# Refraction and light transport in turbid colloids: Is the Poynting vector ill defined?


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**ETOPIM 8**

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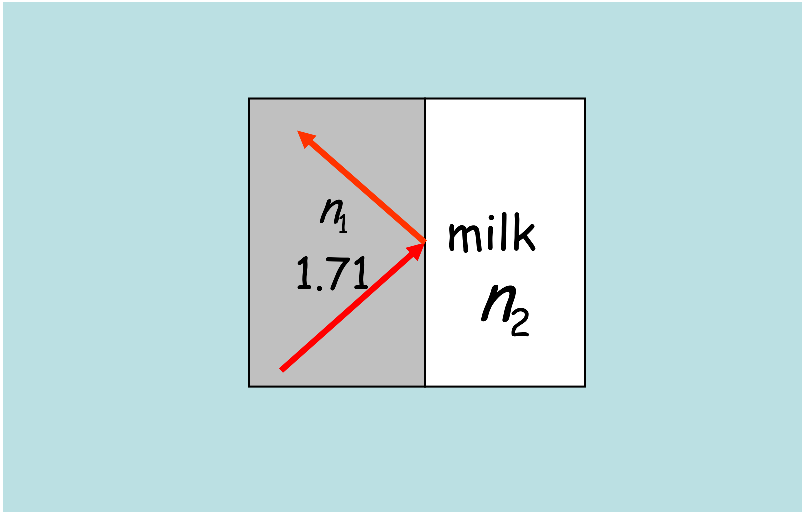
# Motivation 1

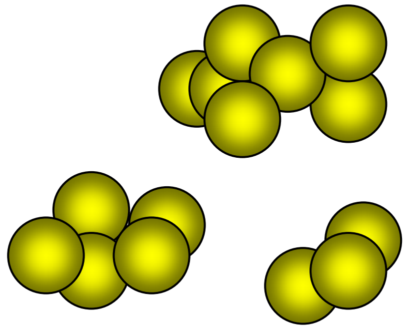
refractive index of milk

critical angle

$$\sin\theta_c = \frac{n_2}{n_1}$$



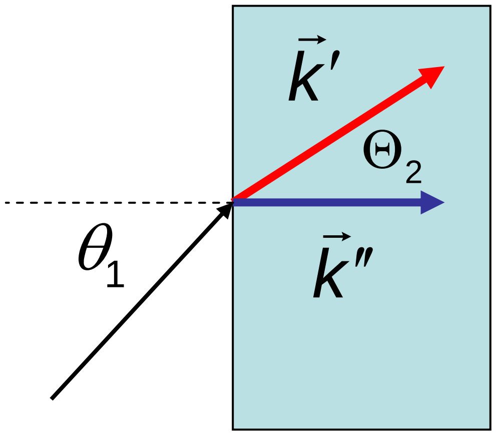

…but is white…and turbid…



# Motivation 2

Absorption

$$\vec{k} = \vec{k}' + i\vec{k}''$$




Inhomogeneous wave

$$\vec{S} = \vec{E} \times \vec{H} \qquad \vec{H} = \frac{\vec{B}}{\mu}$$

$$\vec{S} = \vec{E} \times \frac{\vec{B}}{\mu_0} \quad \rightarrow \quad \underbrace{\vec{E} \cdot \vec{J}^{ind}}_{Q} > 0$$

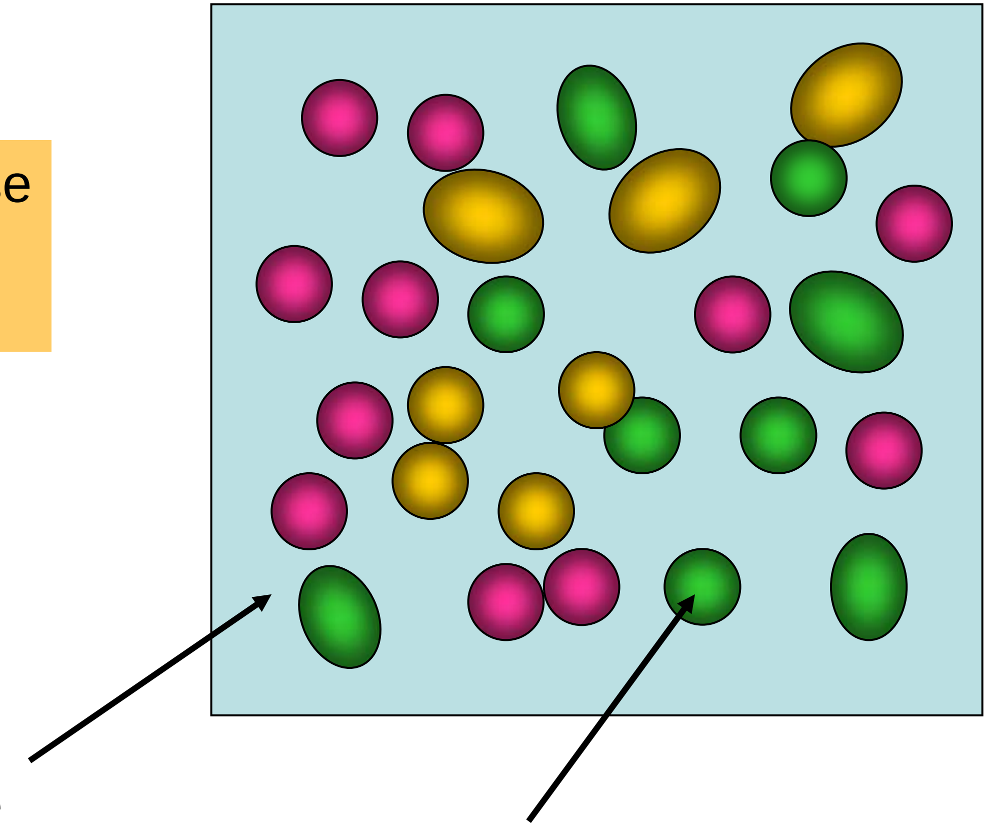
$$\vec{k}' \cdot \vec{k}'' > 0$$

**negative refraction imposible**

V.A. Markel, Optics Express 16 (2008) 19152

# Colloid

Inhomogeneous phase dispersed within a homogeneous one



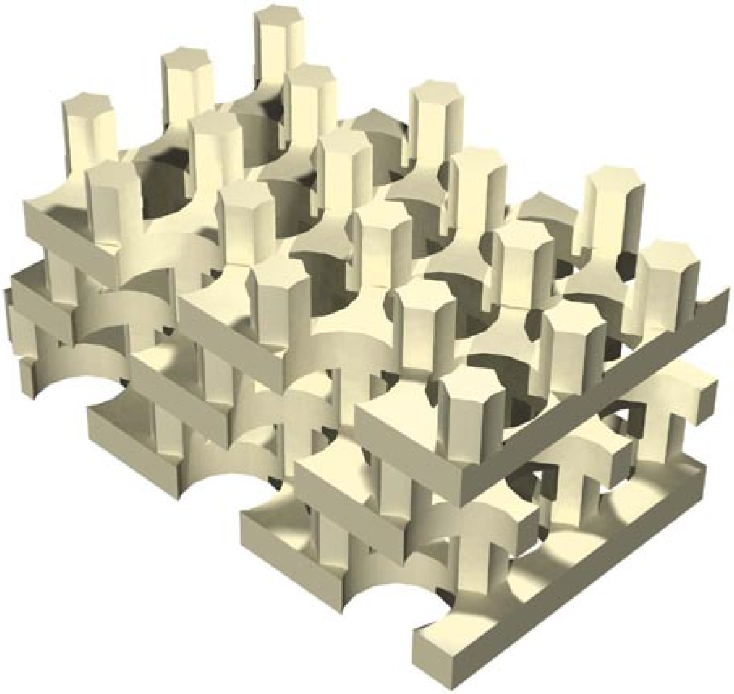
homogeneous phase

colloidal particles

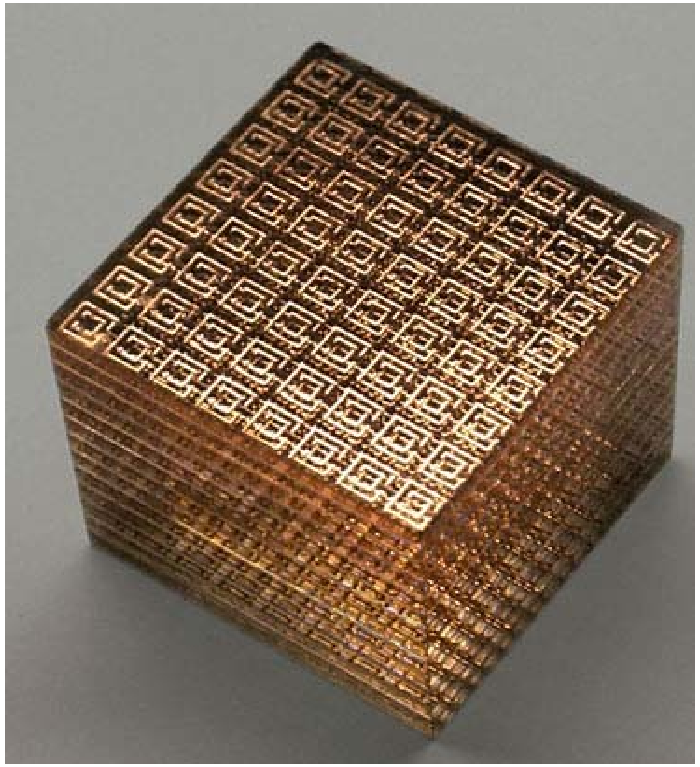
# Colloidal systems

| continuous phase | disperse phase | name | examples |
|---|---|---|---|
| liquid | solid | sol | Milk, paints, blood, tissues |
| liquid | liquid | emulsion | oil in water |
| liquid | gas | foam | foam, whipped cream |
| solid | solid | solid sol | composites, policrystals, rubys |
| solid | liquid | solid emulsion | milky quatz, opals |
| solid | gas | solid foam | porous media |
| gas | solid | solid aereosol | smoke, powder |
| gas | liquid | liquid aereosol | fog |

# “Ordered” colloids

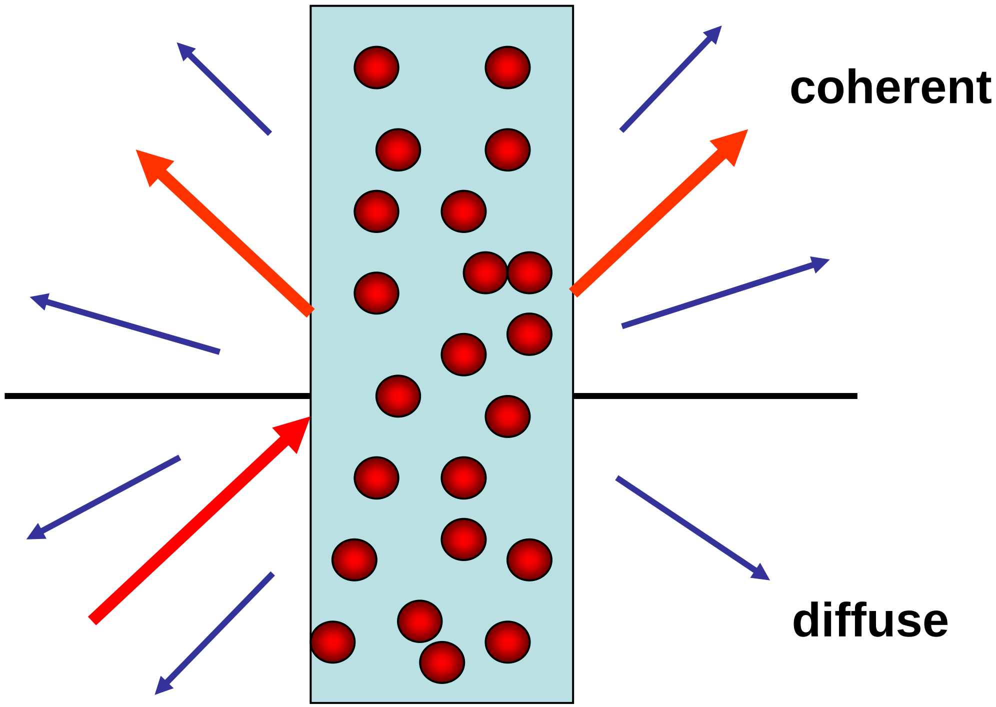


Photonic crystals



Metamaterials

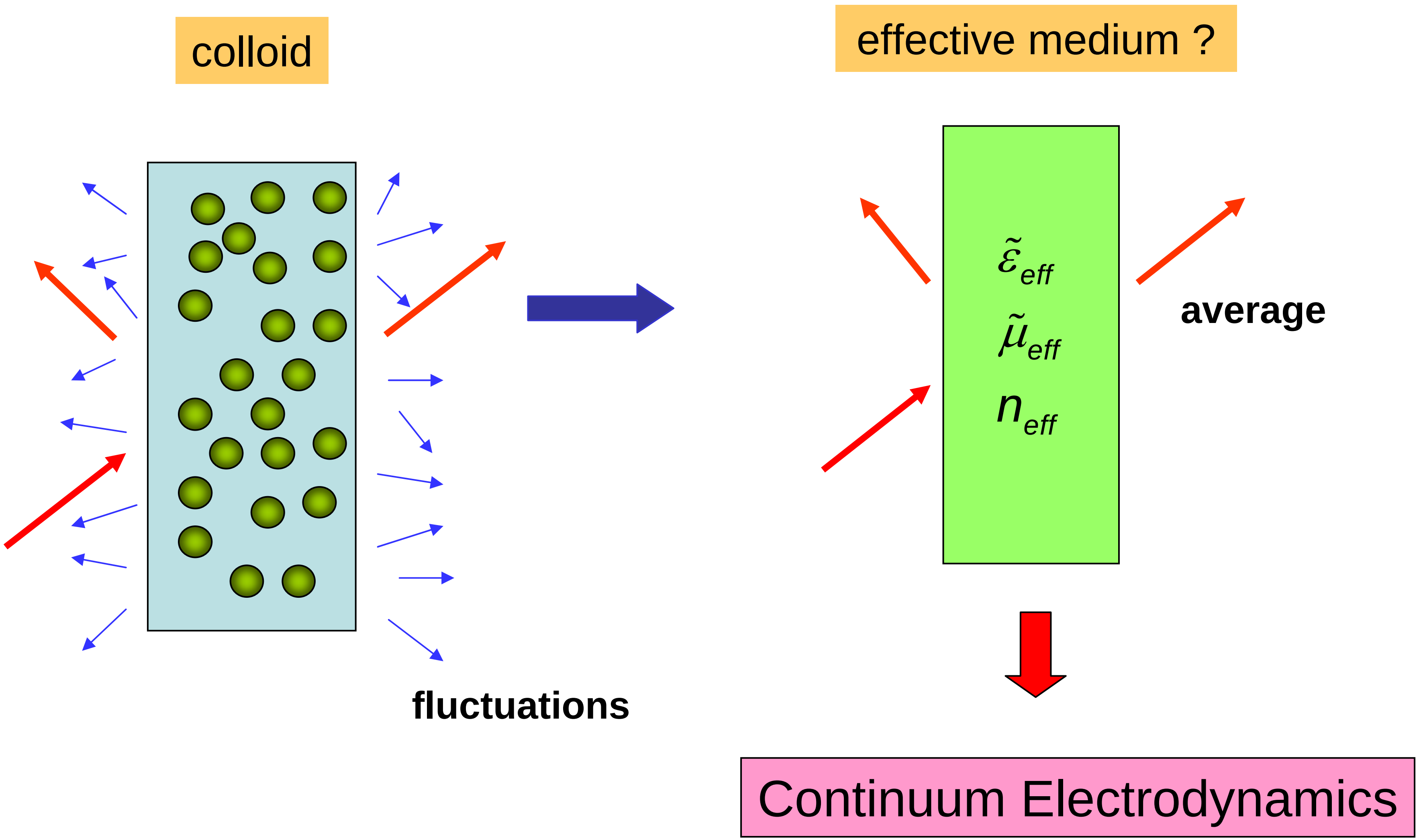
# Light scattering





turbidity

# Effective medium



# Our result

IN TURBID COLLOIDAL SYSTEMSTHE EFFECTIVE MEDIUM **EXISTS**
BUT IT IS **NONLOCAL**

**the probability density Is homogeneous**

$$\left\langle \vec{J}_{ind}(\vec{r};\omega)\right\rangle = \int \overleftrightarrow{\sigma}_{eff}(|\vec{r}-\vec{r}'|;\omega)\left\langle \vec{E}(\vec{r}',\omega)\right\rangle d^3r'$$

total

Spatial dispersion

$$\left\langle \vec{J}\right\rangle^{ind}(\vec{k},\omega) = \overleftrightarrow{\sigma}_{eff}(\vec{k},\omega)\cdot\left\langle \vec{E}\right\rangle(\vec{k},\omega)$$

. **Phys. Rev. 75 (2007) 184202 [1-19].**

# LT scheme

Probability density is homogeneous and isotropic

$$\overset{\leftrightarrow}{\sigma}_{eff}(\vec{k};\omega) = \sigma^{L}_{eff}(k,\omega)\hat{k}\hat{k} + \sigma^{T}_{eff}(k,\omega)(\overset{\leftrightarrow}{1} - \hat{k}\hat{k})$$

generalized effective nonlocal dielectric function

$$\hat{k} \equiv \frac{\vec{k}}{k}$$

$$\overset{\leftrightarrow}{\varepsilon}_{eff}(\vec{k};\omega) = \overset{\leftrightarrow}{1}\varepsilon_0 + \frac{i}{\omega}\overset{\leftrightarrow}{\sigma}_{eff}(\vec{k};\omega)$$

$$\varepsilon^{L}_{eff}(k,\omega) \qquad \varepsilon^{T}_{eff}(k,\omega)$$

$\varepsilon \; \mu$

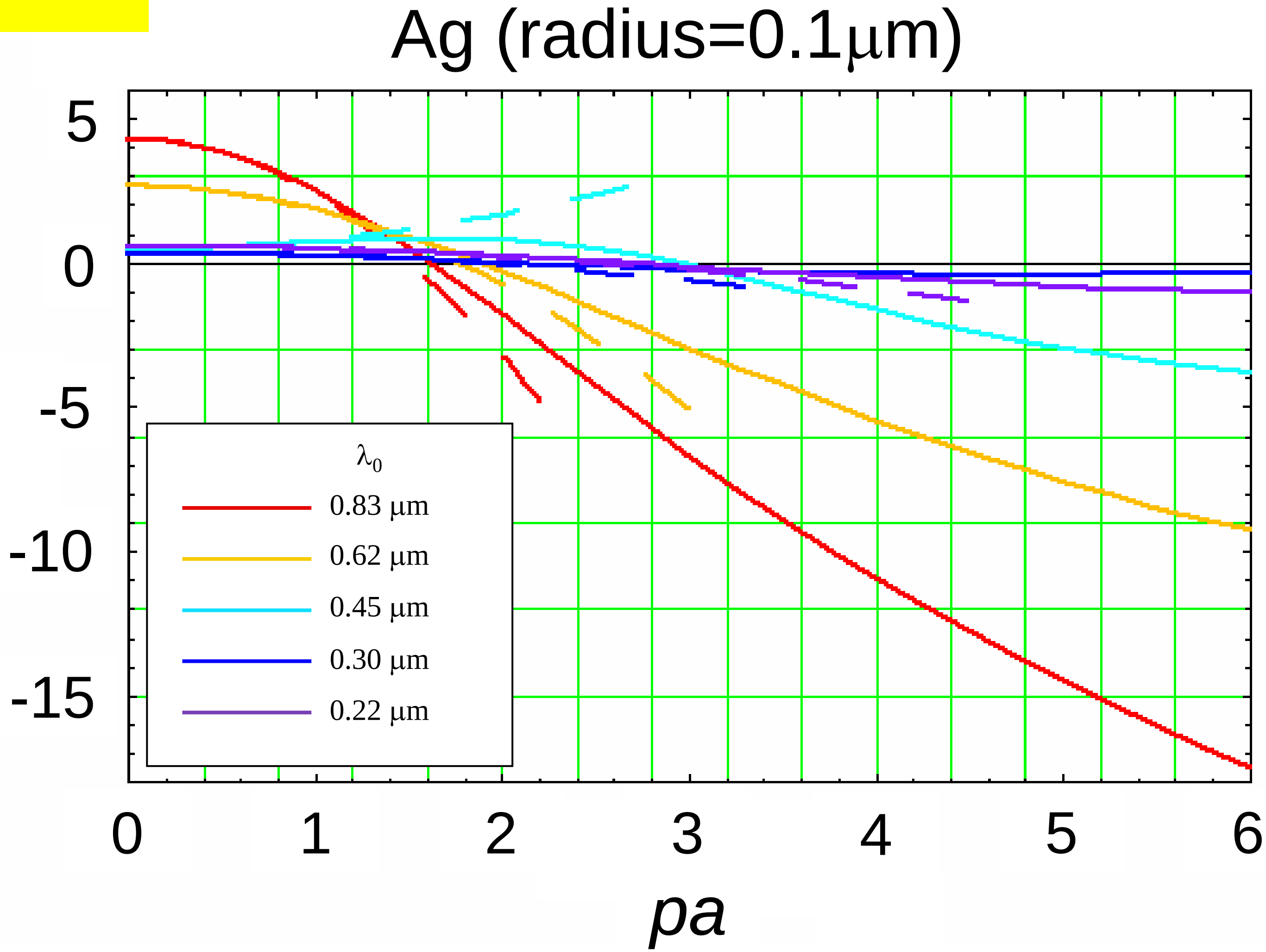
Long wavelength $\left(ka \to 0\right)$

"local limit"

$$\varepsilon^{L}(k,\omega) = \varepsilon^{[0]}(\omega) + \varepsilon^{L[2]}(\omega)\frac{k^2}{k_0^2} + \ldots \qquad \varepsilon^{T}(k,\omega) = \varepsilon^{[0]}(\omega) + \varepsilon^{T[2]}(\omega)\frac{k^2}{k_0^2} + \ldots$$

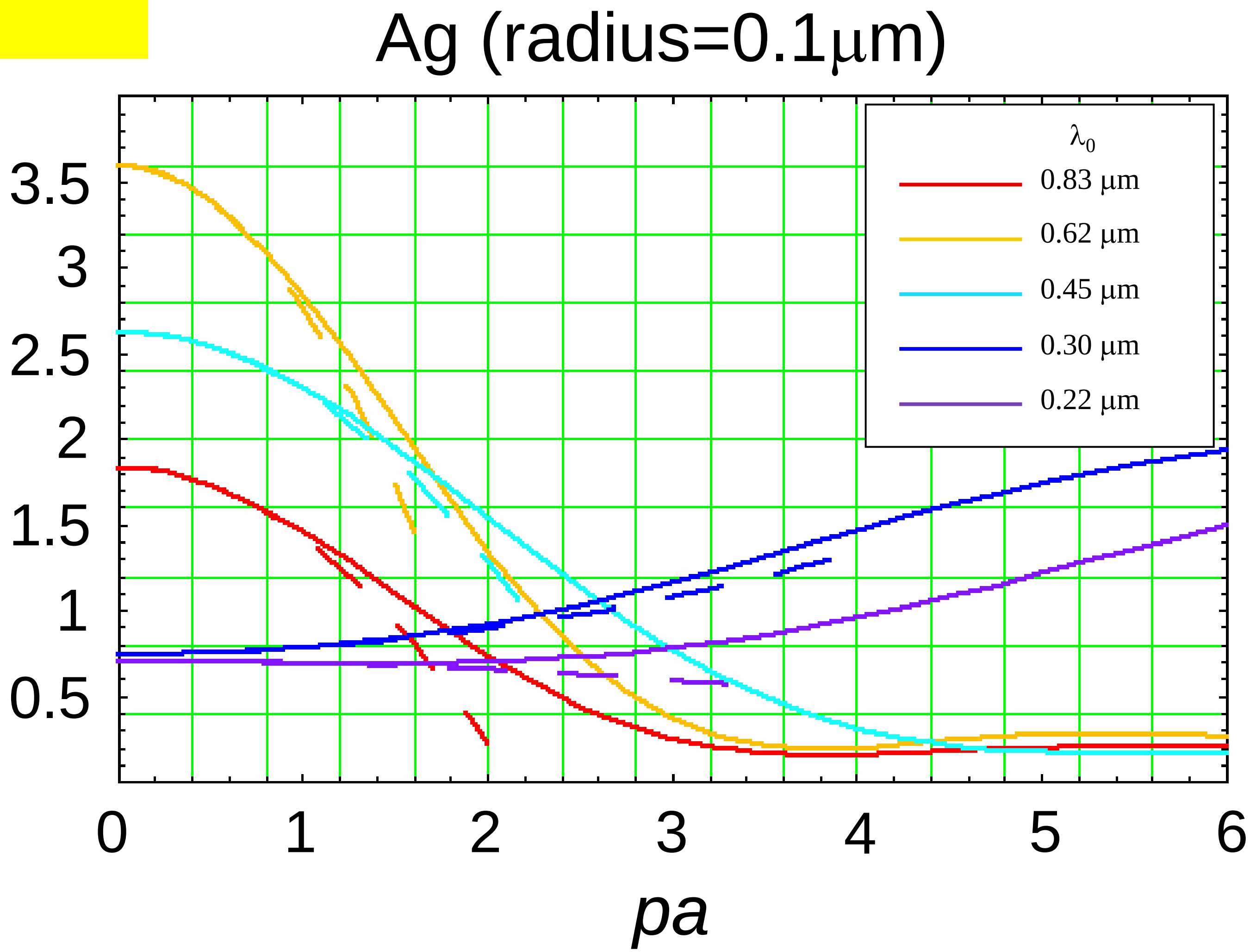
$\omega^2 / c^2$

$$\frac{\mathrm{Re}[\varepsilon^T(p;\omega)]-1}{f}$$




Phys. Rev. 75 (2007) 184202 [1-19].

$$\frac{\mathrm{Im}[\varepsilon^{T}(p;\omega)]-1}{f}$$




$$n_{eff}(\omega)$$

Phys. Rev. 75 (2007) 184202 [1-19].

# Poynting Theorem

Math

$$\nabla\cdot\frac{1}{2}\mathrm{Re}\left(\vec{E}\times\frac{\vec{B}^*}{\mu_0}\right)=\frac{1}{2}\mathrm{Re}\left(\frac{\vec{B}}{\mu_0}\cdot\left(\nabla\times\vec{E}^*\right)-\vec{E}\cdot\left(\nabla\times\frac{\vec{B}^*}{\mu_0}\right)\right)$$

Maxwell

$$=\frac{1}{2}\mathrm{Re}\left(-\frac{\vec{B}}{\mu_0}\cdot\frac{\partial\vec{B}^*}{\partial t}-\vec{E}\cdot\left(\vec{J}^{ext*}+\vec{J}^{ind*}\right)-\varepsilon_0\vec{E}\cdot\frac{\partial\vec{E}^*}{\partial t}\right)$$

$$\nabla\cdot\frac{1}{2}\mathrm{Re}\left(\vec{E}\times\frac{\vec{B}^*}{\mu_0}\right)+\mathrm{Re}\frac{\partial}{\partial t}\left(\frac{\varepsilon_0}{2}|E|^2+\frac{|B|^2}{2\mu_0}\right)=\underbrace{-\frac{1}{2}\mathrm{Re}\left(\vec{J}^{ext}\cdot\vec{E}^*\right)}_{W_{ext}}\underbrace{-\frac{1}{2}\mathrm{Re}\left(\vec{J}^{ind}\cdot\vec{E}^*\right)}_{Q}$$

**Plane wave** $W_{ext}\rightarrow Q$

**WORK**

$$\vec{J}^{ind} = i\omega\varepsilon_0\left(n^2-1\right)\vec{E}$$

**free-propagating mode**

$$W_{ind} = \frac{1}{2}\mathrm{Re}\left(\vec{J}^{ind}\cdot\vec{E}^*\right) = \frac{1}{2}\mathrm{Re}\left[i\omega\varepsilon_0\left(n^2-1\right)\vec{E}\cdot\vec{E}^*\right]$$

$$= \frac{\omega}{2}\varepsilon_0 \underbrace{\mathrm{Im}\, n^2}_{Q} \left|E\right|^2$$

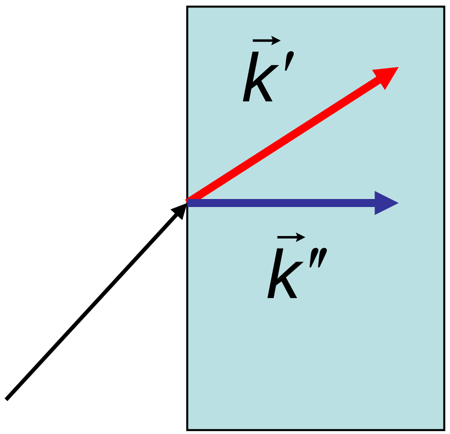
$$\mathrm{Im}\, n^2 > 0$$

dispersion relation

$$k^2 = \vec{k}\cdot\vec{k} = \left(\vec{k}' + i\vec{k}''\right)^2 = k_0^2 n^2$$

$$k'^2 - k''^2 = k_0^2\,\mathrm{Re}\,n^2$$

$$2\vec{k}'\cdot\vec{k}'' = k_0^2\,\mathrm{Im}\,n^2 \quad > 0$$



Inhomogeneous wave

Negative refraction is impossible

*Science* **316**, 430 (2007)

# Negative Refraction at Visible Frequencies

Henri J. Lezec,1,2*† Jennifer A. Dionne,1* Harry A. Atwater1

Nanofabricated photonic materials offer opportunities for crafting the propagation and dispersion of light in matter. We demonstrate an experimental realization of a two-dimensional negative-index material in the blue-green region of the visible spectrum, substantiated by direct geometric visualization of negative refraction. Negative indices were achieved with the use of an ultrathin Au-Si3N4-Ag waveguide sustaining a surface plasmon polariton mode with antiparallel group and phase velocities. All-angle negative refraction was observed at the interface between this bimetal waveguide and a conventional Ag-Si3N4-Ag slot waveguide. The results may enable the development of practical negative index optical designs in the visible regime.

# Nonlocal calculation

$$\frac{1}{2}\mathrm{Re}\left(\vec{J}^{ind}\cdot\vec{E}^*\right)$$

$$\nabla\cdot\frac{1}{2}\mathrm{Re}\left(\langle\vec{E}\rangle\times\frac{\langle\vec{B}\rangle^*}{\mu_0}\right)+\frac{\partial}{\partial t}\left(\frac{\varepsilon_0}{2}\left|\langle E\rangle\right|^2+\frac{1}{2\mu_0}\left|\langle B\rangle\right|^2\right)=-W_{ext}-W_{ind}$$

$$\vec{J}^{ind}(\vec{r};t)=\int dt'\int d^3r'\vec{\vec{\sigma}}(\vec{r}-\vec{r}';t-t')\cdot\vec{E}(\vec{r}';t')$$

**quasi-monochromatic wave**

$$\vec{E}(\vec{r},t)=\mathrm{Re}\,\vec{E}_0(\vec{r},t)\exp[i\vec{k}\cdot\vec{r}-\omega t]$$

$$\left|\nabla E_{0\alpha}\right|\ll kE_0 \qquad \left|\frac{\partial E_{0\alpha}}{\partial t}\right|\ll\omega E_0$$

$$\vec{E}_0(\vec{r}',t')=\vec{E}_0(\vec{r},t)+\left[\left(\vec{r}'-\vec{r}\right)\cdot\nabla\right]\vec{E}_0+\left(t'-t\right)\frac{\partial\vec{E}_0}{\partial t}+\ldots$$

# Induced current

$$J_\alpha^{ind}(\vec{r},t) = \exp[i(\vec{k}\cdot\vec{r}-\omega t)]\Big\{-i\omega\Big(\varepsilon_{\alpha\beta}(\vec{k},\omega)-\varepsilon_0\delta_{\alpha\beta}\Big)E_{0\beta}(r,t)$$

$$-\omega\frac{\partial\varepsilon_{\alpha\beta}(\vec{k},\omega)}{\partial k\gamma}\frac{\partial E_{0\beta}(\vec{r},t)}{\partial x\gamma}+\left[(\varepsilon_{\alpha\beta}-\varepsilon_0\delta_{\alpha\beta})+\omega\frac{\partial\varepsilon_{\alpha\beta}(\vec{k},\omega)}{\partial\omega}\right]\frac{\partial E_{0\beta}(\vec{r},t)}{\partial t}\Big\}$$

$$\overleftrightarrow{\varepsilon}_{eff}(\vec{k};\omega) = \overleftrightarrow{1}\varepsilon_0 + \frac{i}{\omega}\overleftrightarrow{\sigma}_{eff}(\vec{k};\omega)$$

$$\frac{1}{2}\mathrm{Re}\left(\vec{J}^{ind}\cdot\vec{E}^*\right)$$

long wavelength $ka \to 0$

"local"

$$\varepsilon^L(k,\omega) = \varepsilon^{[0]}(\omega) + \varepsilon^{L[2]}(\omega)\frac{k^2}{k_0^2}+\ldots \qquad \varepsilon^T(k,\omega) = \varepsilon^{[0]}(\omega) + \varepsilon^{T[2]}(\omega)\frac{k^2}{k_0^2}+\ldots$$

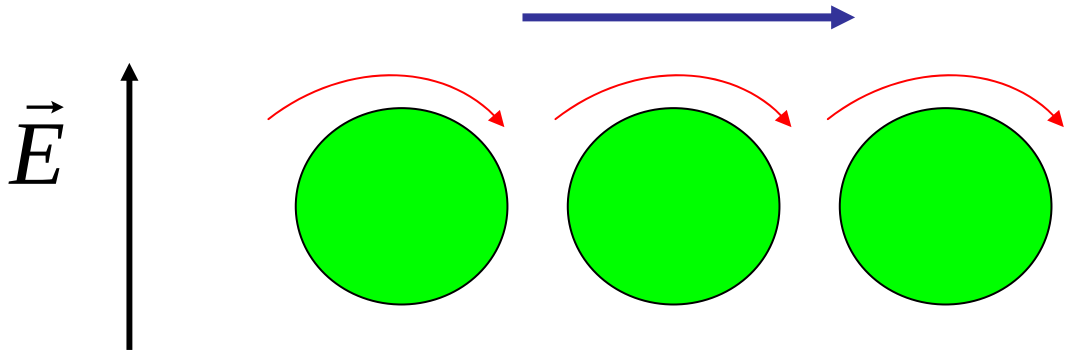
$$\mathrm{Im}\,\varepsilon^L \ll \mathrm{Re}\,\varepsilon^L \qquad \mathrm{Im}\,\varepsilon^T \ll \mathrm{Re}\,\varepsilon^T$$

# Work on the induced currents

$$W_{ind} = \frac{\omega}{2}\left[ \mathrm{Im}\,\varepsilon^{L}(k,\omega)\left|E_0^L\right|^2 + \mathrm{Im}\,\varepsilon^{T}(k,\omega)\left|E_0^T\right|^2 \right]$$

$$-\nabla\cdot\frac{1}{2}\mathrm{Re}\left[\frac{1}{\varepsilon_0\mu_0}\left(\vec{E}_0\times\vec{B}_0^*\right)\varepsilon^{T[2]*}(\omega) + \frac{\varepsilon^{L[2]}(\omega)}{\omega\varepsilon_0\mu_0}\left(\vec{k}\left|E_0^L\right|^2 + kE_0^{L*}\vec{E}_0^T\right)\right]$$

$$+\frac{1}{2}\frac{\partial}{\partial t}\mathrm{Re}\left[\frac{\partial\omega\varepsilon^{[0]}(\omega)}{\partial\omega}\left|E_0\right|^2 + \frac{k^2}{\varepsilon_0\mu_0}\left(\frac{\partial}{\partial\omega}\frac{\varepsilon^{L[2]}(\omega)}{\omega}\left|E_0^L\right|^2 + \frac{\partial}{\partial\omega}\frac{\varepsilon^{T[2]}(\omega)}{\omega}\left|E_0^T\right|^2\right) - \varepsilon_0\left|E_0\right|^2\right]$$




$$\nabla\cdot\frac{1}{2}\mathrm{Re}\left(\left\langle\vec{E}\right\rangle\times\frac{\left\langle\vec{B}\right\rangle^*}{\mu_0}\right) + \frac{\partial}{\partial t}\left(\frac{\varepsilon_0}{2}\left|\left\langle E\right\rangle\right|^2 + \frac{1}{2\mu_0}\left|\left\langle B\right\rangle\right|^2\right) = -W_{ext} - W_{ind}$$

# Energy theorem

Transverse waves

$$\nabla\cdot\frac{1}{2}\mathrm{Re}\left[\frac{1}{\mu_0}\left(\vec{E}_0\times\vec{B}_0^*\right)\left(1-\varepsilon^{T[2]*}(\omega)/\varepsilon_0\right)\right]+\frac{1}{2}\frac{\partial}{\partial t}\mathrm{Re}\left[\frac{1}{\mu_0}\left|B_0\right|^2+\frac{\partial\omega\varepsilon^{[0]}(\omega)}{\partial\omega}\left|E_0\right|^2\right.$$

$$\left.+\frac{k^2}{\varepsilon_0\mu_0}\frac{\partial}{\partial\omega}\frac{\varepsilon^{T[2]}(\omega)}{\omega}\left|E_0\right|^2\right]=-W_{ext}-\frac{\omega}{2}\mathrm{Im}\left[\varepsilon^{[0]}(\omega)+\frac{k^2\varepsilon^{T[2]}(\omega)}{\omega^2\varepsilon_0\mu_0}\right]\left|E_0\right|^2$$

$$\vec{S}_{trans}=\frac{1}{2}\mathrm{Re}\left[\frac{1}{\mu_0}\left(\vec{E}_0\times\vec{B}_0^*\right)\left(1-\varepsilon^{T[2]*}(\omega)/\varepsilon_0\right)\right]$$

## $\epsilon\ \mu$ scheme

$$\vec{J}^{ind}(\vec{r},t) = \frac{\partial \vec{P}(\vec{r},t)}{\partial t} + \nabla \times \vec{M}(\vec{r},t)$$

NOT UNIQUE

quasi-monochromatic

$$\vec{J}^{ind}(\vec{r},t) = \vec{J}_0^{ind}(\vec{r},t)\exp\left[i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right]$$

$$\vec{P}(\vec{r},t) = \vec{P}_0(\vec{r},t)\exp\left[i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right]$$

$$\vec{M}(\vec{r},t) = \vec{M}_0(\vec{r},t)\exp\left[i\left(\vec{k}\cdot\vec{r}-\omega t\right)\right]$$

# Response

$$\vec{J}^{ind}(\vec{r},t) = \frac{\partial \vec{P}(\vec{r},t)}{\partial t} + \nabla \times \vec{M}(\vec{r},t)$$

$$\vec{P}_0(\vec{r},t) = \left[ \left( \varepsilon^l(k,\omega) - \varepsilon_0 \right) \overleftrightarrow{P}^L + \left( \varepsilon^t(k,\omega) - \varepsilon_0 \right) \overleftrightarrow{P}^T \right] \cdot \vec{E}_0(\vec{r},t)$$

$$\vec{M}_0(\vec{r},t) = \left[ \frac{1}{\mu_0} - \frac{1}{\mu(k,\omega)} \right] \vec{B}_0(\vec{r},t)$$

CHOICE I

$$\varepsilon^t(k,\omega) = \varepsilon^t(0,\omega)$$

$$\mu_I(k,\omega)$$

CHOICE II

$$\varepsilon^t(k,\omega) = \varepsilon^l(k,\omega)$$

$$\mu_{II}(k,\omega)$$

# Poynting vector in $\epsilon\mu$

long wavelength limit

$$\left[\frac{1}{\mu_0}-\frac{1}{\mu(0,\omega)}\right]\vec{B}_0=\left[\frac{1}{\mu_0}-\frac{1}{\mu(0,\omega)}\right]\frac{1}{\omega}\vec{k}\times\vec{E}_0$$

$$\vec{S}_{trans}=\frac{1}{2}\mathrm{Re}\left[\vec{E}_0\times\underbrace{\left(\frac{\vec{B}_0^*}{\mu_0}-\vec{M}_0^*\right)}_{\vec{H}_0}-\vec{E}_0\times\frac{\vec{B}_0^*}{\mu_0}\frac{\varepsilon^{t\,[2]*}(\omega)}{\varepsilon_0}\right]$$

CHOICE I

$$\varepsilon^{t\,[2]*}(\omega)=0$$

$$\mu_I(0,\omega)$$

CHOICE II

$$\varepsilon^{t\,[2]*}(\omega)=\varepsilon^{l\,[2]*}(\omega)=\varepsilon^{L[2]}(\omega)$$

$$\mu_{II}(0,\omega)$$

# Poynting vector in LT

The correct expression of the Poynting vector

$$\vec{S}_{trans} = \frac{1}{2}\mathrm{Re}\left[\frac{1}{\mu_0}\left(\vec{E}_0 \times \vec{B}_0^*\right)\left(1-\varepsilon^{T[2]*}(\omega)/\varepsilon_0\right)\right]$$

$\varepsilon^{T[2]*}(\omega)$ is NOT a non-local correction

This is the LOCAL limit

# Conclusions

(i) We used the formalism developed for the treatment of the non-local effective medium associated to turbid colloids to find a general expression for the energy theorem, in the long wavelength limit, in terms of parameters associated to the non-local response of the system.

(ii) We found an explicit expression for the Poynting vector in terms of these non-local parameters and show that there are many ways of writing it in terms of the electric permittivity $\epsilon$ and the magnetic permeability $\mu$, depending on the choice taken to define them.

(iii) This approach can be extended to finite wave vectors and non negligible dissipation as well as energy transport for free-propagating modes