1.-

8.10. Consider an ideal Fermi gas, with energy spectrum $\varepsilon \propto p^s$, contained in a box of "volume" V in a space of n dimensions. Show that, for this system,

(i) $PV = \frac{s}{n} U$;

(ii) $\frac{C_V}{Nk} = \frac{n}{s}\left(\frac{n}{s}+1\right)\frac{f_{(n/s)+1}(z)}{f_{n/s}(z)} - \left(\frac{n}{s}\right)^2 \frac{f_{n/s}(z)}{f_{(n/s)-1}(z)}$;

(iii) $\frac{C_P - C_V}{Nk} = \left(\frac{sC_V}{nNk}\right)^2 \frac{f_{(n/s)-1}(z)}{f_{n/s}(z)}$;

(iv) the equation of an *adiabat* is: $PV^{1+(s/n)} = \text{const.}$,

2.-

8.6. Show that the velocity of sound w in an ideal Fermi gas is given by

$$w^2 = \frac{5kT}{3m}\frac{f_{5/2}(z)}{f_{3/2}(z)} = \frac{5}{9}\langle u^2\rangle,$$

where $\langle u^2\rangle$ is the mean square speed of the particles in the gas. Evaluate w in the limit $z \to \infty$ and compare it with the Fermi velocity u_F.

3.-

7.8. The velocity of sound in a fluid is given by the formula

$$w = \sqrt{(\partial P/\partial \rho)_s},$$

where ρ is the mass density of the fluid. Show that for an ideal Bose gas

$$w^2 = \frac{5kT}{3m}\frac{g_{5/2}(z)}{g_{3/2}(z)} = \frac{5}{9}\langle u^2\rangle,$$

where $\langle u^2\rangle$ is the mean square speed of the particles in the gas.

4.-

4. Consider an idealized sun and earth, both black bodies, in otherwise empty space. The sun is at a temperature of $T_\oplus = 6000$ K and heat transfer by oceans and atmosphere on the earth is so effective as to keep the earth surface uniform. The radius of the earth is $R_\oplus = 6 \times 10^8$ cm, the

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radius of the sun is $R_\odot = 7 \times 10^{10}$ cm, and the earth radius distance is $d_{\oplus-\odot} = 1.5 \times 10^{13}$ cm. *(i)* Find the temperature of the earth. *(ii)* Find the radiation force on the earth. *(iii)* Compare these results with those for an interplanetary "chondrule" in the form of a spherical, perfectly conducting black-body with a radius of $R = 0.1$ cm, moving in a circular orbit around the sun with a radius equal to the earth-sun distance $d_{\oplus-\odot}$.

5.-

8. The universe is pervaded by 3K black body radiation. In a simple view, this radiation arose from the adiabatic expansion of a much hotter photon cloud which was produced during the big bang. *(i)* Why is the recent expansion adiabatic rather than, for example, isothermal? *(ii)* Write down an integral which determines how many photons per cubic centimeter are contained in this cloud of radiation. Estimate the result within an order of magnitude. *(iii)* Show that a freely

6.-

1.-Considere un modelo de sólido bidimensional finito con N átomos donde ω denota las frecuencias de vibración fonónica. Si la densidad de dichos estados vibracionales en el material va como,

$$\rho(\omega) = \begin{cases} A(1-p) + Ap\omega_D\delta(\omega-\omega_0), \text{ si } \omega \leq \omega_D \text{ (donde } \omega_0 < \omega_D) \\ 0, \text{ si } \omega > \omega_D. \end{cases}$$

siendo ω_D una frecuencia de corte constante, A una constante, p un número tal que $0 \leq p \leq 1$, y $\delta(\omega-\omega_0)$ la función delta de Dirac centrada en ω_0, donde $\omega_0 \ll \omega_D$.

a) Encuentre la constante A en términos de p.

b) Encuentre la energía interna (incluya efectos cuánticos) si el sistema está en equilibrio con un baño térmico a temperatura T.

c) Encuentre el calor específico en el límite de altas temperaturas . Compare con la ley de Dulong-Petit y comente sus resultados.

d) Encuentre el calor específico en el límite de bajas temperaturas . Interprete físicamente sus resultados.