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Three-body interactions in sociophysics and their role in coalition forming

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Abstract

An study of the effects of three-body interactions in the process of coalition formation is presented. In particular, we modify a spin glass model of bimodal propensities and also a Potts model in order to include a particular three-body Hamiltonian that reproduces the main features of the required interactions. The model can be used to study conflicts, political struggles, political parties, social networks, wars and organizational structures. As an application, we analyze a simplified model of the Iraq war.

Keywords: Sociophysics; Coalitions; Conflicts; Three-body interaction

1. Introduction

Coalition as a form of aggregation among a set of actors (countries, firms, individuals) has been studied using concepts from the theory of spin glasses [1–5]. A spin glass is a disordered material exhibiting high magnetic frustration due to competing interactions [6,7], and in the model of coalition formation, the type (or sign) of the interactions simulates the respective bilateral propensities of two agents to either cooperation or conflict. If two agents cooperate (have conflict), they tend to be in the same coalition (different coalition). Optimal coalitions can be determined according to a minimum conflict principle. The theory can be applied to many social systems, such as families, internal struggles, political parties in parliments, social networks, and organizational structures.

In the seminal paper of Axelrod and Bennett [1], both the alignment of 17 European nations in the Second World War (WW II), and membership in competing alliances of nine computer companies to set standards for

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Unix computer operating systems were investigated. For the WW II alignment model, the propensities were modelled from past conflict experience, physical borders, religion, etc. [1].

Florian and Galam [3] used the model to describe the fragmentation of former Yugoslavia and showed that the number of optimal coalitions could be more than two.

However, there are two main shortcomings of all these models. The first one comes from the fuzziness in mathematical sociology that arises from trying to quantify some traits in social behavior. Therefore, a better and more detailed quantification of bilateral propensities must be the aim in every particular problem. But the second criticism is a very general one and deals with the lack of the more general "many-body problem interaction" as known to physicists. This could also be called many-body-correlation interaction.

It is easy to motivate its inclusion in the model of formation of alliances since there are many empirical evidence of its importance. Let us just say, in a very colloquial way, that the behavior of two persons (or two animals) can be very different when another person (or animal) is present. The corresponding analogy is a well-known fact in some branches of physics. For example, in contrast to two-body long range coulombic and gravitational interactions, three- or four-body interaction are naturally present in nuclear or high-energy physics [8,9].

Also, in atomic and molecular physics and in polymers, many-body interactions are employed to fit or simulate bending, torsion, and general bonds [10–12]. On the other hand, many-body collisions have to be considered in dense systems [13].

In social sciences there is a previous qualitative effort to deal with behavior rules among friends and enemies employing three-body interactions. The main idea was to change rules such as "the enemy of my enemy is my friend" into "the enemy of my enemy may or not be my friend" [14]. Furthermore, very recently it has been shown the importance of multi-scientist (more than two) collaborations in the social network [15]. This motivates the introduction of generalized networks, where basic connections are not binary, but involve arbitrary number of components, like three-body interaction [15].

In this work, within the spirit of applying spin glass concepts to the formation of alliances, we generalize the theory to include three-body interactions and furthermore we show how it works in an important and recent geopolitical event such as the 2003 war in Iraq. Note that here we present a general failure of the available spin models used in the literature, since three-body effects have not been considered neither in the Ising nor in the Potts like models used by many workers. The structure of the paper is the following, in Section 2 we analyze the two-body models and a simplified model of the Iraq war, in Section 3 we develop the three-body interaction for bimodal coalitions, and in Section 4 we present the case of the Potts model, which allows to treat neutrality in a more natural way. Finally, in the last section, the conclusions are given.

2. Coalition formation and spin glass models

Axelrod and Bennett [1] (AB) first attempted to explain the composition of coalitions by employing the relative pairwise affinity or bilateral propensity p_{ij} between actors i and j to define an "energy" of the system,

$$E(X) = \sum_{i>j} s_i s_j p_{ij} d_{ij}(X), \tag{1}$$

where s_i is a weight positive factor that measures the "power" of the i-actor and $d_{ij}(X)$ is the "distance" from i to j in configuration X which is 0 if i and j both belong to the same coalition and 1 when they are in a different coalition. This model has only two possible coalitions, and thus it is called a bimodal coalition system. By defining $p_{ij} > 0$ when actors i and j tend to be allied and $p_{ij} < 0$ otherwise, then it is postulated that the actual configuration of the system is the one which minimizes the energy. The path followed by the system into the coalition landscape space from an initial configuration, follows the direction of the greatest gradient of energy. Once a minimum is reached the system does not change. The AB model has been applied to the study of both the alliances of the Second World War and UNIX [1]. Galam [2] and Florian and Galam [3], however have criticized the method by which the AB model was constructed, since the ground state is unstable due to a massive degeneration. Galam has shown [3] that in case of bimodal coalitions (A and B), the AB model is totally equivalent to a finite size non-frustrated spin glass at zero temperature. Accordingly, configurations can be expressed by the spin variables η_i, where the spin is $+1$ if the actor i belongs to coalition A, and -1 if the actor belongs to B. By rewriting the distances as $d_{ij}(X) = \frac{1}{2}(1 - \eta_i(X)\eta_j(X))$, the energy becomes

$$E(X) = E_r - \sum_{i>j}^{n} J_{ij}\eta_i(X)\eta_j(X) \tag{2}$$

with

$$E_r \equiv \frac{1}{2}\sum_{i>j} s_i s_j p_{ij}, \quad J_{ij} = \tfrac{1}{2} s_i s_j p_{ij}, \tag{3}$$

which is basically the ground state of a typic spin glass Ising model, given by the following Hamiltonian:

$$H^{(2)} = -\sum_{i>j}^{N} J_{ij}\eta_i\eta_j - \sum_{i}^{N} h_i\eta_i, \tag{4}$$

where the spin η_i at site i can be 1 or -1, with an extra magnetic field (h_i) term. The coalition to which an agent i belongs, is given by the value of the spin. The interaction between agents i and j is J_{ij}. From historical, cultural and economic experience, the interaction J_{ij} favors cooperation if $J_{ij} > 0$, conflict $J_{ij} < 0$, and neutrality $J_{ij} = 0$. According to Galam, the interactions J_{ij} between site i and j have the following form:

$$J_{ij} = (J'_{ij} + \varepsilon_i\varepsilon_j C_{ij}), \tag{5}$$

where ε_i is a natural belonging parameter, i.e., a country has cultural, economic and historic ties to a certain coalition. The epsilons ε_i take values $+1$ for coalition A, -1 for B, and the value $\varepsilon_i = 0$ marks no a priori propensity or preference. The amplitude of the natural belonging is given by the parameter C_{ij}. J'_{ij} is the exchange parameter which is usually set as a constant $-J'$ that sets the energy scale. As a result, all spins are connected and the network contains all possible connections. Finally, the magnetic field term,

$$h_i = \beta_i b_i \tag{6}$$

measures the forces (like military or economic mechanisms) by which each coalition as a whole couples to the orientation of a given actor expressed in terms of an external magnetic field. $\beta_i = \pm 1$ represents the direction of the magnetic force on actor i (towards A or B), while b_i is the amplitude of this force.

In terms of these parameters, several scenarios are possible [4]: local coalitions ($|J'_{ij}| \approx C_{ij}$), global cold war scenario ($|J'_{ij}| \ll C_{ij}$) or unique leader, in which $|J'_{ij}| \ll C_{ij}$ for the powerful leader coalition and $|J'_{ij}| \approx C_{ij}$ for the others actors that interact locally and with the leader.

However, in all of these scenarios the effect of three-body interactions are neglected although they are very important. Let us consider the recent war in Iraq. To illustrate the point, we use a simplified model of four actors: Iraq (Q), Israel (I), Muslim Coalition (M) and the United States (U). Countries of the European Union are not considered here simply because they lead to essentially the same results as this simplified model, so we decided to keep the simplest model with all the substantial features. In this model, the natural belonging are,

Country	ε_i
Q	-1
I	$+1$
M	-1
U	$+1$

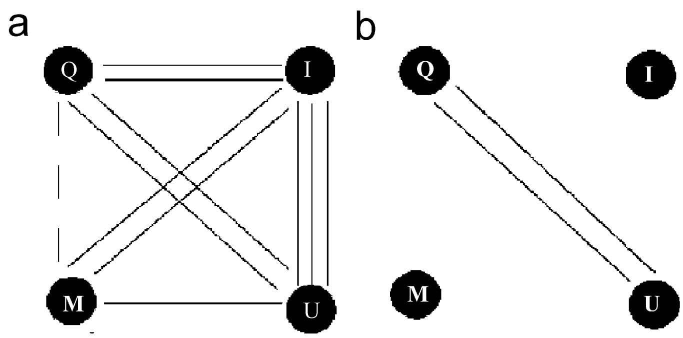

Fig. 1. (a) Simplified network of the Iraq war. Strong friendship (C_F) is represented by three lines, strong enemy (C_E) by two lines, and enemies (friends) with weak interaction by a solid (dashed) line (C); (b) the same network with neutral M and I.

The main feature of this war, is the strong degree of natural belonging of I and U which leads to a big parameter C_{U-I}. In fact, one can make a table with a reasonable choice of interaction parameters C_{ij} as follows:

	Q	I	M	U
Q	0	C_E	C	C_E
I	C_E	0	C_E	C_F
M	C	C_E	0	C
U	C_E	C_F	C	0

, (7)

where each entry of the table is the propensity between two given countries. C_F, C_E and C are three parameters that have a hierarchy: $C_F > C_E > C \gg |J'_{ij}|$ (such interactions are shown in Fig. 1a)). Using these tables, we found the ground state of the Hamiltonian $H^{(2)}$ with many different settings of the parameters, and for all the resulting coalitions, the minimal energy predicts that I enters into the war allied with U, against M and Q. For example, it is easy to show by using an effective spin $\tau_i = \varepsilon_i \eta_i$ that the minimal energy corresponds to a coalition determined by the natural belongings in which U and I are in coalition A, and M and Q in coalition B. The corresponding ground state has an energy $E_0^{(2)} = -(C_F + 3C_E + 2C)$. The only relevant change in the solution, is achieved by assuming that a strong leader, like U, has a huge magnetic field, $b \gg 1$, that enforces M to enter into a coalition with U.

In spite of these calculations, in the real war, Muslims and Israel stayed as neutral countries. Even more impressive, in the 1991 Gulf war, Iraq sent missiles to Israel, which remained neutral. Within the $H^{(2)}$ model, to account for the possibility of neutrality of I and M, we need to compare the minimal energy of the previous network with the case in which M and I are disconnected (see Fig. 1b). In a scenario of strong enemies and friendships, assuming all $C_{ij} \gg J'$, the solution with neutral M and I has energy $E_{0,U-Q}^{(2)} = -C_E$. Since $C_F > C_E$ is clear that $E_0^{(2)} < E_{0,U-Q}^{(2)}$, so the best solution is to keep all countries fighting. Why this result was not observed? The answer is that in principle, one should use a three state Hamiltonian, like a Potts model, since coalition forming is no longer bimodal. However, this eludes the deep question: why the interaction leads to neutrality of some actors when the best solution seems to be a fight?

The main reason is the three-body interaction, and the associated damages due to war. If I goes into the coalition with U, the reaction of M will be very strong against U. Thus, the interaction between U and M depends also on I. In terms of the original idea of "distances between countries", if the distance X between two of them is reduced, the other distance is increased.

3. Three-body interactions

From the previous discussion, is clear that we need to modify the spin models to include the three-body interaction. Since two-body models still capture many of the ingredients of the coalition forming process, the three-body interaction can be introduced as an extra Hamiltonian $H^{(3)}$ to the original one $H^{(2)}$,

$$H = H^{(2)} + \alpha H^{(3)}, \tag{8}$$

where α is a parameter that measures the magnitude of the three-body effects. The most simple form of the perturbation is

$$H^{(3)} = \sum_{i,j,k}^{N} \frac{t_{ijk}}{3} \eta_i \eta_j \eta_k, \tag{9}$$

with a coupling parameter t_{ijk} for each triangle of actors i, j and k that occurs in the lattice. The parameters are given by

$$t_{ijk} \equiv \gamma_{ijk} J_{ij} J_{jk} J_{ki}, \tag{10}$$

where γ_{ijk} is the magnitude of the conflict or damage associated with a three-body interaction. However, this simple Hamiltonian has to be modified to include the main ingredients of the three-body interactions:

(a) When three actors interact between them forming a triangle, a conflict arises if two actors do not have the same natural belongings, given by their corresponding ε_i's. We call this a natural conflict. As a result, the energy must be increased. For example, when the triangle *U*, *M* and *I* is formed, a natural conflict arises due to their different natural belongings. Eq. (9) can be fixed by using a function that is zero when all actors in a triangle have the same natural belongings, and one in any other case. The corresponding Hamiltonian is,

$$H^{(3)} = \sum_{i,j,k}^{N} \frac{t_{ijk}}{3} \left(\frac{3 - |\varepsilon_i + \varepsilon_j + \varepsilon_k|}{2} \right) \eta_i \eta_j \eta_k. \tag{11}$$

(b) If a natural conflict appears in a triangle, the increase in energy depends upon the relative configuration of spins. But note that in Eq. (9), the Hamiltonian is not invariant against the same relative orientation of the spins. For example, the energy of the state $\eta_i = \eta_j = \eta_k = 1$, is not the same as the one obtained from $\eta_i = \eta_j = \eta_k = -1$, although both states have the same relative orientation between them (all parallel). This problem is solved by using the absolute value function

$$H^{(3)} = \sum_{i,j,k}^{N} \frac{|t_{ijk}|}{3} \left(\frac{3 - |\varepsilon_i + \varepsilon_j + \varepsilon_k|}{2} \right) |\eta_i \eta_j \eta_k|. \tag{12}$$

Since $\eta_i = \pm 1$, $|\eta_i \eta_j \eta_k| = 1$, it follows that

$$H^{(3)} = \sum_{i,j,k}^{N} \frac{|t_{ijk}|}{6} (3 - |\varepsilon_i + \varepsilon_j + \varepsilon_k|). \tag{13}$$

(c) However, in a natural conflict, one can imagine three configurations: either all actors are in the same coalition (all spins up or down), two actors are allied against the third one, or an actor prefers to "break" the triangle and stays neutral by leaving the network. We need to assign a penalty in energy for each of these situations. In the case of the real Iraq war, the system is more stable when the triangle is broken, instead of trying to build an artificial coalition or a fight. This is the less costly solution, but the penalty is automatically taken into account by $|t_{ijk}|$, which is zero when the triangle is broken. The next penalty occurs when $\eta_i = \eta_j = \eta_k$; the conflict is solved by an artificial coalition. To assign such penalty with energy W_1, let us first introduce an auxiliary function $f_1(\eta_i, \eta_j, \eta_k)$, with value one when all actors are in the same coalition, and zero in the other case,

$$f_1(\eta_i, \eta_j, \eta_k) = \left(\frac{|\eta_i + \eta_j + \eta_k| - 1}{2} \right). \tag{14}$$

When two of the actors are in the same coalition and the other is an enemy, we use a function $f_2(\eta_i, \eta_j, \eta_k)$ which one if two actors are in the same coalition and zero in the other case,

$$f_2(\eta_i, \eta_j, \eta_k) = 1 - f_1(\eta_i, \eta_j, \eta_k) = \left(\frac{3 - |\eta_i + \eta_j + \eta_k|}{2}\right). \tag{15}$$

An energy penalty W_2 is assigned when $f_2(\eta_i, \eta_j, \eta_k) = 1$. Therefore,

$$H^{(3)} = \sum_{i,j,k}^{N} \frac{|t_{ijk}|}{6}(3 - |\varepsilon_i + \varepsilon_j + \varepsilon_k|) \tag{16}$$

$$\times[W_1 f_1(\eta_i, \eta_j, \eta_k) + W_2 f_2(\eta_i, \eta_j, \eta_k)]. \tag{17}$$

Finally,

$$H^{(3)} = \frac{\delta W}{6} \sum_{i,j,k}^{N} |t_{ijk}|(3 - |\varepsilon_i + \varepsilon_j + \varepsilon_k|)|\eta_i + \eta_j + \eta_k| + E_r^{(3)}, \tag{18}$$

where $\delta W = (W_1 - W_2)/2$, and $E_r^{(3)}$ is a shift of the energy that only depends on the number of triangles with natural conflicts,

$$E_r^{(3)} = \sum_{i,j,k}^{N} \frac{|t_{ijk}|}{6}(3 - |\varepsilon_i + \varepsilon_j + \varepsilon_k|)\left(\frac{3W_2 - W_1}{2}\right). \tag{19}$$

The effect of $H^{(3)}$ is to increase the energy of triangles for which a natural conflict is present. If a coalition is artificially set in, it has a penalty W_1, while if the natural conflict is solved by fighting against the common enemy, the penalty is W_2. In Fig. 2, we present the evolution with α of the states with lower energy for the case of the Iraq war with $|J'_{ij}| = 0$, using all $|t_{ijk}| = C_{ij}C_{jk}C_{ki}$, $C_F = 10, C_E = 3, C = 1$, for all i, j, k, $W_2 = 1$ and $W_1 = 0$. The condition $W_1 = 0$ imply no extra cost if a coalition is artificially set. Since $H^{(3)}$ does not change the energy of states with triangles of actors in the same coalition, the corresponding energy of such states is not affected by α, as observed in Fig. 2. In other words, in the presence of strong three-body effects, the solution

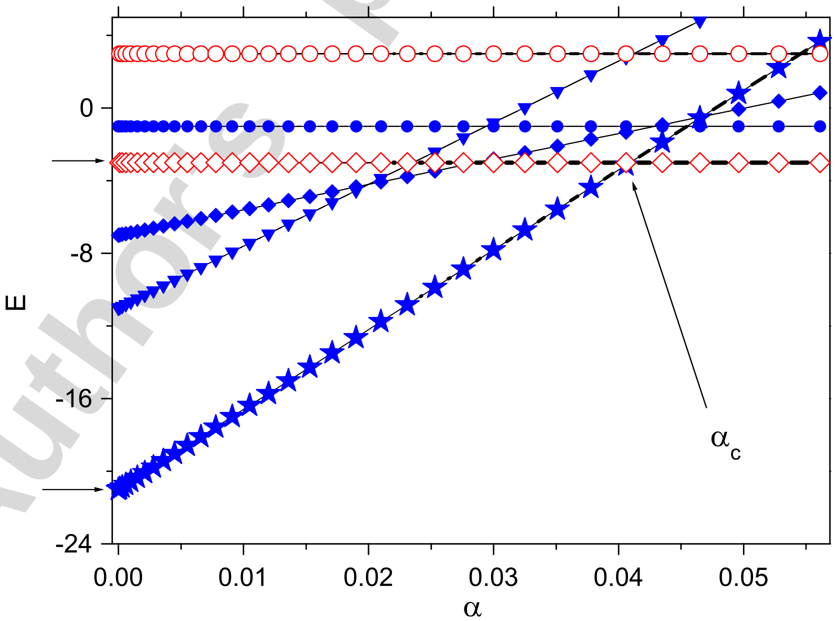

Fig. 2. Evolution of the states with lowest energy as a function of α, for the Hamiltonian H with $C_F = 10, C_E = 3, C = 1$, $|J'_{ij}| \approx 0$, $\delta W = -\frac{1}{2}, b = 0$. The ground state is shown with stars, while the first, second and third excited states are shown with filled triangles, squares and circles, respectively. The lowest energy solution of $H^{(2)}$ with I and M neutral ($E = -3$) is also shown with diamonds, while the first excited state is shown with open circles. The ground states of H and $H^{(2)}$ with I and M neutral, are indicated with arrows; they join each other at α_C.

can be interpreted as "not to fight", which is a first interesting effect, akin to a clustering effect observed in the Potts model [16].

To get further understanding of how the Hamiltonian works, consider the case of small α. The eigenenergies are obtained by taking the expectation value of $H^{(3)}$ in a state $|\Phi\rangle$ of $H^{(2)}$, with energy $E^{(2)}$,

$$E \approx E^{(2)} + \langle \Phi | H^{(3)} | \Phi \rangle \tag{20}$$

and

$$\langle \Phi | H^{(3)} | \Phi \rangle = \frac{\alpha \delta W}{6} \left(2 \sum_{\Delta} |t_{ijk}| + 6 \sum_{\Delta'} |t_{ijk}| \right) + \alpha E_r^{(3)}, \tag{21}$$

where Δ is a sum over all conflicts with two actors in one coalition, and Δ' is a sum over natural conflicts with actors in the same coalition. $\langle \Phi | H^{(3)} | \Phi \rangle$ produces different energy shifts for different states.

To test our modified model, it is necessary to compare the three-body case with the solution considering M and I neutral. When M and I are neutral, there is only a bond with energy $-C_E$ and no triangles appear. The solution with $H^{(3)}$ can have much more energy because there are 4 triangles with natural conflicts. The ground state $E_0 \approx E_0^{(2)} + \langle \Phi_0 | H^{(3)} | \Phi_0 \rangle$ with all countries involved is

$$E_0 \approx E_0^{(2)} + \frac{\alpha \delta W}{3} \left(\sum_{\Delta} |t_{ijk}| \right), \tag{22}$$

The case of M and I neutral is favorable when

$$E_0^{(2)} + \frac{\alpha \delta W}{3} \sum_{\Delta} |t_{ijk}| > -C_E, \tag{23}$$

that leads to a condition for the parameter α,

$$\alpha > \frac{3}{\delta W} \left(\frac{C_F + 2C_E + 2C}{\sum_{\Delta} |t_{ijk}|} \right), \tag{24}$$

and for this particular case,

$$\sum_{\Delta} |t_{ijk}| = \sum_{\Delta} C_{ij} C_{jk} C_{ki} = C_E (C_F + C)(C_E + C). \tag{25}$$

If this result is applied to predict the α in which there is a crossing between the solution with neutral countries and the three-body case, the value $\alpha_C = 18/44 = 0.409\ldots$ is obtained for the set of parameters used in Fig. 2. This value agrees with the crossing of the three-body and M–I neutral solution, which is an horizontal line at $E = -C_E = -3$ in Fig. 2. In fact, we can get further insights on the nature of the solution in the general case, by supposing that $C_F \gg C_E \gg C$ from where

$$\alpha_C \approx \frac{3}{\delta W C_E^2} \left(1 + 2\frac{C}{C_F} + 2\frac{C_E}{C_F} \right) \approx \frac{3}{\delta W C_E^2}. \tag{26}$$

This result means that the three-body interaction basically leads to war when the conflicts are small compared with the biggest friendship, since $C_E \to 0$, $\alpha_C \to \infty$. If $C_E \to C_F$, we have that

$$\alpha_C \to \frac{9}{\delta W C_F^2}. \tag{27}$$

For a big friendship, the limit is $\alpha_C \to 0$. This leads to the neutrality of some actors, because the crossing between solutions is located at a smaller α. In other words, if the conflict grows compared with the biggest friendship, neutrality is preferred by some actors, as observed in the real war. One could cite here the former secretary of US defense D. Rumsfeld in 2003, that the governments of Cuba, Lybia and Germany made clear that they will not send troops to Iraq.

4. Potts model

In this section, we show that the procedure presented previously, can be used for the Potts model, for which it is possible to have more than two coalitions. This allows to treat the case of neutrality without changing the connectivity of the network. In that sense, it is a more realistic model. The cost function associated with the p-state Potts model is given by [16]

$$H^{(2)}_{Potts} = -\sum_{i<j}^{N} J_{ij}(p\delta_{\sigma(i),\sigma(j)} - 1), \tag{28}$$

where the $\sigma(i)$ Potts states can take the $0, 1, 2, \ldots, p-1$ values. The sum is extended over all $N(N-1)/2$ pairs, with $\delta_{m,n} = 1$ if $m = n$ and $\delta_{m,n} = 0$ otherwise. To simplify the presentation, let us consider the case $p = 3$, in which $\sigma(i) = 1$ for i in the coalition A, and $\sigma(i) = 2$ when i belongs to coalition B. The agent or country i is neutral when $\sigma(i) = 0$. This three-state model allows a country to chose between coalitions A, B or neutrality. This model has been studied in the context of two-body interactions. To construct the three-body term $H^{(3)}_{Potts}$, we follow the same path outlined in the previous section. First we identify all the triangles with a natural conflict. This is done by considering the propensities as in Eq. (11),

$$H^{(3)} = \sum_{i,j,k}^{N} \frac{|t_{ijk}|}{3}\left(\frac{3 - |\varepsilon_i + \varepsilon_j + \varepsilon_k|}{2}\right) f(\sigma(i), \sigma(j), \sigma(k)), \tag{29}$$

where $f(\sigma(i), \sigma(j), \sigma(k))$ is a function not yet determined. This function must reflect the costs assigned to solve the natural conflict in different ways as follows:

(a) An artificial coalition can be set in with a cost W^p_1. In this case, all spins are in the same direction, thus, the first contribution to $f(\sigma(i), \sigma(j), \sigma(k))$ has the form $\delta_{\sigma(i),\sigma(j)}\delta_{\sigma(i),\sigma(k)}\delta_{\sigma(j),\sigma(k)}$. This combination is one when all of the countries are in the same coalition and zero otherwise. However, if the three countries are neutral, is clear that there is no conflict, although the spins are the same. To account for this, we multiply the previous function by a factor that is zero when the three countries are neutral, so $f(\sigma(i), \sigma(j), \sigma(k))$ has a contribution given by

$$W^p_1 \delta_{\sigma(i),\sigma(j)}\delta_{\sigma(i),\sigma(k)}\delta_{\sigma(j),\sigma(k)}(\delta_{\sigma(i),1} + \delta_{\sigma(i),2}). \tag{30}$$

(b) Two countries are in the same coalition, and the third one can be in the opposite coalition or neutral. Since neutrality does not have any associated three-body cost, we only need to worry for the case in which the third is in the opposite coalition. A cost W^p_2 is assigned to this state, with a function that is one when two spins have the value 1 or 2 while the other third spin is in the opposite coalition,

$$W^p_2 \begin{bmatrix} (\delta_{\sigma(i),1} + \delta_{\sigma(i),2})(\delta_{\sigma(j),1} + \delta_{\sigma(j),2})(\delta_{\sigma(k),1} + \delta_{\sigma(k),2}) \\ -\delta_{\sigma(i),\sigma(j)}\delta_{\sigma(i),\sigma(k)}\delta_{\sigma(j),\sigma(k)}(\delta_{\sigma(i),1} + \delta_{\sigma(i),2}) \end{bmatrix}. \tag{31}$$

Finally,

$$f(\sigma(i), \sigma(j), \sigma(k)) = \begin{bmatrix} (W^p_1 - W^p_2)\delta_{\sigma(i),\sigma(j)}\delta_{\sigma(i),\sigma(k)}\delta_{\sigma(j),\sigma(k)}(\delta_{\sigma(i),1} + \delta_{\sigma(i),2}) + W^p_2(\delta_{\sigma(i),1} \\ +\delta_{\sigma(i),2})(\delta_{\sigma(j),1} + \delta_{\sigma(j),2})(\delta_{\sigma(k),1} + \delta_{\sigma(k),2}) \end{bmatrix}. \tag{32}$$

The analysis of the resulting Hamiltonian is similar to the one presented for the Ising case. The results are also similar, i.e., in the case of strong conflicts, some actors remain neutral.

5. Conclusions

In conclusion, we have shown that three-body effects are important in a coalition forming system. Then, a modified Hamiltonian was presented to take into account the three-body interactions in a conflict. The Hamiltonians are obtained by adding an extra term to the Ising or Potts models. Such Hamiltonians predict interesting effects, as for example, which are the limits of friendship. It is worthwhile mentioning that here we only considered three-body effects due to triangles of actors. However, is clear that not all three-body effects arise in triangles as a first instance. An actor can modify the interaction of two others just by indirect influence. The best example are gossips, in which a third person can affect a relation between two others. In spite of this, one can always renormalize the interaction in order to get an effective triangle. We are aware that our model is very simple but we hope that this work will stimulate further research of three body effects in social systems. These effects are real and very important, thus, the value of this work is to make people aware that three or more many body effects must be included for a better description of most systems.

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