are peculiar, and arise only in special contexts. Just as power laws yield linear log–log charts, so do SE distributions yield linear log(log)–log plots, so given enough scientometric data it is possible to distinguish

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* Corresponding author.
    E-mail address: jcphillips8@comcast.net (J.C. Phillips).

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between the two distributions, but what is enough? This question has been discussed in the context of many typically sparse databases, and it was concluded that with only sparse databases it is difficult to distinguish between power laws and heavy-tailed ("black swan") distributions, such as stretched exponentials [4].

While power laws are widely used to describe sparse data, many scientists are unjustifiably suspicious of successful SE fits even to very large databases with hundreds of millions of entries. For very large databases one can appeal to objective physical models. Mathematically equilibrium models have been defined in the context of homogeneous graphs [5], but a genuine objective physical model should not only be microscopically homogeneous, but it should also correspond to the observable properties of real materials, which makes the task of finding suitable objective physical models much more difficult, as real materials are much more complex than graphs or even sand piles or coffee grounds (wet or dry). Specifically glasses are much more complex than even homogeneous random graphs, and network glasses are no longer described by theorists as "continuous random networks".

A 2009 very large scale scientometric study [3] of 20th century citations, involving 25 million papers and 600 million citations, with 10^6–10^7 more entries than typical power-law databases and unique in the history of epistemology, found that stretched exponentials (SE) give better fits than power laws to citation chains for both low and intermediate citation levels $n < n_1$, where n_1 is the "crossover to fame". Here $n_1 \sim 40$ earlier in the century, and $n_1 \sim 200$ later. These "best fits" bifurcate into two well-separated groups when scaled with respect to the total number of citations for each decade. (This is an instructive point, because many sparse data sets appear to collapse onto a single universal power law curve when normalized, and this is often taken as evidence for scale-free character.) The middle SE region with $n < n_1$ accounts for 95% of the citations, and it, not the 5% scale-free or power law high end, probably objectively represents the essential features of working research citation patterns. The parameters involved in the stretched exponentials appear to follow rules already recognized [6] as characterizing relaxation of ideal (microscopically homogeneous) glasses. This coincidence is hardly likely to be accidental: it suggests that a universal, objective physical model for systems described by SE distributions does exist, which is analogous to the well-established equilibrium critical point models for power-law distributions. This objective physical model must be based on glasses, which are not in physical equilibrium even when they are microscopically homogeneous.

2. Correspondence between glass networks and citation chains

The quantitative correspondences between the stretched exponential fractions β of citation chain distributions and glass network relaxation are quite precise, and already break down for even deeply supercooled liquids, with viscosities millions of times that of water, where β is much larger (typically ~ 0.8). The characteristic features of glass networks are that the atoms are confined to cages, within which they vibrate for long times without escaping [7]. The cages are formed during quenching because there is a near equality between the number of short-range force constraints and the number of spatial degrees of freedom d; the usual momentum degrees of freedom, also d and found in gases and liquids, are absent in glasses. The implications of this topological equality between constraints and spatial degrees of freedom are especially clear for covalent network glasses, in terms of bond-stretching and bond-bending forces [8]. They have been developed in great detail for chalcogenide and some oxide glasses, where they not only describe the phase diagrams with an accuracy of $\sim 1\%$, but also they have led to the discovery of a new "intermediate phase", which is rigid yet unstressed, and which exhibits nearly reversible glass transitions [9,10], with sharp compositional boundaries. (These quantitative topological concepts have replaced the old and topologically qualitative description of network glasses as "continuous random networks".) Quenching produces a few traps (empty cages) even in the "intermediate phase", which is nearly stress-free, but there is always some residual internal stress, and some residual non-reversibility of the glass transition. The traps cause the glassy medium to be multiply (not simply) connected, and fundamentally alter its relaxation properties.

When an article is written, it utilizes the "rigid" existing literature as sources of its citations, so the cited articles correspond to the caged atoms in a glass. The citing article corresponds to a mechanically, thermally or optically excited atom of the glass, which as its excitation diffuses through the glass, undergoes collisions with caged atoms, each collision representing a citation. When the excitation stops, the citation chain ends, and the last citing article occupies a fixed (immobile) trap (formerly empty cage). The trap captures the excited atom or excitation, which becomes immobile and part of the immobile glass network. The correspondence between citation statistics and glass relaxation occurs because the latter is also described most accurately by a diffusion-to-immobile traps model. By contrast, in supercooled liquids the traps are mobile; their sweeping motion accelerates their capture of excitations, shortening lifetimes and increasing the stretching fraction β well above its glass value until it eventually approaches 1 in normal liquids (simple exponential relaxation), destroying the correspondence with the citation stretching fractions. There are no "mobile" published articles, so an objective physical model for SER involving some material other than glasses, such as deeply supercooled liquids, is unreasonable.

3. Microscopic trap model of relaxation of strongly disordered solids

Stretched exponential relaxation (SER) was first recognized by Kohlrausch in 1847 as providing a very good fit (for example, far superior to two exponentials, although the latter involve four parameters) to the residual glassy decay of charge

on a Leyden jar. The stretched exponential function $A\exp(-(t/\tau)^{\beta})$ contains only three parameters, with τ being a material-sensitive parameter and $0 < \beta < 1$ behaving as a kinetic fractional exponent. SER is regarded by some as the oldest unsolved problem in science.

As sample homogeneity and data improved, the accuracy of SE three-parameter fits also improved by orders of magnitude, and over time ranges t as large as 10^{12}, leaving no doubt that SER is intrinsic to microscopically homogeneous glasses, after rigorous derivations of SER emerged in the 1970's for short-range forces [6]. It turns out that the key to understanding field-free, time-resolved SER, which is characteristically observed most accurately in microscopically homogeneous glasses, is to recognize that such glasses, unlike most crystals and normal liquids where critical point behavior is observed for second-order equilibrium phase transitions, are not topologically simply connected; instead they are multiply connected. Multiple connectivity is a topological property that is characteristic of systems that cannot be described by mean-field approximations, except in a restricted sense when the point defects are randomly distributed. Multiplied connected configuration spaces inevitably lead to non-equilibrium, hysteretic glassy behavior, as phase transitions can occur non-coincidentally among multiply inequivalent and chronologically tangled percolation paths. Because the structural evolution and annealing of even microscopically homogeneous glasses is arrested at the glass transition temperature T_g, mean-field approximations must be handled with care in describing glassy relaxation.

During the quenching process, an excellent glass-former avoids crystallization, and it may appear to be microscopically homogeneous, but it actually contains quenched-in, randomly distributed defects that function as traps for diffusive electronic or molecular excitations. When excitations decay, they diffuse freely to these traps and disappear. The excitations that are closest to the traps disappear first, and the excitations that are further away take longer times to reach the traps. It is this easily calculated effect (a kind of memory effect, characteristic of non-equilibrium systems) that produces SER [6], and is ultimately responsible for many hysteretic effects.

While the overwhelming majority of relaxation studies are made on microscopically inhomogeneous glasses [11], fortunately there are many glasses that can be made microscopically homogeneous, especially network glasses (like silica and window glass) and molecular glasses (like glycerol) where accurate temporal relaxation data are available (at least 50 (not just one or two) well-studied cases, including a few polymers, where homogeneity is harder to achieve). There the microscopic theory based on excitations diffusing to traps has proved to be extremely accurate, explaining multiple (material family, pump, probe) concordances in the modern data [6]. The odds against the theory (which has not been rigorously proved) being successful by accident are of order 10^{50} to 1 [12,13].

4. Anomalous (super-)diffusion

Ideally one would like to be able to simulate citation chains realistically, but this is both impossible and unnecessary. It is impossible to construct a virtually omniscient program that will simulate 25 million papers and 600 million citations. It is unnecessary, because the key diffusion process in glasses with traps has already been simulated, as we will see later in discussing Eq. (4).

The key result of the diffusion to traps model [6] is that residual relaxation in glasses is not described by a power law, but by SER with $\beta_g = d/(d+2)$, where d is the effective dimensionality of configuration space. Moreover, SER with β well below 1 is found primarily in glasses and deeply supercooled liquids [6]. For short range forces d $= 3$ and $\beta_g = 3/5$, a result confirmed by both numerical simulations and many experiments [6,12,13]. However, the long-chain nature of polymers introduces long-range elastic forces, and in some other materials (like solid electrolytes and partially ionic semiconductor alloy quantum dots) there are strong long-range Coulomb forces. In all these long-range cases it was found without exception that $\beta_g = d^*/(d^* + 2)$, with $d^* = fd$, and $f = 1/2$. Because space-filling itself involves a delicate balance between short-range constraints and long-range attractive forces, such an equalized kinetic balance between a short-range and long-range diffusion steps seems quite plausible. In this case $d^* = 3/2$ gives $\beta_g = 3/7$, a value observed in experiments on microscopically homogeneous polymers and solid electrolytes. The crossover from 3/5 to 3/7 has been observed in the case of monomer (PG) to polymer (PPG) propyl glycol [12]; it occurs when the polymeric chain contains around 5 monomers.

Suppose that the diffusive motion is determined by competition between short- (**r**) and long-range (**R**) forces, and that the respective length scales are l and L. Also suppose that the average distance between traps is λ (also the radius of the spherical capture construct [14]).

$$l < \lambda < L. \tag{1}$$

Here λ defines a characteristic length scale in the multiply connected medium, and this (together with the intrinsically non-equilibrated cage character of glasses) means that glassy relaxation is not scale-free. The two inequalities in (1) divide the set of all diffusive motions into two subsets, those with short Brownian steps only, and those with at least one long step (called a Lévy flight in the mathematical literature). In the second subset the diffusive motion becomes entangled, and the simple short-range solution $\beta_g = d/(d + 2)$ is no longer valid. To see what happens in the entangled case, consider the following example from polymer science. Dense packing of polymer glasses tends to favor polymer bundles, with individual polymer chains having variable lengths. Here excitations diffuse to polymer chain ends, which act as traps, but not all the chain ends are at bundle ends; many chains are broken within bundles (length scale λ). Diffusing excitations can reach these internal traps by diffusing with large steps to the end of a polymer bundle (length scale L) bypassing many adjacent traps, or by hopping between chains (short steps, length scale l) to reach traps on other chains. This entangled process implies

composite relaxation paths that combine small l and large L steps. The large steps along polymer chains are similar to the Lévy flights often discussed in statistical mechanical models of foraging [15].

Heuristically we analyze composite relaxation paths by dividing the excited density $\rho(t)$ harmonically:

$$\rho(t) = \rho_s(t/\tau_s)\rho_l(t/\tau_l). \tag{2}$$

The local diffusion equation in normal liquids is based on short-range **r** forces and is written

$$\partial\rho/\partial t = D\nabla^2\mathbf{r}\rho. \tag{3}$$

In glasses with short-range forces the diffusion to traps model leads [9] to $\beta = d/(d+2)$, which is in excellent agreement with earlier results [4,7,8] and those discussed here. How should (3) be generalized to describe the non-local case of mixed short- (**r**) and long-range (**R**) forces? The physical content of (3) is that the spatial fluctuations on an **r** scale drive the temporal diffusion. In the mixed **r**:**R** case, short-(long-)range forces drive spatial fluctuations on an **r** (**R**) scale, with the two processes competing to drive temporal diffusion. This picture is the dynamical relaxation aspect of the "multiple length scale" approach to polarizable static inhomogeneities [15]. Alternatively ρ could be represented as a Fourier sum, and evaluated by an Ewald separation into **r** and **R** parts. Such non-local competition between short- and long-range forces (and small (Brownian) and large (Lévy flight) diffusive steps) suggests the anomalous diffusion equation with superposed fluctuation channels

$$\partial\rho/\partial t = D'(\nabla^2\mathbf{R} + \nabla^2\mathbf{r})\rho. \tag{4}$$

The superposition approximation in (4) has been studied and validated by extensive numerical simulations in the context of anomalous diffusion probe trajectories described as continuous random walks impacted by bath statistics [16]. Substituting (2) in (4) causes the left hand side to separate into two terms for $\partial\rho_s/\partial t$ and $\partial\rho_l/\partial t$. Because of the factorization assumption (2), the right hand side also separates, and the optimal combination of relaxation by combined short- and long-range forces is determined by the principle of maximum entropy production [17]. One can assume that at the variational extremum this entropic optimization produces a separation of the right hand side of (3) fully parallel to that of the left hand side. This leads to decoupled equations for $\rho_l(\mathbf{R})$ and $\rho_s(\mathbf{r})$, each equation formally identical to (3). With $\rho_l(\mathbf{R})$ scaling as $L^{-f d}$, and with $\rho_s(\mathbf{r})$ scaling as $L^{-(1-f)d}$, (4) can then be solved by applying the capture sphere methods of [14] twice to the separately local **r** and **R** equations to give

$$I(t) \sim \exp\left[-(t/\tau_1)_l^\beta\right]\exp\left[-(t/\tau_2)_s^\beta\right] \tag{5}$$

with $\beta_l = fd/(fd+2)$ and $\beta_s = (1-f)d/((1-f)d+2)$. The fastest relaxation (maximum entropy production [10]) at long times will occur along paths for which $f = 1/2$ and $\beta_1 = \beta_s = \beta_{ls} = d/(d+4)$. The two relaxation times collapse into one, with

$$\tau^\beta{}_{ls} = (\tau_1)_{ls}^\beta(\tau_2)_{ls}^\beta/[(\tau_1)_{ls}^\beta + (\tau_2)_{ls}^\beta]. \tag{6}$$

The extra "push" given to short-range fluctuations by long-range forces explains the anomalously enhanced (by a factor of 100!) surface diffusion reported for fragile molecular glass formers like OTP [18]. Even macromolecules diffusing through small pores exhibit [19] enhanced diffusion, with $\beta \sim 0.8$. The reduced dimensionality of surfaces or pores enhances the effects of long-range forces. It is striking that while the measured short-time diffusivity is enhanced by long-range interactions, its long time-dependent relaxation is slowed, as β is reduced.

Mathematical note: the physics behind the ideal model describes particles diffusing to traps, whereas the derivation given in Ref. [14] utilizes a different geometry, namely the particle is at the sphere center initially and the nearest trap is represented by a spherical shell at $r = \lambda$. However, the final result for β is topological ($\beta = \beta(d$ only$)$), so the change in geometry is irrelevant. Alternatively, we can consider a random distribution of traps, and partition space into Voronoi polyhedra. We can now construct a topologically isomorphic model, in which the excitations are initially distributed on the polyhedral surfaces. Then we convert this model to its dual, in which the excitations are initially at the polyhedral centers, and the traps are distributed on the surfaces. The dual model preserves d, and yields the same value for β, which proves the correctness of the spherical cluster construction [14]. The bifurcation of β into 3/5 and 3/7 channels is so universal in ideal glasses because of these topological isomorphisms.

5. Scientometric citation results

The startling result found in the scientometrics study [3] is shown in Fig. 1. Not only do the intermediate distributions exhibit SER, but they also separate into two bunches, which shift abruptly in 1960, with stretching exponents β that match (within a few %) the $\beta_g = (\beta_{g11}, \beta_{g22}) = (3/5, 3/7)$ values predicted in the 1996 abstract of Ref. [6]: 3/5 holds < 1960, while 3/7 holds > 1960! More precisely, the fitted values are $\beta_c = (\beta_{c11}, \beta_{c22}) = (0.57, 0.47)$. The task of theory, to explain these results, goes well beyond noting their agreement with the d = 3 glass values identified in 1996, as one must justify using d = 3, and also explain how the microscopically homogeneous conditions necessary for glass networks become appropriate for the world-wide 20th century citation network. One must also explain how these conditions changed abruptly in 1960, and here this is done without using adjustable parameters.
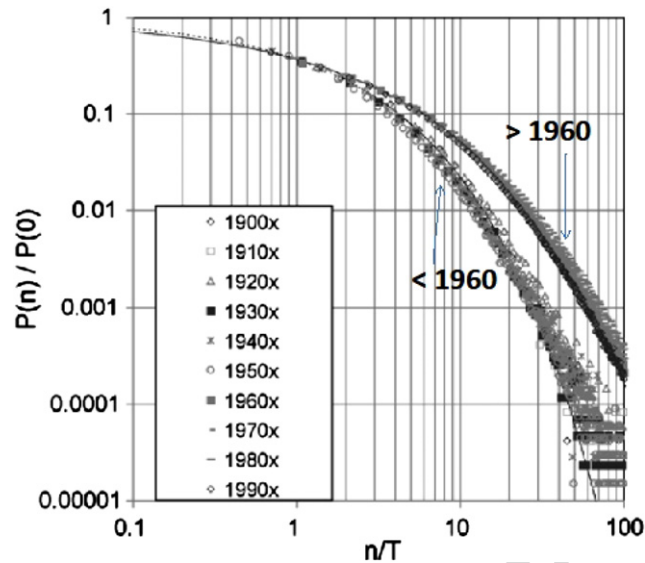
Fig. 1. Evolution of the number of articles n with a number of citations τ (or T) as a function of the decade in which the article was published. The distributions for the decades up to 1960 bunch and are well fitted by an SE with $\beta = 0.57$, while the decades after 1960 also bunch, but are fitted with $\beta = 0.47$. There is no crossover decade; the crossover at 1960 is unambiguous [3], as are the accuracies of the SE fits.

6. Disordered crystalline alloys

What happens when the sample is disordered but is not a conventional glass? This case should surely test the limits of the bimodal mixed forces theory, and by studying it we can go beyond facile comparisons to a deeper understanding of the origin of the extremely close correspondence between the citation network and network glasses.

This question has been elegantly answered in a recent study of luminescence from 16 single-crystal isoelectronic $ZnSe_{1-x}Te_x$ alloys of commercial quality (these microscopically homogeneous alloys are used in orange light-emitting diodes) [20], with results for $\tau(x)$ and $\beta(x)$ shown in Fig. 2. After optical excitation has produced free electrons and holes, there are many channels for radiative recombination. In the pure crystal ($x = 0$ and $x = 1$) the free electron and hole can recombine, or they can first form an excitonic (hydrogenic) bound state and then recombine, with simple exponential kinetics ($\beta = 1$). In the alloy, $0 < x < 1$, either the electron or the hole or both can be partially or strongly localized, adding complex recombination channels. The competition between this multiplicity of unavoidably disordered channels always leads to $\beta < 1$ in the alloys, in other words, SER. The accuracy of SER fits to commercial semiconductor luminescence data has improved by a factor of order 100 from the best data of 15 years ago [21], and [20] used SER to monitor the quality of their samples.

A close inspection of Fig. 2 reveals two previously unnoticed features associated with the extrema of $\tau(x)$ (near $x \sim 0.1$) and $\beta(x)$ ($x \sim 0.25$). With increasing x, Te anion clusters attracting holes begin to form localized states (the ionization potential of ZnTe is smaller than that of ZnSe). The holes also have much heavier masses than the electrons, which means that the holes can localize at these isolated Te-rich clusters while the electrons remain delocalized, a situation favorable for long lifetimes, which explains the maximum in $\tau(x)$ at $x \sim 0.1$. The localized holes will be strongly disordered, and when the clusters are large enough to localize nearly all the holes, these localized hole states will nearly fill space – in other words, near the maximum in $\tau(x)$ at $x \sim 0.1$, the Te clusters and their bound holes resemble a conventional glassy state. For $x > 0.1$, the clusters will begin to overlap, and the holes will begin to delocalize, losing the glassy, caged character of the hole distribution.

Very near the maximum in $\tau(x)$, we expect that radiative recombination will be dominated by states in which the electron and hole have formed an exciton state bound to the Te-rich clusters. The radius of this exciton state defines a characteristic length for short-range interactions, and thus the value of $\beta(0.1)$ is close to $3/5$, as shown in Fig. 2.

Although the energy gaps at $x = 0$ and $x = 1$ are nearly equal, the alloy electron–hole energy gap is strongly bowed downward, and the band edge shifts from 2.7 eV ($x = 0$) to 2.05 eV for $x = 0.35$. In 1993 [22] observed a continuous Localized–Extended (L–E) transition from the recombination through free and bound exciton states to the recombination of excitons localized by the compositional fluctuations of the mixed crystal in the concentration region of about $x = 0.25$. This L–E transition is the two-particle analog of the better known single-particle metal-insulator transition. The transition occurs when the phase spaces accessible to the localized and extended states are nearly equal. This is again a kind of dual space-filling condition, and it is the kind of condition where one expects $f = 1/2$, and $\beta(0.25) = 3/7$. As shown in Fig. 2, this is what actually is observed at the β minimum. Size-dependent effects have been observed in luminescence from pure compound quantum dots; $\beta = \beta_{11}$ or β_{22}, depending on the size of excitons compared to dot size [12,13].
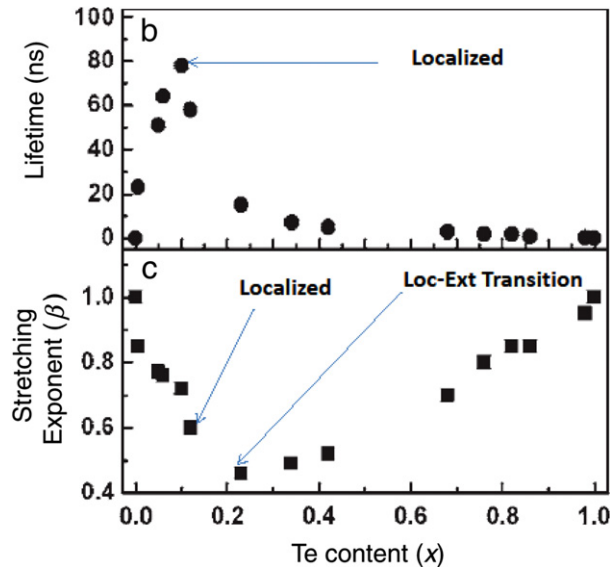
Fig. 2. Annotated data [20] on luminescence in isoelectronic Zn(Se,Te) alloys. The peak in relaxation time τ occurs near $x = 0.10$, where there is a break in $d\beta/dx$. These are the maximally localized states, which behave as quasi-particles subject to short-range forces only, with $\beta = \beta_1 = 3/5$. The localized-extended transition occurs at $x = 0.22$, where the long- and short-range forces are equally weighted, and $\beta = \beta_2 = 3/7$. The β fitting errors (0.2%) are about ten times smaller than the size of the data points, as expected from semiconductor samples of commercial homogeneity. In this case, because the crystalline alloys are not true glasses, glassy relaxation is obtained only near the τ and β extrema.

7. Physical ranges of citation interactions

Ref. [3] surveyed 6.10^8 citations, but this number, although very large (much larger than in previous studies of "only" a few million citations [2]), is still small compared to Avogadro's number. Moreover, scientific culture is far from microscopic homogeneity, with significant differences between North and South America, Europe, Africa and Asia. If we regard β_c and β_g as the diagonal elements of β_{ij} matrices ($i, j = 1, 2$) in citation and glass spaces respectively, then $\beta_{c11} + \beta_{c22} = \beta_{g11} + \beta_{g22}$, in other words, the traces of the β_c and β_g matrices are equal to within 1%. To explain the reduction in the splitting of the informetric citation values relative to physical values found in glasses, we must assume that while the glass matrix can be taken as diagonal except in rare crossover cases [12], the citation matrix is anti-Hermitian, with identical imaginary off-diagonal values $\beta_{c12} = \beta_{c21} = ai$, with $a \ll 1$.

Most scientists are familiar with Hermitian matrices, which arise in quantum theories of equilibrium. Anti-Hermitian matrices arise in the context of a new and very efficient way to calculate two-electron excited states of many-electron atoms and molecules, without doing configuration interaction, by using an anti-Hermitian contracted Schrodinger equation (ACSE) [23]. In view of the well-known parallels between temperature-dependent Hamiltonian H dynamics for the partition function Z ($dZ/d\gamma = -HZ$, with $\gamma = 1/kT$) and the Schrodinger equation for the time-dependent wave function ψ ($d\psi/dt = iH\psi$), this suggests a way of understanding both citation distributions and glassy relaxation.

Within the ACSE one calculates two-body charge density matrices that can be said to be the classical analog of citer–cited pair interactions. The ACSE is an optimized reduction of the exponentially complex problem of quantum configuration interaction. In principle the two-body charge density matrices are related to the three- and four-body charge density matrices, which in turn are related to still higher order charge density matrices. Because interference effects are not included, these classical correlation functions in general do not correspond to actual wave function solutions of the Schrodinger equation itself, which is consistent with a non-equilibrium model. In many examples that restricting the higher order corrections to the leading anti-Hermitian third-order term yields excellent results, at least an order of magnitude better than obtained from the full (Hermitian + anti-Hermitian) third-order term [23]. Classically speaking, the anti-Hermitian interaction corresponds to damping of the pair interaction by a third particle. Apparently such damping seldom occurs in glasses (for example, in quantum dot luminescence it would require accidental coincidence of exciton and dot radii). It does occur in citation space (albeit only weakly); it incoherently admixes a small fraction of additional long-range interactions into the "pure" short-range case, and vice-versa.

8. Sociology and dimensionality

Scientific collaboration occurs on multiple levels, from details to general ideas and motivation [2]. Direct collaboration requires frequent contacts of individuals, and appears quite analogous to short-range interactions in glasses. The apparent dimension of the environment is likely to be $d = 3$ (latitude, longitude, and time t). Since there is a strong tendency to cite

articles that one knows best, and these are often the articles of one's closest colleagues, and possibly their closest colleagues, the citation number for a given article will be limited largely to this circle. This corresponds to $\beta_1 = 3/5 \sim 0.57$, which gives the best fit to low and intermediate citation levels prior to 1960.

Because the detailed aspects of a research article are best known only to this small space–time circle, only the more general aspects will give rise to a larger number of citations. After 1960, scientific conferences became popular and grew steadily in size, for a wide variety of practical reasons – an upsurge in funding for science after Sputnik (suddenly many research scientists were offered all-expenses paid intercontinental travel), rapid moderation of commercial jet fares (the first successful commercial jet, the Boeing 707, was delivered in waves, with the first delivery wave peaking in 1960), etc. At international conferences one encounters a wider range of ideas in a much less detailed format, which corresponds to long-range interactions, and appears to give a broader distribution of citations. The wider range of ideas can survive longer, the article can continue to accumulate citations, and because $f \sim 1/2$ the diffusive interactions affect only details, while leaving the larger ideas intact and still capable of accumulating citations. Here it is quite striking that the balance between short- and long-range effects on relaxation in glasses should be so closely echoed (similar values of f!) in the competition between narrower and wider influences on citation patterns.

9. Informalogic of science

In one sense our conclusion that long-range interactions changed the nature of scientific research abruptly around 1960 agrees well with the common perception of cultural "globalization". However, even the term "globalization", as reflected in the Science Citation Index, first appeared only in the mid 1980's. By 1990(2000)[2009] it was appearing annually in 30(1000)[1600] articles. Here we have found that citation statistics show that in scientific research the actual transition took place around 1960. It appears that cultural recognition of this transition took more than 30 years!

The most difficult question is, why does the 2009 distribution of scientific citations follow the 1996 model of SER *in glasses* so well, and what does this tell us about science and scientific research? Quite naturally [4,7,8] emphasized the universality of SER in microscopically homogeneous glasses, including polymers. However, current research is showing that polymer blends [24,25] often exhibit multiple SER (not single SER, as shown in Fig. 1 here for citations, and in Refs. [6,12,13] for many microscopically homogeneous glasses, including polymers). Thus there must be internal factors in science that give scientific research a compacted glassy character that is nearly ideal, and specifically correspond to microscopic homogeneity. A possible candidate for compacted glassy character is the search for novelty, which expands to fill the technologically accessible space densely, just as a glass, subject to stronger direct forces, is compacted by weaker residual (Van der Waals) forces. A possible candidate for microscopic homogeneity is the insistence that scientific theories either be validated by experiments (or in the case of some theories, gain wide acceptance without experimental support) – this is what ultimately limits branching balkanizations such as Lysenkoism.

This leaves one last question. Granted that careful scientific research leads to SER, why does it lead either to β_1 or to β_2? In Fig. 2 we notice that although there is a wide range of values of β in pseudobinary crystalline semiconductor alloys, the two common "glassy" values β_1 and β_2 are metastable extrema, either in τ or in β. The characteristic feature of extrema is their stability, which in turn favors consensus. While scientists seek originality, they also attempt to place their new results in the context of established results (local or long-range), which is most readily accomplished with articles that have stabilized their informational contexts. This stability is analogous to the frozen or immobilized diffusive kinetic energy of the glass compared to a liquid. Thus (of course only in retrospect!) the appearance of SER in citation patterns, and even its extremely abrupt change in character around 1960, appears obvious.

10. Broad implications

Although power-law scaling with accurate exponents γ is observed very widely ([1] and 37,000+ articles on "power laws", or 4000+ articles on "power laws" AND scaling* in the Science Citation Index for the last 25 years), at present there exist very few examples where those exponents have been predicted from objective physical models. A noteworthy exception is the evolutionary exponents found in the solvent-accessible surface areas of the twenty protein amino acids [26], which correctly predict hydrophobicity trends [27]. It is striking that often $1.5 \leq \gamma \leq 3$ [28], which is topologically suggestive of fd, with d $= 3$ and $1/2 \leq f \leq 1$; the short- and long-range interaction model may also be applicable to power-law scaling for topologically simply connected (mean-field equilibrium) networks.

The success of the microscopically homogeneous diffusion-to-traps model of the epistemology of modern science has taken many people by surprise. It is based on experimental data from ideal glasses, by which we mean modern microscopically homogeneous glasses [6,12,13], as distinguished from the examples found in older uncritical compilations that included polymer blends, resins and other samples of dubious homogeneity dating back many decades [29]. Nothing succeeds like success, and given the remarkable success of the model, it is important to realize which issues are really surprising. Diffusion is a very common phenomenon, but its exact behavior can easily be masked by sample inhomogeneities, as is the case for most academically prepared glass samples, and great care is required to achieve microscopic homogeneity in ideal glasses.

The wide range of ideally glassy models discussed here show that (in the broadest sense) all non-equilibrium multiply connected networks are similar, as their properties are topological and depend only on dimensionality, even in the

most complex cases (mixed short- and long-range forces). There are many simpler non-topological theoretical models that incorporate some aspects of glass physics, but that either do not derive SER, or do not calculate the stretching fraction β explicitly. Using only Boolean algebra, Monasson et al. have shown [30] that with increasing complexity the computability of k-SAT logical trees (primitive models of citation networks) undergoes two phase transitions that are parallel to those involved in the Boolchand transition observed in the phase diagrams of covalent network glasses with increasing coordination number [8,9]. Excellent theoretical work can be done anywhere (for instance, Lebanon) without large support [31], indicating that global science is microscopically more homogeneous than is often supposed, as implied by the success of the "ideal" glass model in predicting not only SE but also specific values of β.

The question is often raised, will the Internet erase the meaning of distance, leaving time as the only relaxation coordinate? If it does, then the citation distribution will retain its SE form, but with d and d* reducing towards 1 and β reducing towards 1/3. The answer to this question should be known by 2020. A progressive decrease from $\beta = 3/5$ (<1960) to 3/7 (1960–2000) to 1/3 (>2000) would be most impressive, but it appears unlikely: most scientists still prefer short-range (face-to-face) interactions over long-range ones. Note that in resistive dielectric relaxation experiments, where mobile ions become entrained by the d = 1 applied electric field, there is evidence for $\beta = 1/3$ [32].

Lévy flights arise in superdiffusion models of foraging with a length cutoff R [33], which involves an optimized power-law step distribution with exponent $\mu = 2$ [34]. The topological significance of μ is brought out by writing $\mu = 1 + f(d - 1)$. The first term represents minimal ballistic radial motion, and the second term represents diffusion on the foraging cutoff surface R, which is optimized by a random walk ($f = 1/2$). Competition between short- and long-range relaxation also occurs in simulations of Josephson junction arrays, where $\beta = 0.45$ was found over a wide range $4/9 < T/T_c < 2/3$ [35]. The long-range interactions involve release or capture of vortex–anti-vortex pairs by nanodomain walls (length L), as illustrated in their Fig. 3. The arrays represent an excellent realization of the geometry assumed in Ref. [14], but with long L hops included. The checkerboard patterns observed in their Fig. 2 are similar to those reported for rigidity percolation in network glasses [36]. The identification made here of superdiffusive motion with Lévy flights agrees with recent simulations of bi-dispersive grains at the jamming transition [37].

It is striking that modern distributions of country population sizes (except China and India outliers) are fitted over nearly two decades [1] by $\beta = 0.42$; this implies extensive "long hops" (large scale migration). It also implies that the world is not "empty" (like a gas, $\beta = 1$), nor is it even "full" (like a liquid, $\beta \sim 0.8$), it is a compacted "overfull" world that has solidified (like a glass, $\beta = 0.60$ or 0.43) [38]. It is also consistent with the existence of a broad class of compacted "glassy" networks that includes strategic virtual global water trade (stretched exponential distribution of node strengths of 184 nations with $\beta = 0.42$) but not free global food trade (simple exponential, $\beta = 1$) [39].

Subjective axiomatic anthropomorphic individual or social assumptions [40] tend to give simple exponential functions ($\beta = 1$) below the "crossover to fame" n_1, which do not explain fat tail distributions [1], notably the 600 million citations discussed in Ref. [3]. Moreover, these random models yield a value of $n_1 \sim 25$, which is too small by a factor of 8. However, one does find simple exponentials in time delays of responses to email messages [41], an isolated and much simpler task than preparing interlocking networked citation lists for research papers. This difference can even be interpreted as an objective definition of the differences between the much-used (but seldom quantified) objective/subjective dichotomy (in the abstracts of $\sim 10^3$ papers/year over the last 25 years). Simple exponentials correspond to ballistic (random or non-interacting) spreading of excitations, while stretched exponentials correspond to diffusion with strong interactions. In the last 25 years, there have been 140 scientific abstracts discussing "diffusion of knowledge" [42], and none discussing "ballistics of (or random) knowledge". The common assumption of simple exponential distributions associated with random citations appears unjustified [40]. The value of the crossover to fame n_1 which it suggests (about 25 (25), instead of 200 [3]) is too small by a factor of 8.

An historical note: Eq. (3) is well-known for normal liquids [43], while Eqs. (2), (4) and (5) seem to be new for glasses. Microscopic homogeneity dramatically affects many properties of network glasses at the molecular level [44]. A decisive experiment confirming β glass bifurcation into 3/5 and 3/7 in the best industrial-quality glass has appeared [45]. Here microscopic homogeneity is necessary for commercial success.

Uncited references

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