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Physica A 369 (2006) 789–798
PHYSICA A
www.elsevier.com/locate/physa

Phase transition and diffusivity in social hierarchies with attractive sites

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Received 7 October 2005; received in revised form 23 December 2005
Available online 28 February 2006

Abstract

We study the effects of including a distribution of valuable or attractive sites in a two-dimensional lattice in self-organizing social hierarchies. Agents move aleatorily except in the case where an attractive site is located in their neighborhood. We find that the transition between an egalitarian society at low population density and a hierarchical one at high population density strongly depends on the distribution and percolation of strategic sites. Also, it is shown how agent diffusivity is closely related to the amount of inequality. The proposed model introduces an optimization aspect to the problem of social hierarchies since the system tends to maximize the occupation of attractive sites (wealth per capita). However, when the density of attractive sites is small, the system fails to reach this state, and is trapped in a local minimum, as in a glass or jam transition.

PACS: 89.65.−s; 87.23.Ge; 89.75.Fb

Keywords: Sociophysics; Social hierarchies; Phase transitions

1. Introduction

In recent years many efforts have been devoted to the study of complex social systems by applying methods of statistical physics. Modeling such systems as sets of interacting dynamical elements or agents has proven to be a fruitful procedure that captures the essential mechanisms which lead to generic forms of emergent collective behavior. One of the simplest ways to investigate animal and human communities is by introducing simple rules to produce artificial societies [1], as the model of social hierarchies of Bonabeau et al. [2]. This model can describe the transition from egalitarian hunting and gathering societies to more hierarchical agricultural and city-life ones, which occurred thousands of years ago. In the Bonabeau model [2], agents (animals, individuals, communities, countries, etc.) move on a two-dimensional (2D) territory represented by a square lattice. A certain population of agents is initially placed in the lattice aleatorily, in such a way that two

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0378-4371/$ - see front matter
doi:10.1016/j.physa.2006.01.069

agents never occupy the same position. Then agents move randomly and when an agent jumps onto an occupied site, a fight breaks out between both and a memory function stores the outcome of the fight. Initially, the probability of each agent to win is 50%; but later on it is modified according to the memory function and used for the next fight. The basic feature of the model is the introduction of a feedback mechanism of the agent fitness based on its memory that evolves with the following rule: whenever an agent wins, its probability to win increases and when it loses, its probability to win decreases. Other works share the same spirit of the model described before [3–8]. However, the phase transition between an egalitarian and hierarchical regime (or society) observed at a critical density was very weak [2] and it was reinforced [5,6] by introducing a feedback mechanism on the probability of an agent to rise or to fall in the hierarchy. Since these models produced the same number of weak (low fitness) and powerful (high fitness) agents in the hierarchy, a slight modification was made to reproduce the realistic case of more weak agents [8]. Numerical simulations have been reported on the Bonabeau model on a fully connected graph, where a ‘‘forgetting’’ control parameter is crucial and spatial degrees of freedom are absent [9]. Later on, the model was applied to agents that move in scale-free networks [4] exhibiting a sharp transition from an egalitarian to an hierarchical society, with a very low population density threshold which depends on the network size. More recently, it has been shown that the Stauffer version [8] of the Bonabeau model exhibits a fixed point structure [10]. Finally, an analytical model within a mean-field theory is developed with a distribution obeying a nonlinear master equation which also exhibits a phase transition [11]. As another example of self-organizing hierarchical model, Kohring [12] used a theory of social impact extended to include learning in which the final opinions were highly correlated with those of a single agent (leader). In previous models of self-organizing social hierarchies, it is assumed that the sites in the lattice are all equivalent, in the sense that they have the same strategic value, so the agents decide the direction of motion at random. Although this assumption leads to an important understanding of the basic mechanism of social hierarchy creation, in many environments not all sites are equivalent. In other words, in all the previous models, agents fight just for the purpose of fighting. In reality, some sites are more valuable than others due to several factors, as for example the availability of natural resources or a valuable geostrategic site. That is, all sites have not the same value (or they are not axiologically [13] equivalent) since they depend on the particular system. Thus, fights in real systems usually take place in order to dominate high-value territories, and it is unlikely that diffusion takes place at random. To improve upon the Bonabeau model, in a previous article [14] we modified the model to take into account the different values of territory's sites. Then agents move in such a way that they try to reach high-value sites. Such sites act like attractive sites to the agents. It is important to remark that this model introduces an optimization aspect to the problem of social hierarchies, i.e., fights take place to conquer the valuable sites. In the previous work [14], the results showed that in general, inequality is enhanced and the critical agents concentration for a phase transition observed by Stauffer [6] is moved toward lower values of the population density. In the present article, we present new results of the modified Bonabeau model by studying the ability of the society to reach wealth, which can be defined as the number of agents that occupy valuable sites. Furthermore, we show that there is a clear relationship between agent diffusivity, inequality and wealth due to a feedback mechanism. This allows to do analytical estimations of the critical value for the densities where social phase transitions occur. Finally, we find a new phase that is akin to a glassy behavior, where the artificial society can be trapped in states that do not correspond to the global optimum as happens in glass or jam transitions [15]. The layout of this work is the following: In Section 2, for the sake of clarity, we give the details of the model and in Section 3 we present the results, divided in two subsections, depending on the percolation of attractive sites. Finally, the conclusions are given in Section 4.

2. The model

In the model of Bonabeau et al. [2], agent diffusion occurs in a planar 2D square lattice, where agents can only jump to first neighboring sites. The agents diffuse randomly and when agent i wants to move into the site of agent k, a fight breaks out between them. If i wins, i and k exchange sites on the lattice; if k wins none of them move. Therefore these rules imply that the population density (p) is kept constant through the whole evolution. Following Stauffer [6], the probability q for i to win is given by a

Fermi-like distribution

$$q = \frac{1}{1 + \exp(\sigma[h(k) - h(i)])}, \tag{1}$$

where $h(i)$ counts the weighted number of victories minus the weighted number of loses of person i, and the amount of inequality is measured by

$$\sigma = \sqrt{\langle q^2 \rangle - \langle q \rangle^2}, \tag{2}$$

and the average $\langle\,\rangle$ is performed over all the agents. Initially the probability of losing is 50% since all h are equal. This weight h is initially unity, but decreases at 10% at each iteration [6]. To improve over the Bonabeau model, very recently we proposed to keep the fighting rules but change the way in which agents move by assigning a certain value to each site of the lattice [14]. The new rules of agent movements are the following: (1) If all the neighbor's site values are less than the value of the site occupied originally by the agent, then the agent tries to remain in the site, since we assume that it is not worthwhile to take a less valuable site. (2) If condition (1) is not fulfilled, the agent tries to move to the neighboring site with the highest value. However, it can happen that more than one of the neighboring sites have the same high value. In such a case, one site is chosen at random among the most valuable ones. Finally, it can also happen that the original site has the same value as the maximum value of the neighbors, then the agent always moves to one of the randomly chosen most valuable neighbors. We set the last rule in order to recover the limit of the Bonabeau model for a local homogenous neighborhood, since under Bonabeau rules it is always necessary to jump. Thus, if in the present model all sites are rich or poor, our rules are reduced to the Bonabeau case. The next step is to produce a territory in which the agents move. To understand the effects of the new rules, the most simple territory that we can imagine is one with two kinds of sites: non-attractive or poor sites, say with value equal to zero, and attractive or valuable, with value equal to one. The distribution of sites can be made at random, with a probability x of having an attractive site, and $1 - x$ for non-attractive sites. Thus, the lattice is akin to a percolation problem in 2D or to a binary square lattice in the split band limit [16]. It is worthwhile remarking that the percolation transition occurs for $x = 0.59$, i.e., for $0.59 \leqslant x \leqslant 1.0$ there is an infinite spanning cluster of valuable sites in the lattice and the agents are able to cross the lattice inside rich clusters.

3. Results

In this section we discuss the results of the simulations for the proposed model. The simulations were performed on 250×250 sites square lattices, with 10 000 time steps (in each step, all agents are considered for a possible displacement) for each population density. For low concentrations of attractive sites, the lattices were of size 300×300. In all the cases, the size of the symbols in each graph is proportional to the maximal error in the evaluation of a given quantity for a fixed p. This error was evaluated by obtaining the standard deviation during the productive run of the simulation, which in this case was set to 5000 time steps. Due to the percolation properties of the lattice, it is natural to divide the discussion of the results in two subsections: percolation and non-percolation of attractive sites.

3.1. Percolation of attractive sites

Fig. 1(a) shows the results of inequality as a function of the density p of agents when valuable sites percolate in the lattice. The first thing to notice is that for $x = 1.0$, we recover the results of Stauffer [6], where a phase transition is observed around $p = 0.35$, with an inequality of 0.45 for high populations. As the concentration of rich sites is diminished, two important features are observed. The first is a gradual shift of the phase transition towards low-density populations. When $x = 0.60$ the phase transition almost occurs at $p = 0$. In that sense, the introduction of valuable sites tend to enhance inequality for low p. However, at high populations, the inequality decreases as valuable sites are introduced. How is this possible? To answer this

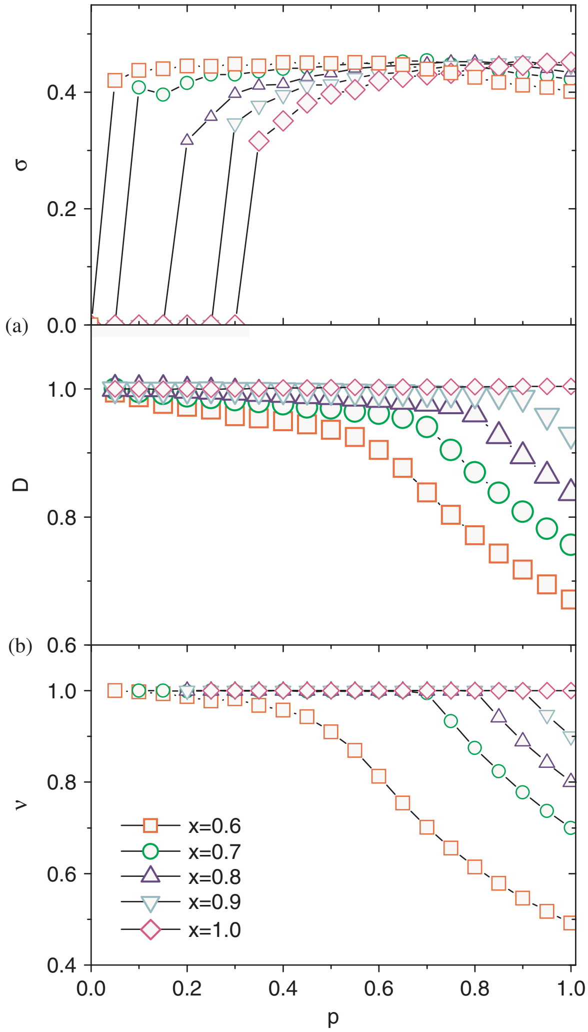

Fig. 1. (a) Inequality σ as a function of the population density p, for different values of the valuable (attractive) site concentration x. The symbol code for each value of x is in the inset of (c). (b) Diffiusion constant as a function of p, for the same values of x as in (a). The symbol code is the same as the inset of (c). (c) Average value of occupied attractive sites (v) per agent for different values of x from $x = 0.6-1.0$. The code for each symbol is given in the inset.

question, in Fig. 1(b) we plot the agents diffusion constant, defined by,

$$D = \frac{1}{N}\sum_{i=1}^{N}\left(\lim_{t\to\infty}\frac{\langle x_i^2\rangle_t}{t}\right),$$

where $\langle x_i^2\rangle_t$ is the average quadratic displacement of agent i at time t and N is the total number of agents. To avoid the introduction of extra parameters, the space between sites is taken as 1 and the time is measured in units of simulation steps. One can think that D is the average of the agents diffusivity, as in liquid theory where there is an average and a tagged diffusivity [17]. Fig. 1(b) shows D as a function of p for different values of the valuable site concentration and in Fig. 2 we show an amplification of Fig. 1(b). Notice that Fig. 2 reveals an important fact, even for the homogeneous model: the phase transition in the inequality is preceded by a transition in the diffusivity. In this case, the transition to inequality occurs around the critical concentration $p_c^\sigma = 0.35$ while diffusivity begins to grow at $p_c^D = 0.25$. We can understand, in an approximate way, this value by obtaining a lower bound using the following heuristic argument. First, there is a relationship between

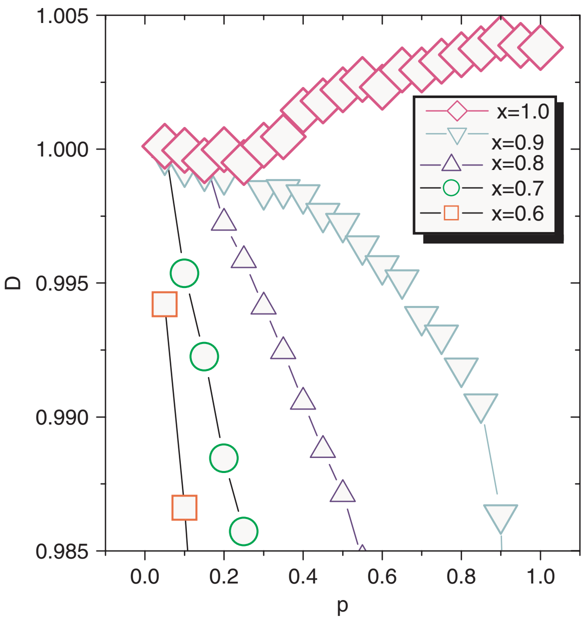

Fig. 2. Amplification of the diffusivity. Notice that for $x = 0.6$, D does not reach 1 since x is very close to the percolation limit and thus agents cannot diffuse inside valuable clusters.

diffusion and inequality, although not in a straightforward manner, since there is a certain degree of feedback between them. On the one hand, when a fight takes place, the displacement of the involved agents depends on the results of the fight, while on the other hand, fight between agents is encouraged by diffusivity. For example, when an agent attacks the neighboring site, the diffusivity can be zero if the fight is lost, or increased if the agent wins the fight and takes over the place. In order to have a fight, an agent needs to have at least one neighbor. As a first approximation, we can suppose that for each unit cell of the lattice, at least one agent is needed in a site for each cell, but each site is shared by 4 contiguous cells. This corresponds to a population density $p_c^\sigma = 1/4$, exactly the place where D changes its behavior. From this argument, $p = 1/4$ is minimum to have fights. However, observe that given this population, still one needs to take into account that each agent can jump into any of the contiguous 4 sites, of which only one is occupied. Thus, $1/16$ extra agents are needed to assure fights at almost all times. This gives the value $p_c^\sigma \approx (1/4) + (1/16) = 5/16 \simeq 0.31$, which is very close to the place where σ jumps. A second observation from Fig. 1, is that p_c can be tuned by the concentration of valuable sites. This happens because an important aspect of the present model is that the agents try to stay or to move towards valuable sites. A rough estimation of this change is to suppose a lattice of rich sites with $(1 - x)$ impurities that decrease diffusion, in this case the critical concentration is given by $p_c^D \approx x/4$. This predicts, decreasing of the critical density, but the agreement with the numerical values is not so good, for example, the transition for $x = 0.9$ and 0.8 seems to occur at the same p. Also, for $x = 0.7$ the prediction is $p_c^D \approx 0.18$ while the numerical value is $p_c^D \approx 0.10$. To improve the estimation, it is necessary to take into account the different configurational probabilities of having impurities and the boundary effects around impurities. Another interesting feature observed in Fig. 1(a), is the increase in σ with increase-x at high populations. The reason is that non valuable sites decrease the diffusivity since agents try to avoid such sites. As can be observed, for a given p, D increases when x augments and fights become less common. In this limit, it is useful to remind the picture of electrons or phonons in a crystal, which are scattered by impurities that decrease the diffusivity [18]. Then, as D decreases, fights are less common and σ diminishes. Notice that the place where D diminishes at high population, corresponds nearly to $p = x$. Again, this effect is a consequence of agent's tendency to reach valuable sites, as we will explain by studying another important variable of the system that we define below: the average value of occupied attractive sites per agent (v). It is also worthwhile mentioning that in Fig. 2, for $x = 0.6$ which is very close to the percolation limit, D never reaches the value one even for low concentrations of agents, when infinite clusters still are rare. Since agents try to reach valuable sites, we can quantify the success of the system to reach a state where all agents occupy attractive sites. Let us consider the quantity of value v_i of agent i, where $v_i = 1$ if the agent is in an attractive site, and 0 otherwise. The average value of occupied attractive sites v per agent is

$$v = \frac{1}{N}\sum_{i=1}^{N} v_i.$$

It is important to remark that a distribution of valuable sites introduces an optimization aspect to the Bonabeau problem, in the sense that agents try to maximize their v_i, although as we will see, this is not always possible. Fig. 1(c) shows the value of v as a function of p for different values of x. A plateau with maximal value $v = 1$ is observed when $p < x$ if $x = 0.7, 0.8, 0.9$ and 1.0. In that case, it is possible to sit all the agents in valuable sites. When the population density exceeds the valuable site density ($p > x$), v has the approximated value $v \simeq x/p$. Notice that $p = x$ is also a critical density for the model, since it divides the behavior between enough resources for all agents to a lack of resources for the population. This critical value is also reflected in Fig. 1(a) and (b), since diffusivity decreases when there are not enough rich sites for the agents, and thus most of the population moves in a lattice with an effective reduced average coordination $\langle Z \rangle$ less than four and given by [16] $\langle Z \rangle = 4x$. For $x = 0.6$, the plateau of occupied attractive sites is never observed below $p = 0.6$. This means that near the percolation threshold, the system fails in the search for the state of optimal occupation of attractive sites. These phenomena are best understood in the limit of low x, so we will discuss later the corresponding analysis. For the moment, we can say that this phenomenon certainly reminds the case of glass or jam transitions, where the system is trapped in a state that is not the global optimum.

3.2. Non-Percolation of attractive sites

In this subsection, we will discuss the result when valuable sites do not percolate, which corresponds to $x = 0-0.59$. From Fig. 3, the most important feature of this limit is the absence of a phase transition for almost all x, i.e., hierarchies are almost always observed, except when $x \to 0$, where the Bonabeau model is recovered. However, the transition to the Bonabeau model is not as smooth as in the case $x \to 1$, as we will analyze later. For the moment, we leave aside the case $x = 0$ and concentrate ourselves on values between $x = 0.1-0.5$. Other feature is that σ depends strongly on the value of p. However, for $p < 0.2$, σ decreases when x diminishes for a given p. This tendency is also reflected in D, and is a consequence of the asymmetry between the probabilities of establishing agents fight around valuable sites. Agents in non valuable sites are always trying to jump into rich sites, while agents in valuable sites just stay in their places. As a consequence, diffusivity is decreased. Fig. 3(c) shows the corresponding distribution of occupied attractive sites. Again, v is reduced in a systematic way when x decreases. As seen in Fig. 3(c), the system never reaches the plateau for $x < 0.4$, and there is a progressive deviation from the ideal state. This strong departure from the state of maximal v is a consequence of the trap in a local minimum of the system, that does not allow the system to attain the global minimum, like in many other systems that break ergodicity [15]. This effect can be easily understood in terms of local traps, and using snapshots of the system. To represent in a graphical manner the state of the system, we use the following code: non-valuable empty sites are represented by 0 and valuable empty sites by 1. Agents in a non-valuable site are represented by 8 and in valuable sites by a 9. Thus, a snapshot of a piece of the lattice can look like

$$\begin{matrix} 0 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & 0 & 8 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 9 & 0 & 0 & 0 & 0 \end{matrix}$$

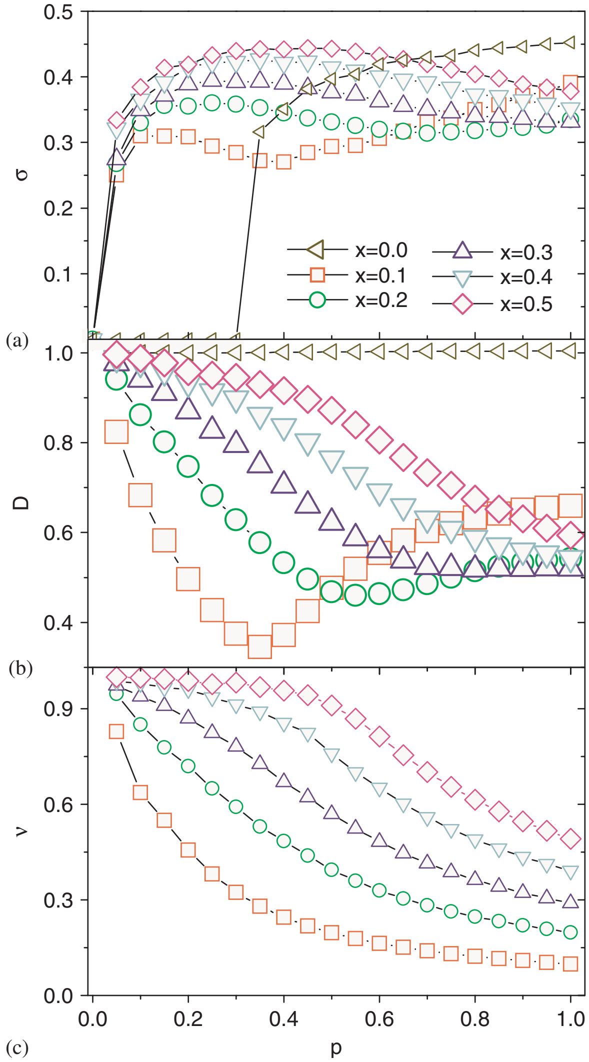

Fig. 3. (a) Inequality σ as a function of the population density p, for different values of the valuable (attractive) site concentration x in the non-percolative regime. The symbol code for each value of x is in the inset of (a). (b) Diffusion constant as a function of p, for the same values of x as in (a). The symbol code is the same as the inset of (a). (c) Average value of occupied attractive sites (v) per agent. Notice that the system never reaches a plateau for $x>p$.

Since in principle there are as many empty valuable sites as "poor agents", the state with maximal v is the following:

$$
\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 9 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 9 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 9 & 0 \\
0 & 9 & 0 & 0 & 0 & 0
\end{matrix}
$$

However, if during their random path poor agents find a non-valuable site that is connected with a valuable site, it will remain there trying to capture it, thus always fighting in the same local environment. It is easy to see that once the following state is reached, the situation is not changed anymore, except for local fights around

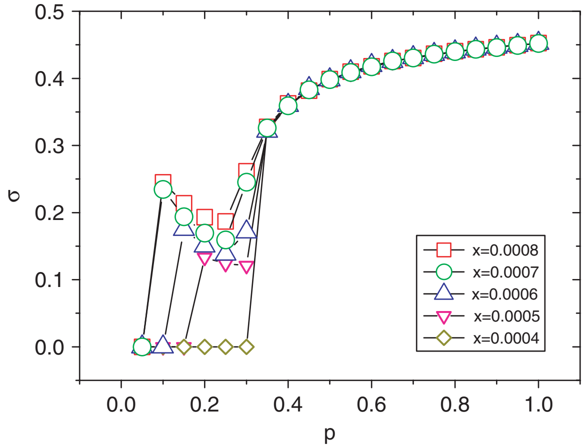

Fig. 4. Evolution of the order parameter σ as a function of p for very low concentration of attractive sites. Notice that the Bonabeau model is recovered for very low concentration of attractive sites.

valuable sites:

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 8 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 9 & 0 & 0 & 0 & 0 \end{matrix}$$

Thus, valuable sites have a core that tends to attract random walkers, and that does not allow to reach a global optimum of v. The effects of local trapping are dramatic for σ and v, explaining why the transition to the Bonabeau model is very different from the case $x \to 1$. Fig. 4 shows the evolution of σ when a very small amount of valuable sites are present in the lattice. Even at those concentrations, σ remains different from zero at small p, although it is clear that a second phase is observed. Such results are due to the fact that around the few valuable sites, agents are fighting all the time between them. Thus, agents in valuable sites tend to fight more often and thus increase their experience, which eventually results in a big inequality. In these cases, the v per agent is nearly zero.

4. Conclusions

An inhomogeneous distribution of sites in the Bonabeau model of social hierarchies can be interpreted as either an attractive interaction between some sites and agents or, more realistically, sites with different values. In the model the diffusion of agents depends on the distribution of attractive sites, but their memory (which counts the weighted number of victories minus the weighted number of losses) does not depend on such distribution. The results show that in general, the inequality is enhanced and the critical concentration for a phase transition observed by Stauffer [6] is moved towards lower values of the agent's population density. The transition to the homogenous case is very sharp in the case of few attractive sites and clearly the introduction of an inhomogenous distribution of valuable or attractive sites has very important effects upon the artificial society. Also, we have shown the relationship between agent diffusivity and social hierarchies, and the number of occupied attractive sites per agent. Our results show that a small amount of valuable sites can trap the system in states where the average value of occupied attractive sites is not well distributed, even if the proportion of valuable sites is bigger than the population. This effect is due to the agents narrow horizon which means that a better or more just distribution of wealth [19] could be achieved if weaker agents have more extended or general information. The model studied here is quite general and could be applied to different systems such as animals, individuals, communities, countries, etc. and there are many ways in which one could find evidence of the phenomenology presented here. The first thing is to consider a suitable order parameter for social inequality. If the agents were animals, the order parameter could be obtained from the distribution of dominant males versus non-dominant ones for a given territory and population density (ρ). For example, it has been observed that for spider monkeys in the Yucatan peninsula, the origin of complex social structures should consider the heterogeneity and complexity of the environment in which social animals live [20–22], although how ecological factors influence animal social structure is far from clear [22]. In the case of monkeys, they have either a complete or a partial knowledge of the location and size of food patches, and maximize the ratio between the size of the next visited patch and the distance traveled to it, ignoring previously visited patches [22]. Similar observations have been made in chimpanzees [23] and dolphins [24]. For human societies, one is very much tempted to apply this model to economy, where the attractive sites would represent rich positions and the average value of occupied attractive sites per agent would be the average income or wealth per capita. As a possible order parameter that resembles the one used here, one can use the economic income inequality index (EIII), defined as the ratio between the income of the 10% richest population and 10% of the poorest population of a given country [26]. However, modern economy is not a zero sum game since to trade is not a fight, and it is clear that the original Bonabeau model is not able to explain the observed social inequality versus population density.Countries with small inequalities, like Japan (EIII = 4.5), Finland (EIII = 5.1), Norway (EIII = 5.3), Sweden (EIII = 5.4), and the Netherlands (EIII = 9), can have very wide variations in their population density ($\rho = 337, 15, 14, 383\,\mathrm{inhabitants/km^2}$, respectively) [26], while countries with high inequalities also display wide variations of the population density, such as Brazil that has almost the same population density as Sweden, but with an EIII of 65.8. Our modification of the original Bonabeau model improves the situation upon the original one, since the behavior obtained is much more subtle, but still is clearly insufficient. In spite of this, it can serve to understand the transition from egalitarian hunting and gathering societies to hierarchical agricultural ones. Although our model is very simple, the unequal distribution of wealth per capita obtained here is an interesting result without appealing to much more complex theories [25,27]. It would be interesting to study the effects of more complicated territories than the one assumed here. On the other hand, it is important to observe that the model presented here has the drawback of having absorbing states, in the sense that the artificial society does not present any further evolution with time after reaching the attractor state. This fact do not seem to be realistic since real societies are always evolving. However, to take into account such further evolution, one needs to consider other elements in the model. For example, the model can be improved by using the realistic fact that site resources diminish as they are used to sustain agent wealth and fight capacity (as happens with food for animals). Thus, attractive sites can decrease in value as they are used (sometimes the site value could be increased due to the construction of infrastructure like a railroad line or an airfield) while non-attractive sites can become rich, as happens when a land is not used during many years. The introduction of these ideas requires a lattice with a dynamical distribution of attractive sites coupled with the agent distribution. In further works, we will consider such variants of the present model. In this first version, we isolated the most basic ingredients which lead to social hierarchies before considering the most complex case. However, the present model is valid to study the society transient time relaxation towards a state of inequality. This approach works fine when there are two well separated time scales: one is the creation and renewal of resources and the other is agent mobility (compare the time required by nature to create oil fields with the very short time scale in which it is consumed by humans). In this article we have shown that, even in the static version of attractive sites, the model is able to produce a rich behavior. We hope that this work may stimulate further research on inhomogeneous self-organizing hierarchical systems.

Acknowledgements

This work was partially supported by National Science Foundation-CONACyT joint project 41538 and DGAPA-UNAM IN108502-3. We would like to thank Denis Boyer for his suggestions and discussions in part of this work.

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