Optical properties of colloidal systems

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Tonanzintla, 2006

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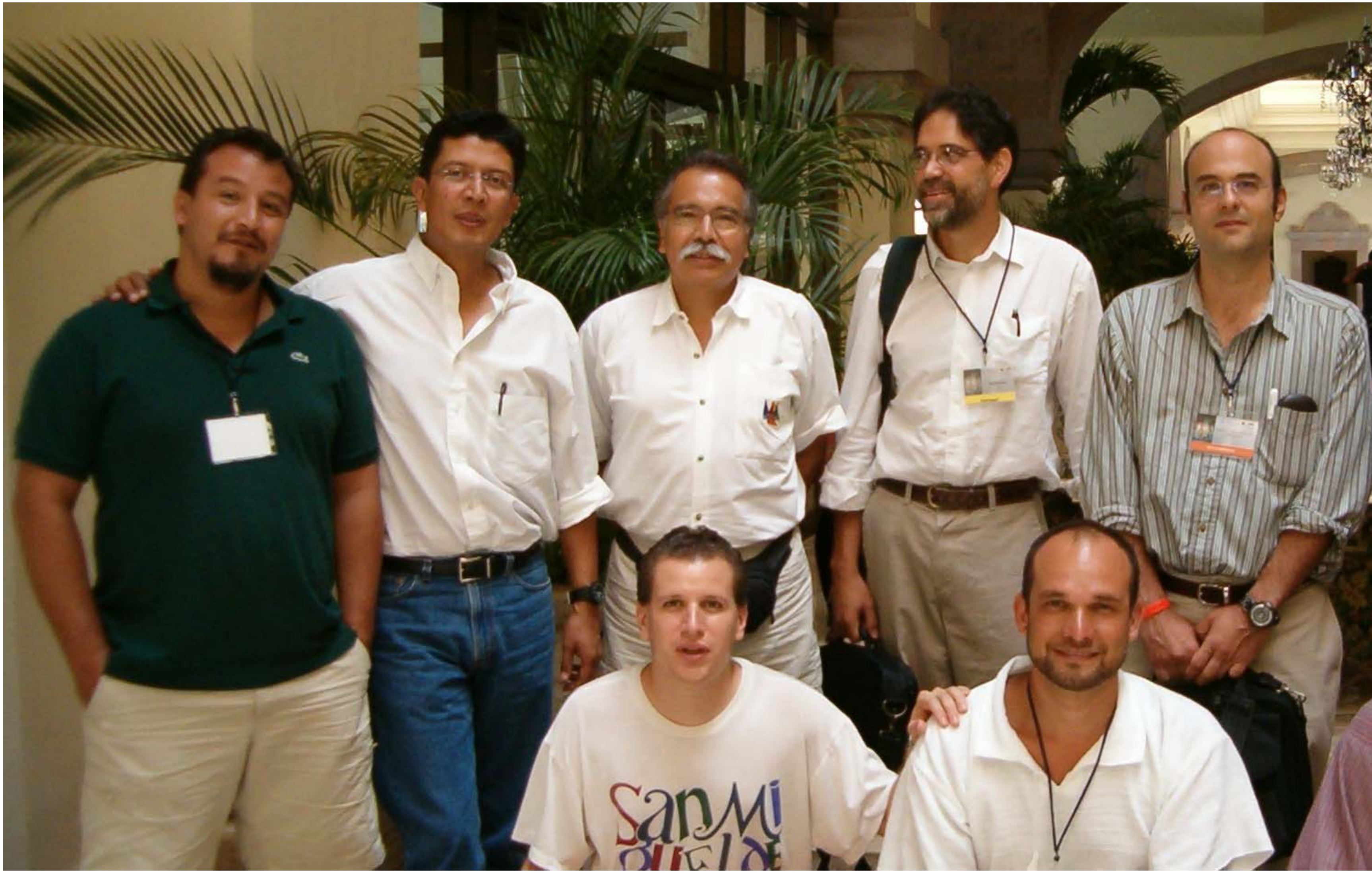


JA Olivares F Curiel RG Barrera E Méndez A García

A Reyes G Ortiz

Electrodynamics in material media

SI units

Conventional approach

material fields

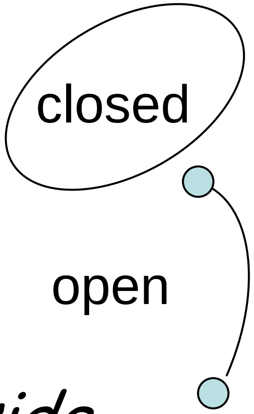
$$\vec{J} = \vec{J}^{ext} + \left\langle \vec{J}^{ind} \right\rangle$$

material

$$\vec{J}_M = \nabla \times \vec{M}$$

$$\vec{J}_P = -i\omega\vec{P}$$

$$\left\langle \vec{J}^{ind} \right\rangle = \vec{J}_M + \vec{J}_P$$



$$\vec{M} = \vec{P} = 0 \quad \textit{outside}$$

$$\vec{D} = \left\langle \vec{E} \right\rangle + \vec{P}$$

$$\vec{H} = \frac{1}{\mu_0}\left\langle \vec{B} \right\rangle - \vec{M}$$

$$\nabla \cdot \vec{D} = \rho^{ext}$$

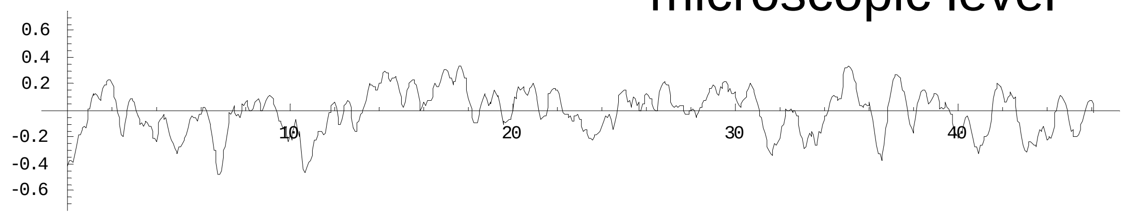
$$\nabla \times \vec{H} = \vec{J}^{ext} + \frac{\partial \vec{D}}{\partial t}$$

Average (macroscopic level)

$$\vec{E} = \langle \vec{E} \rangle + \vec{E}^{fluc}$$

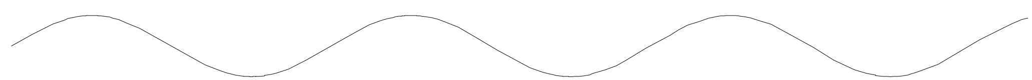
$$\vec{E}, \vec{J}$$

microscopic level



$$\langle \vec{E} \rangle = \hat{P}_{AV} \vec{E}$$

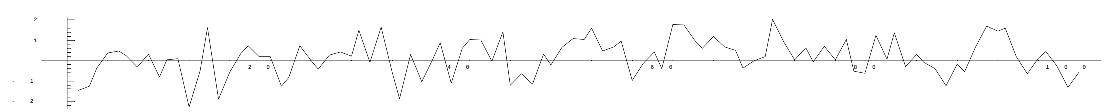
coherent



Length scale

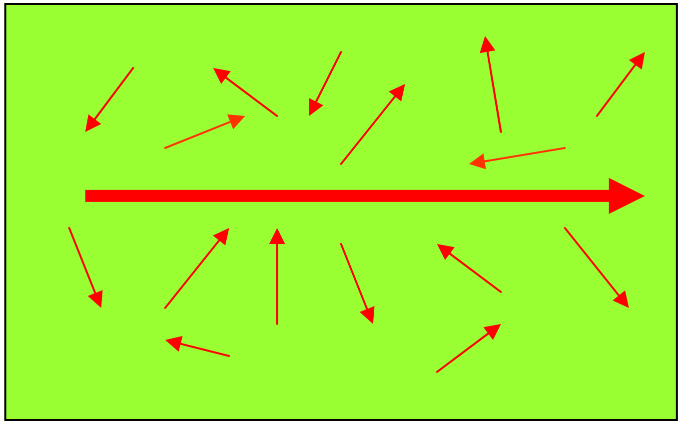
$$\vec{E}^{fluc} = \vec{E} - \langle \vec{E} \rangle$$

diffuse



Continuum Electrodynamics

$$\vec{E} \approx \langle \vec{E} \rangle$$



Power

$$\vec{S} = \langle \vec{S} \rangle + \vec{S}^{fluc}$$

$$S^{fluc} \ll \langle S \rangle$$

Linear approximation

$$\vec{P} = \varepsilon_0 \chi_E \langle \vec{E} \rangle$$

$$\vec{M} = \left(\frac{1}{\mu_0} - \frac{1}{\mu} \right) \langle \vec{B} \rangle$$

Local approximation

$$\chi_E(\omega)$$

$$\mu(\omega)$$

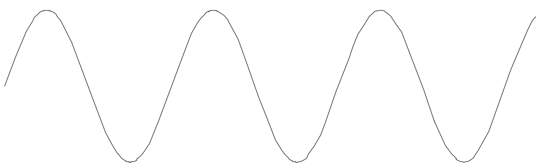
Light in material media

Plane waves

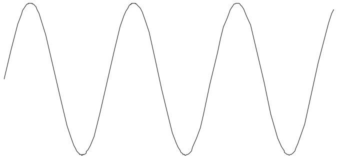
polarization

$$\vec{E} = E_{0i} \exp\left(i\vec{k}\cdot\vec{r} - \omega t\right)\hat{e}_i$$

$$k_0 = \frac{\omega}{c}$$



$$k = k_0\, n(\omega)$$



index of refraction

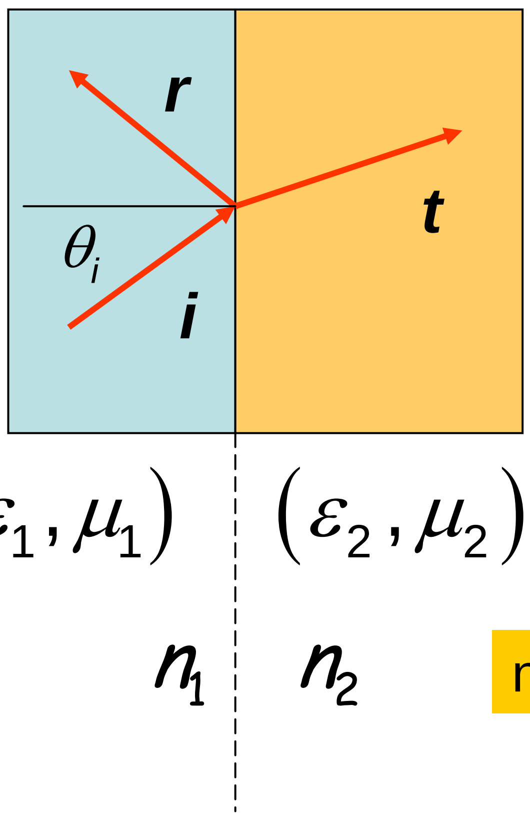
$$n(\omega) = \sqrt{\frac{\varepsilon(\omega)\mu(\omega)}{\varepsilon_0(\omega)\mu_0(\omega)}}$$

$$\varepsilon(\omega) = \varepsilon_0[1 + \chi_E(\omega)]$$

induced currents

Inhomogeneous systems

Fresnel (1820)



(ε_1, μ_1) (ε_2, μ_2)

n_1 n_2

non-magnetic

reflection amplitude

$$r = \frac{E_r}{E_i}$$

polarization

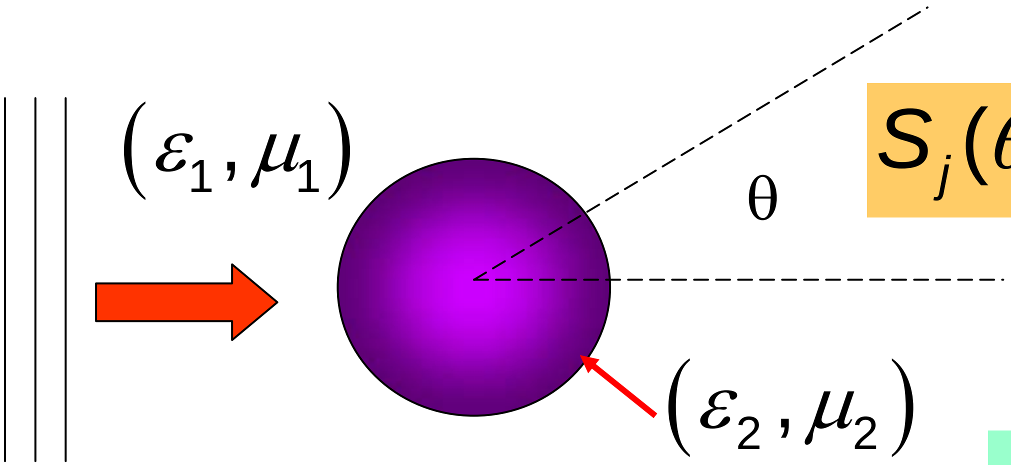
$$(TE, TM)$$

$$r(\varepsilon_1, \mu_1, \varepsilon_2, \mu_2, \theta_i)$$

$$R = |r|^2$$

$$r(n_1, n_2, \theta_i)$$

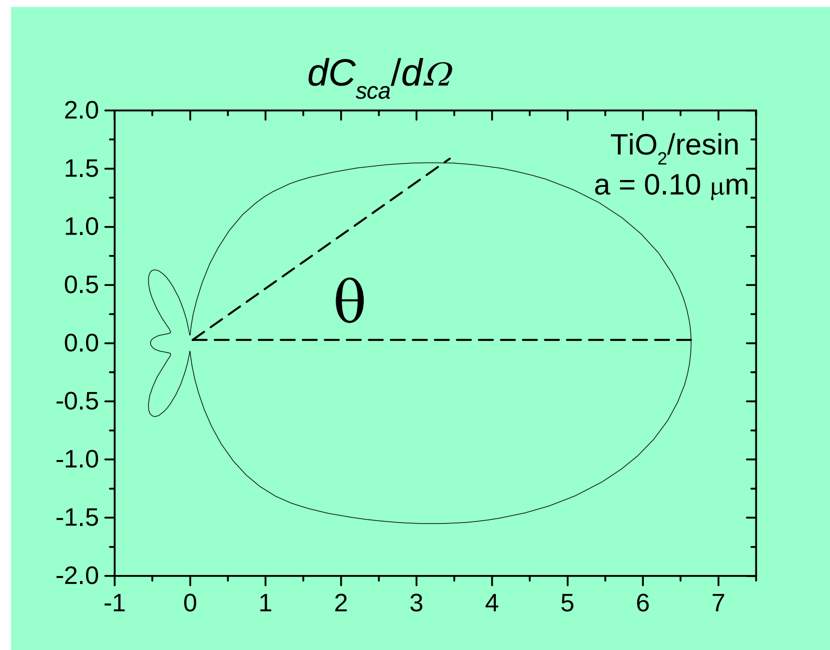
GUSTAV MIE (1905)



$S_j(\theta)$

SCATTERING

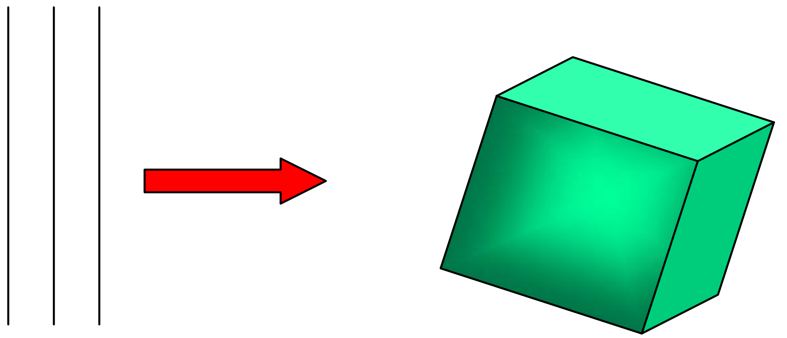
$\dfrac{dC_{sca}}{d\Omega}$

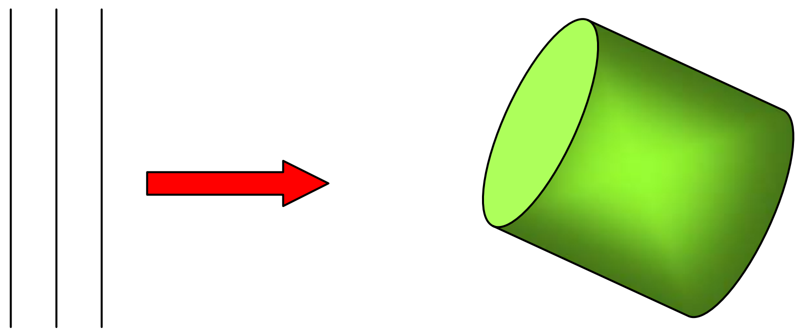


Scattering matrix

$$\begin{pmatrix} E_{\parallel}^{s} \\ E_{\perp}^{s} \end{pmatrix} = \frac{e^{ikr}}{-ikr} \begin{pmatrix} S_2 & 0 \\ 0 & S_1 \end{pmatrix} \begin{pmatrix} E_{\parallel}^{inc} \\ E_{\perp}^{inc} \end{pmatrix}$$

Numerical solution (2006)



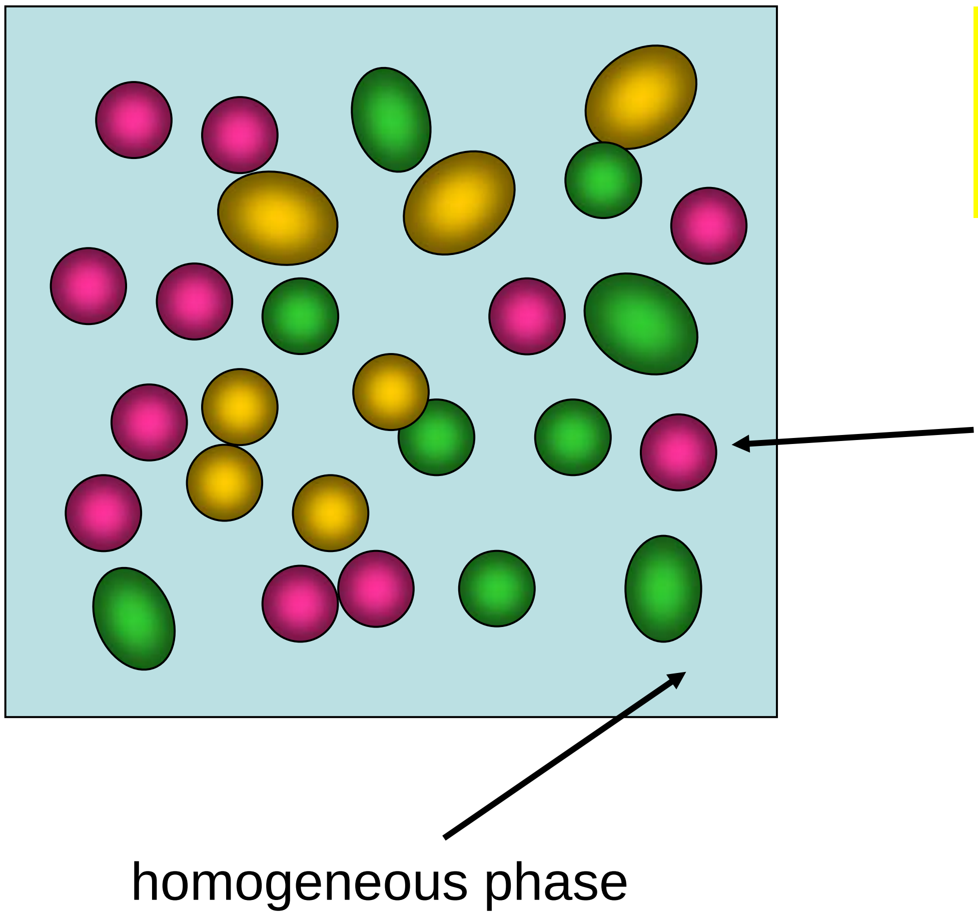


Scattering matrix

$$\begin{pmatrix} S_2 & S_3 \\ S_4 & S_1 \end{pmatrix}$$

$$S_j(\theta,\phi)$$

Colloidal Systems



Inhomogeneous phase dispersed within a homogeneous phase

Examples

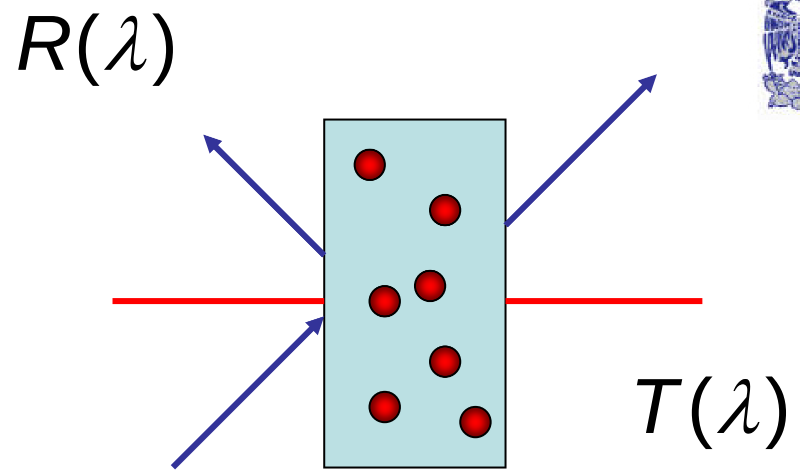
continuous phase	disperse phase	name	examples
liquid	solid	**sol**	paints, blood, tissues
liquid	liquid	**emulsion**	milk, water in benzene
liquid	gas	**foam**	foam, whipped cream
solid	solid	**solid sol**	composites, policrystals, rubys
solid	liquid	**solid emulsion**	opals, milky quartz, …
solid	gas	**solid foam**	porous media,…
gas	solid	**solid aereosol**	smoke, powder,…
gas	liquid	**liquid aereosol**	fog,…

Optical properties

Propagation

Transmission

Reflectión



optical spectroscopy

$400 \leq \lambda \leq 800$ nm

INVERSE PROBLEM

$R(\lambda)$

$T(\lambda)$

structure of the colloid

METAMATERIALS

Effective medium

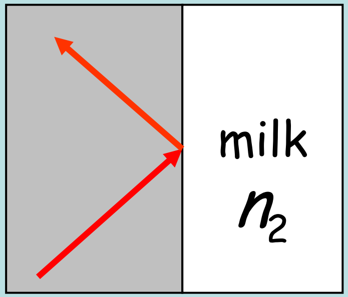
Example

reflection spectroscopy

critical angle

$$\sin\theta_c = \frac{n_2}{n_1}$$

$n_1 = 1.50$



effective index of refraction

δn_2

state of aggregation
particle sizing

Colloids

size parameter: $x = \frac{2\pi a}{\lambda}$ $400 \leq \lambda \leq 800$ nm

SMALL $x \ll 1$ ~ 100 nm $S_{diffuse} \ll S_{coh}$

BIG $x \geq 1$

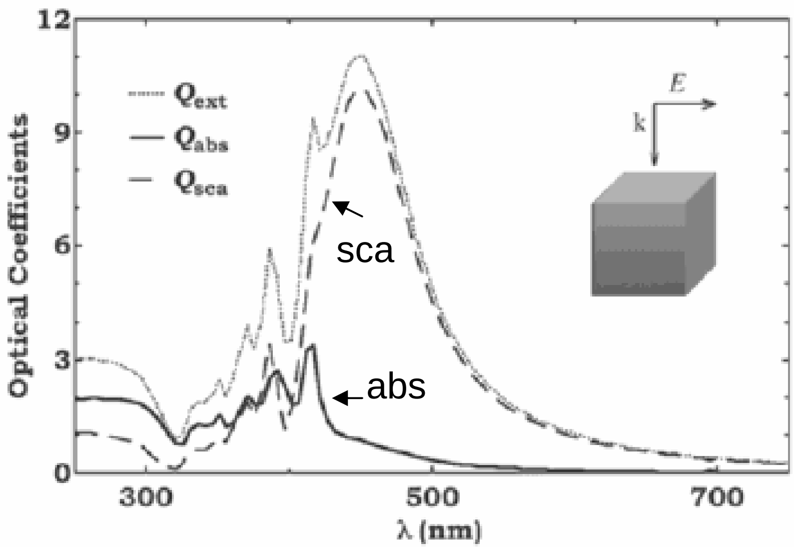
$S_{diffuse} \geq S_{coh}$

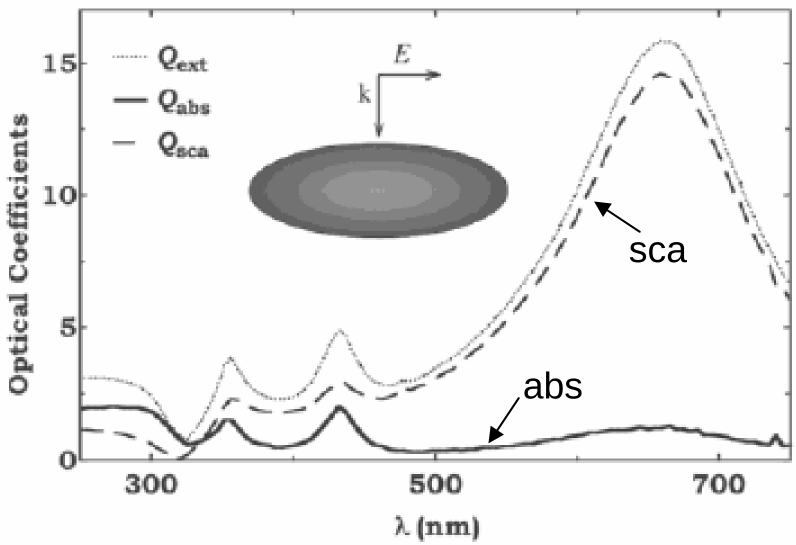
scattering

Size

6272 *J. Phys. Chem. B, Vol. 107, No. 26, 2003*

Sosa, Noguez and Barrera





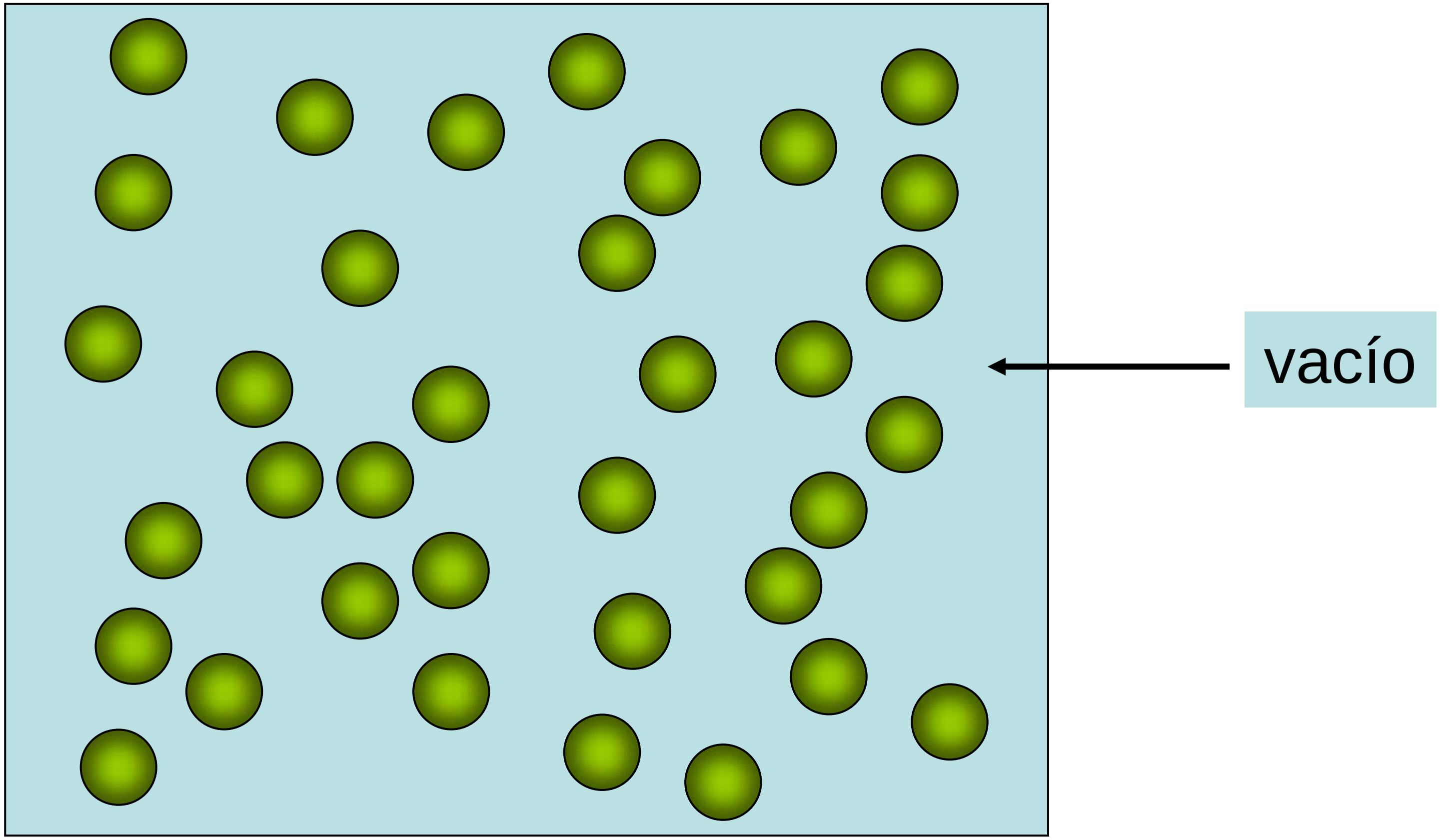
$$V = \frac{4\pi a_{eq}^3}{3}$$

silver

$a_{eq} = 50$ nm

Modelo para el coloide

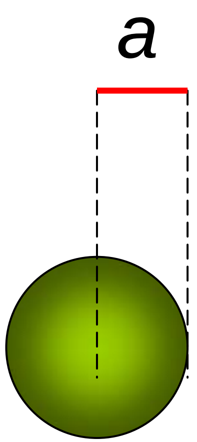
esferas idénticas en suspensión



En promedio: homogéneo e isotrópico

parámetros

parámetros geométricos



parámetros ópticos

no-magnéticas

$$\tilde{\varepsilon}_S \equiv \frac{\varepsilon_S}{\varepsilon_0}, \quad \tilde{\mu}_S \equiv \frac{\mu_S}{\mu_0}$$

$$\tilde{\mu}_S = 1$$

$$n_S = \sqrt{\varepsilon_S}$$

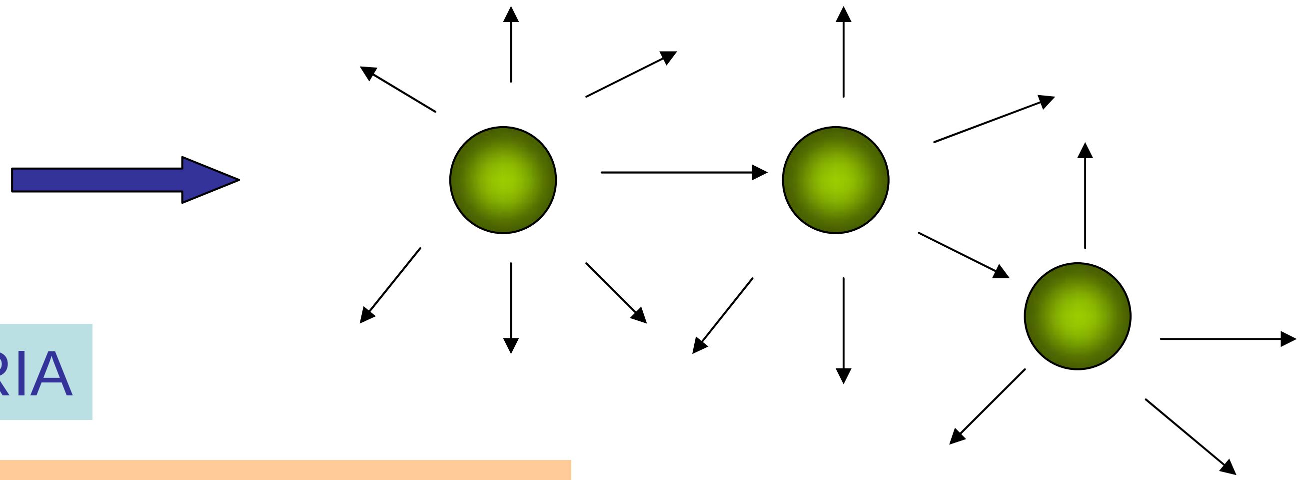
índice de refracción

parámetros estadísticos

$$f = \frac{N}{V}\frac{4\pi a^3}{3}, \rho^{(2)}(r_{12}), \rho^{(3)}(r_1, r_2, r_3), \ldots$$

Cálculo de propiedades ópticas

modelo → TEORIA → **propiedades ópticas**



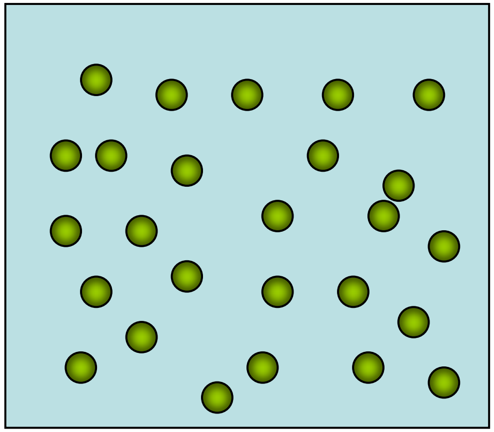
TEORIA

ECUACIONES DE MAXWELL

Esparcimiento múltiple
Difusión de “fotones”
Transferencia radiativa

El medio efectivo

"homogéneo"



¿posible?

$\tilde{\varepsilon}_{eff}$ $\tilde{\mu}_{eff}$

suspension coloidal

medio efectivo

Electrodinámica continua

Propiedades ópticas

Antecedentes

parámetro de tamaño

$$x = ka = \frac{2\pi a}{\lambda_0}$$

partículas pequeñas

$$x \ll 1$$

el medio efectivo siempre existe!

$$\tilde{\mu}_{eff} = 1$$

(no-magnético)

$$n_{eff} = \sqrt{\varepsilon_{eff}}$$

ópticos microestructura

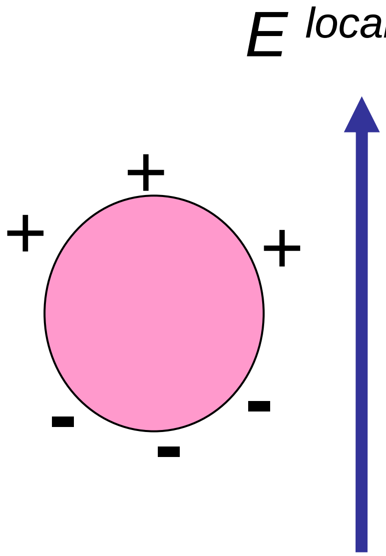
$$\varepsilon_{eff}(n_S, n_0, \lambda; a, f, \rho^{(2)}, \rho^{(3)}, \ldots)$$

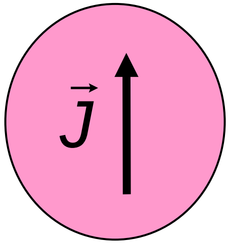
"reglas de mezclado"

Example

non-magnetic

Isolated sphere $\left(\varepsilon_p, \mu_p = \mu_0\right)$





Induced current

$$\langle \vec{J} \rangle = \frac{1}{v_a}(-i\omega)\vec{p}$$

Induced dipole

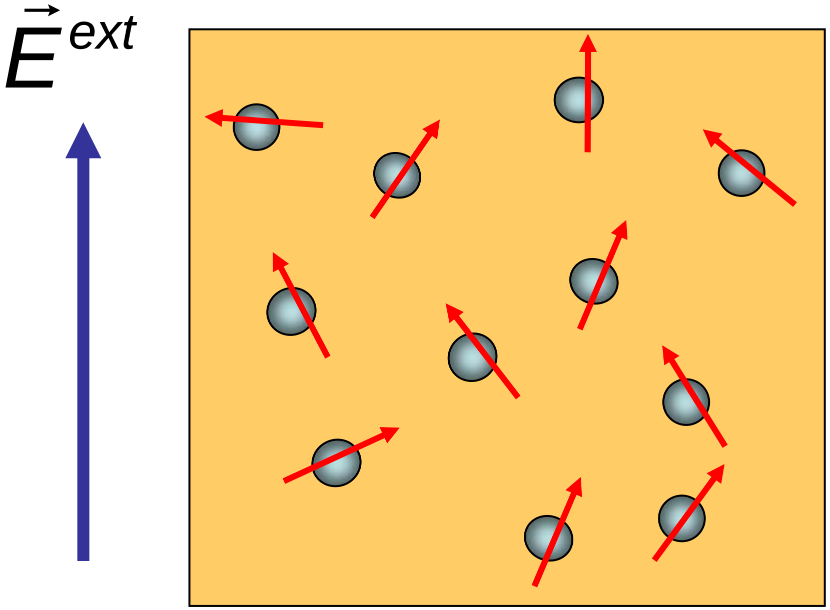
$$\vec{p} = \varepsilon_0 \, \alpha \, \vec{E}^{local}$$

$$\alpha = 4\pi a^3 \frac{\tilde{\varepsilon}_p - 1}{\tilde{\varepsilon}_p + 2}$$

resonancia $\tilde{\varepsilon}_p(\omega) = -2$

Colloid

random locations



$$\vec{p}_i = \alpha \left[\vec{E}^{ext} + \sum_j \overleftrightarrow{T}_{ij} \cdot \vec{p}_j \right]$$

LOCAL FIELD

$$\langle \vec{p} \rangle = \frac{1}{N} \sum_i p_i \quad \Rightarrow \quad \vec{P} = \frac{N}{V} \langle \vec{p} \rangle$$

$$\langle \vec{E} \rangle [\vec{E}^{ext}] \quad \Rightarrow \quad \vec{P} = \varepsilon_0 \chi_{eff} \langle \vec{E} \rangle$$

JC Maxwell Garnett

(1905)

Mean-field approximation

$$\vec{p}_i \approx \langle \vec{p} \rangle$$

$$\mu_{eff} = \mu_0$$

non-magnetic

$$\frac{\varepsilon_{eff}}{\varepsilon_s} = \frac{1+2f\tilde{\alpha}}{1-f\tilde{\alpha}} \approx 1+3\tilde{\alpha}f$$

dilute limit

$$f = \frac{N}{V}\frac{4\pi a^3}{3}$$

filling fraction

$$\tilde{\alpha}(\omega) = \frac{1}{f}$$

shift of resonances

$$\tilde{\alpha} = \frac{\tilde{\varepsilon}_p(\omega)-1}{\tilde{\varepsilon}_p(\omega)+2}$$

dimensionless polarizability

optical properties

Further improvements

PHYSICAL REVIEW B VOLUME 38, NUMBER 8 15 SEPTEMBER 1988-I

Renormalized polarizability in the Maxwell Garnett theory

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(Received 10 July 1987; revised manuscript received 11 February 1988)

We develop a simple theory for the macroscopic dielectric function of a system of identical spheres embedded in a homogeneous matrix within the dipolar long-wavelength approximation. We obtained a relationship similar to the Clausius-Mossotti relation, but with a renormalized polarizability for the spheres instead of the bare polarizability. This renormalized polarizability obeys a second-order algebraic equation and it is given in terms of the bare polarizability, the volume fraction, and a functional of the two-particle correlation function of the spheres. We calculate the optical properties of metallic spheres within an insulating matrix and we compare our results with previous theories and with experiment.

α^* **renormalized polarizability** (1988-94)

$$\frac{\varepsilon_{eff}}{\varepsilon_s} = \frac{1 + 2f\tilde{\alpha}^*}{1 - f\tilde{\alpha}^*}$$

$$\frac{\alpha^*}{2} = \frac{1 - \sqrt{1 - f_e\tilde{\alpha}^2}}{f_e\tilde{\alpha}}$$

$$f_e = f_e^{(2)} + f_e^{(3)}$$

$$f_e^{(2)} = 3f\int_0^\infty \frac{\rho^{(2)}(2ax)}{x^4}\,dx$$

$$f_e^{(3)} = \frac{9}{4\pi^2} f^2 \int \hat{q}\cdot\overleftrightarrow{T}_{12}\cdot\overleftrightarrow{T}_{23}\cdot\hat{q}\,\Delta\rho^{(3)}(\vec{R}_1,\vec{R}_2,\vec{R}_3)\,d^3R_2\,d^3R_3$$

$$\Delta\rho^{(3)} = \rho^{(3)}(\vec{R}_1,\vec{R}_2,\vec{R}_3) - \rho^{(2)}(\vec{R}_1,\vec{R}_2)\rho^{(2)}(\vec{R}_2,\vec{R}_3)$$

Experiment

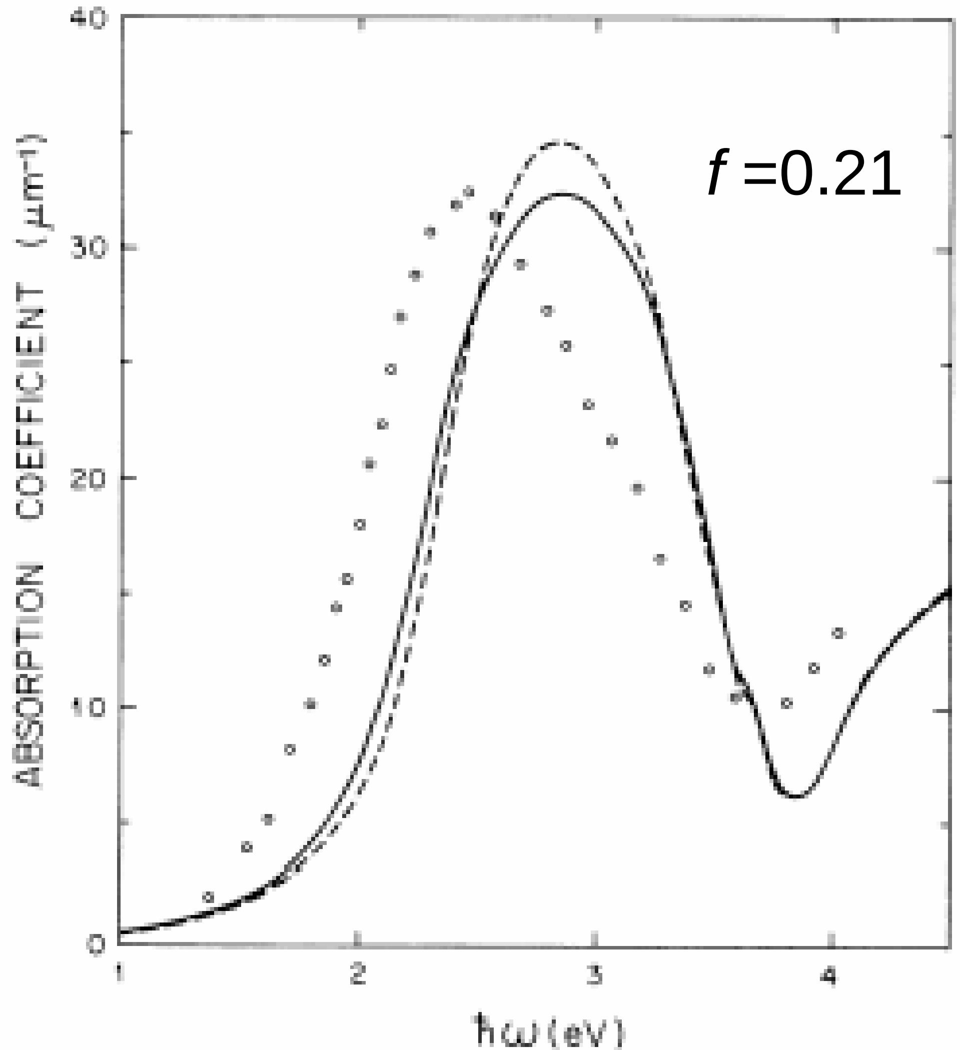
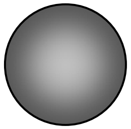


FIG. 7. Absorption coefficient $(2\omega/c)\mathrm{Im}\sqrt{\epsilon_M}$ of Ag particles embedded in gelatin with $f=0.21$. The solid (dashed) line correspond to the PY (HC) correlation function. The dots are the experimental results arbitrarily normalized.



Ag

Further improvements

A new diagrammatic summation for the effective dielectric response of composites

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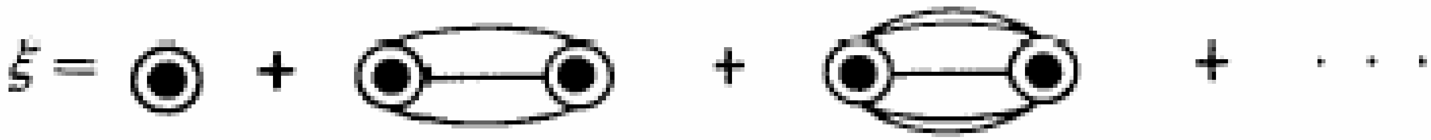
(Received 4 April 1991; accepted 11 October 1991)

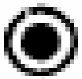
We extended a previously developed diagrammatic formulation for the calculation of the effective dielectric response of composites prepared as a random, homogeneous, and isotropic distribution of small spherical inclusions in an otherwise homogeneous matrix. This is done within the long-wavelength, dipolar approximation in the low-density regime of inclusions. We propose a new diagrammatic summation and we compare our results with two recently reported computer simulations.

J. Chem. Phys., Vol. 96, No. 2, 15 January 1992

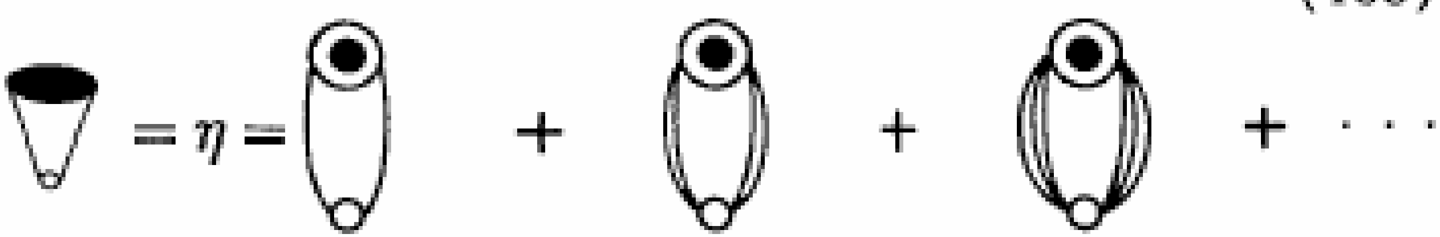
Summation of elementary polarization processes



(18a)

where the renormalized vertex  $= \Delta$ is given by the self-consistent solution of the following diagrammatic equations:

(18b)



(18c)

Numerical

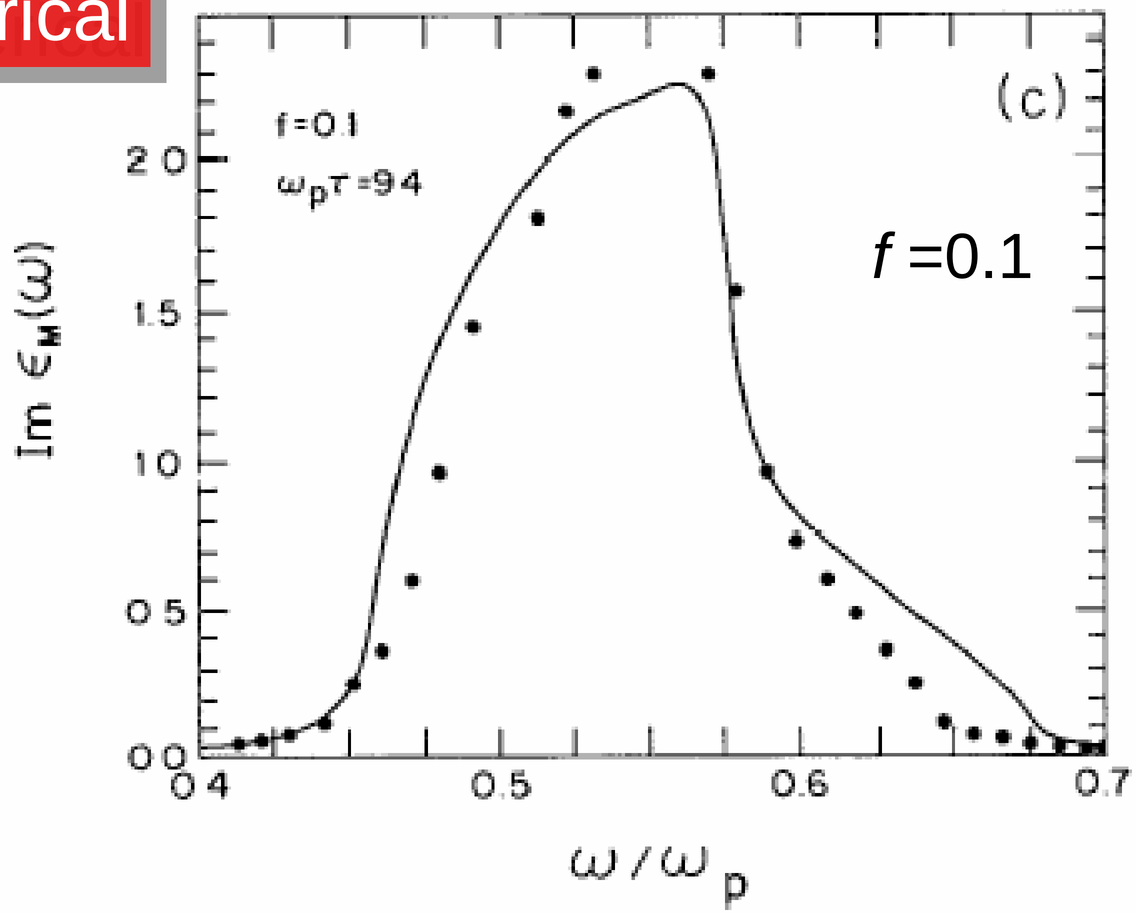
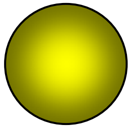


FIG. 2. Im ϵ_M as a function of ω/ω_p for filling fractions 0.01 (a), 0.03 (b), and 0.1 (c). The solid line corresponds to Eq. (8a) with ξ given by Eq. (25) and the dots are the results of the computer simulation of Ref. 11.



Drude

S. Kumar and R.I. Cukier, J. Phys. Chem, 93, 4334 (1989)

Expansion in *ka*

CA Grimes and DM Grimes, PRB **43**, 10780 (1991)

spheres $\left(\varepsilon_p, \mu_p\right)$

Mean-field approximation

$ka \ll 1$

$k_p a = k_0 \sqrt{\varepsilon_p \mu_p}\, a \quad \textit{unrestricted}$

$$\tilde{\varepsilon}_{eff} = \frac{1+2fA}{1-fA} \qquad A \approx \frac{\tilde{\varepsilon}-1}{\tilde{\varepsilon}+2} + 3(ka)^2 \frac{\mu_p \tilde{\varepsilon}_p^2 + \tilde{\varepsilon}_p^2 - 6\tilde{\varepsilon}_p + 4}{10(\tilde{\varepsilon}_p + 2)^2} + \ldots$$

$$\tilde{\mu}_{eff} = \frac{1+2fB}{1-fB} \qquad B \approx \frac{\tilde{\mu}-1}{\tilde{\mu}+2} + 3(ka)^2 \frac{\tilde{\varepsilon}_p \tilde{\mu}_p^2 + \tilde{\mu}_p^2 - 6\tilde{\mu}_p + 4}{10(\tilde{\mu}_p + 2)^2} + \ldots$$

Non-magnetic case

$$\tilde{\mu}_p = 1$$

$$\tilde{\varepsilon}_{eff} = \frac{1 + 2fA}{1 - fA}$$

$$A \approx \frac{\tilde{\varepsilon}_p - 1}{\tilde{\varepsilon}_p + 2} + 3(ka)^2 \frac{2\tilde{\varepsilon}_p^2 - 6\tilde{\varepsilon}_p + 4}{10(\tilde{\varepsilon}_p + 2)^2} + \ldots$$

$$\tilde{\mu}_{eff} = \frac{1 + 2fB}{1 - fB}$$

$$B \approx 3(ka)^2 \frac{\tilde{\varepsilon}_p - 1}{90} + \ldots$$

$$\tilde{\varepsilon}_{eff} = \frac{4\sqrt{2} + 2fA}{4\sqrt{2} - fA}$$

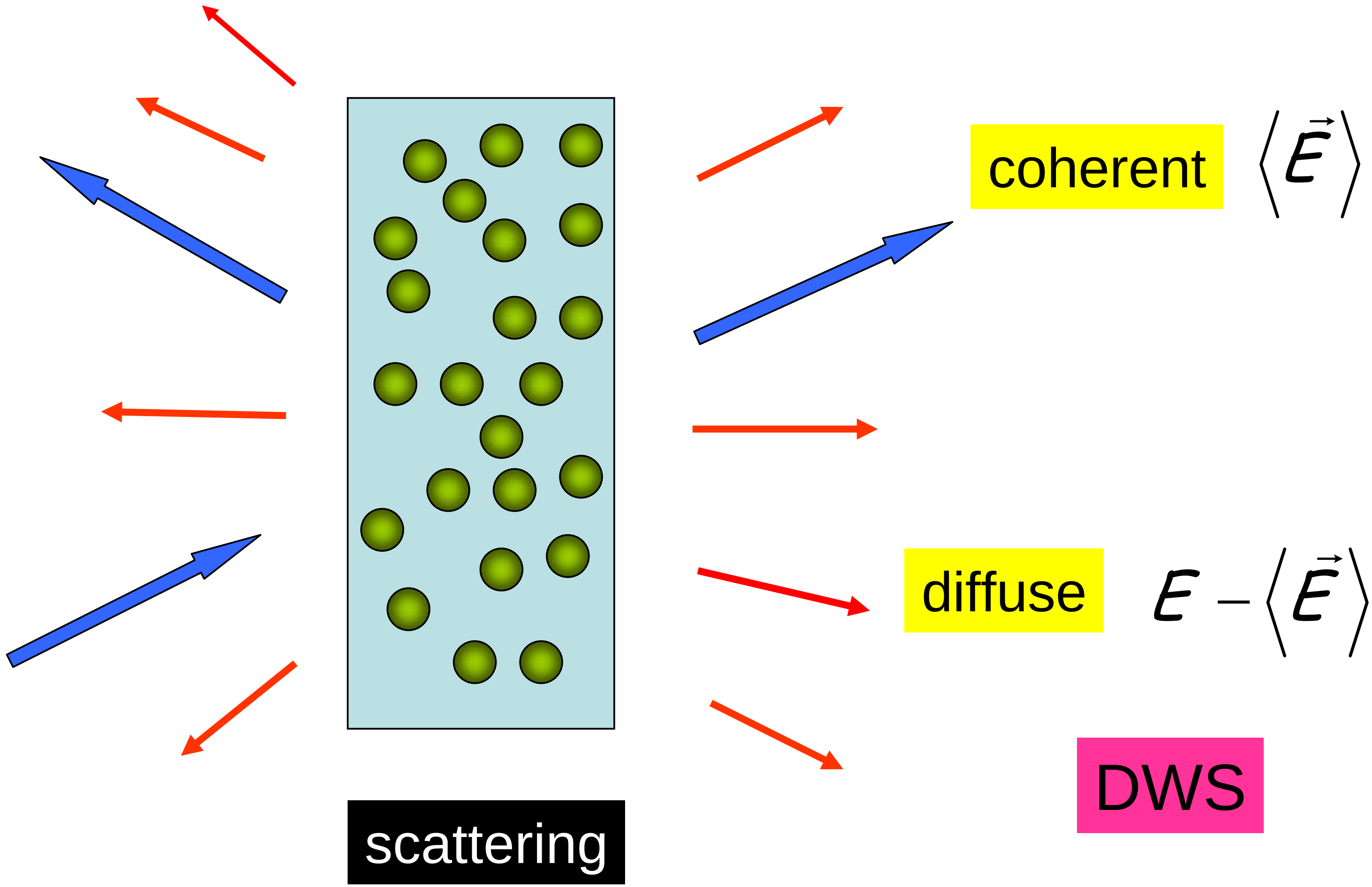
Photonic crystals

Close packed fcc

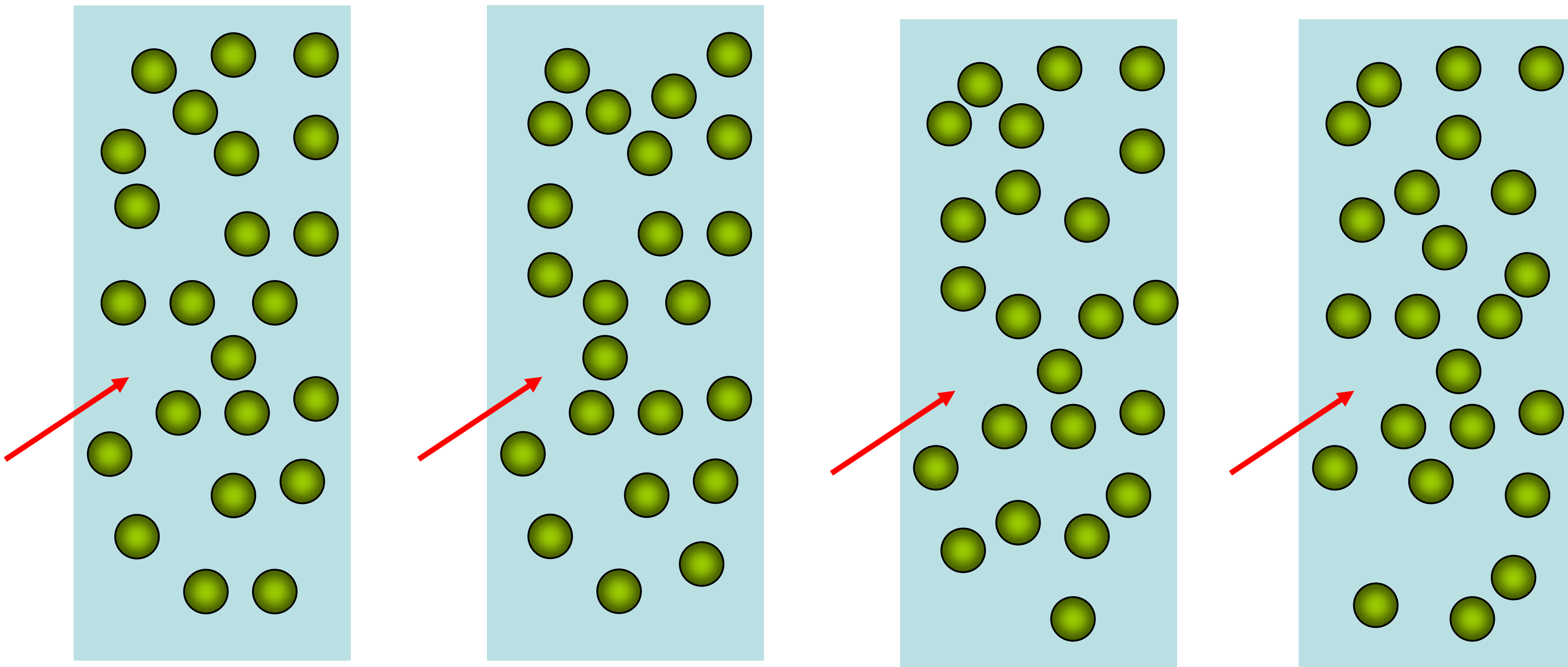
$$\tilde{\mu}_{eff} = \frac{4\sqrt{2} + 2fB}{4\sqrt{2} - fB}$$

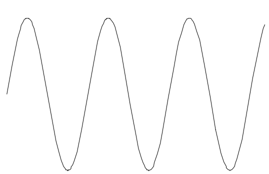
Big particles

$$ka \sim 1$$



Configurational average

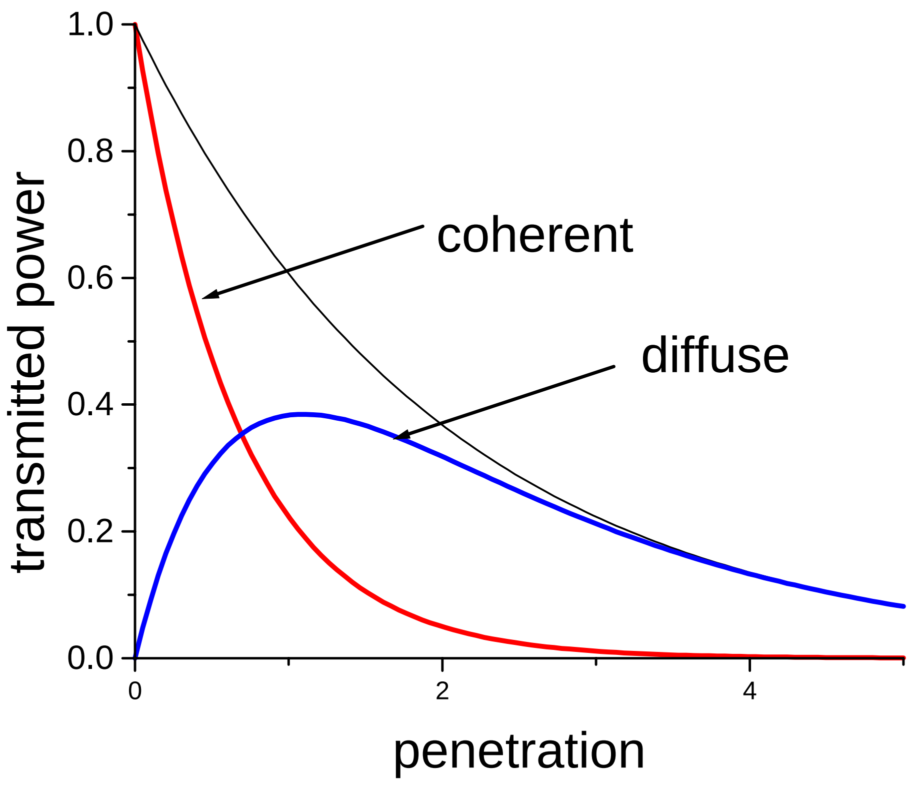




$$\left\langle \vec{E} \right\rangle = \frac{1}{N}\sum_{\{C_n\}} \vec{E}\left(\vec{r};\left\{\vec{R}_1,\vec{R}_2,\ldots\vec{R}_N\right\}_{C_n}\right)$$

The power

$$Power \propto |E|^2 = |E_{AV}|^2 + |E_{fluc}|^2$$



$$ka \sim 1$$

Attempts

Van de Hulst

complex

$$\frac{n_{eff}}{n_S} = 1 + i\gamma S(0)$$

δn_{eff}

$$\mu_{eff} = \mu_0$$

Teoría de MIE

$$\gamma = \frac{3}{2}\frac{f}{x^3}$$

$$f \ll 1$$

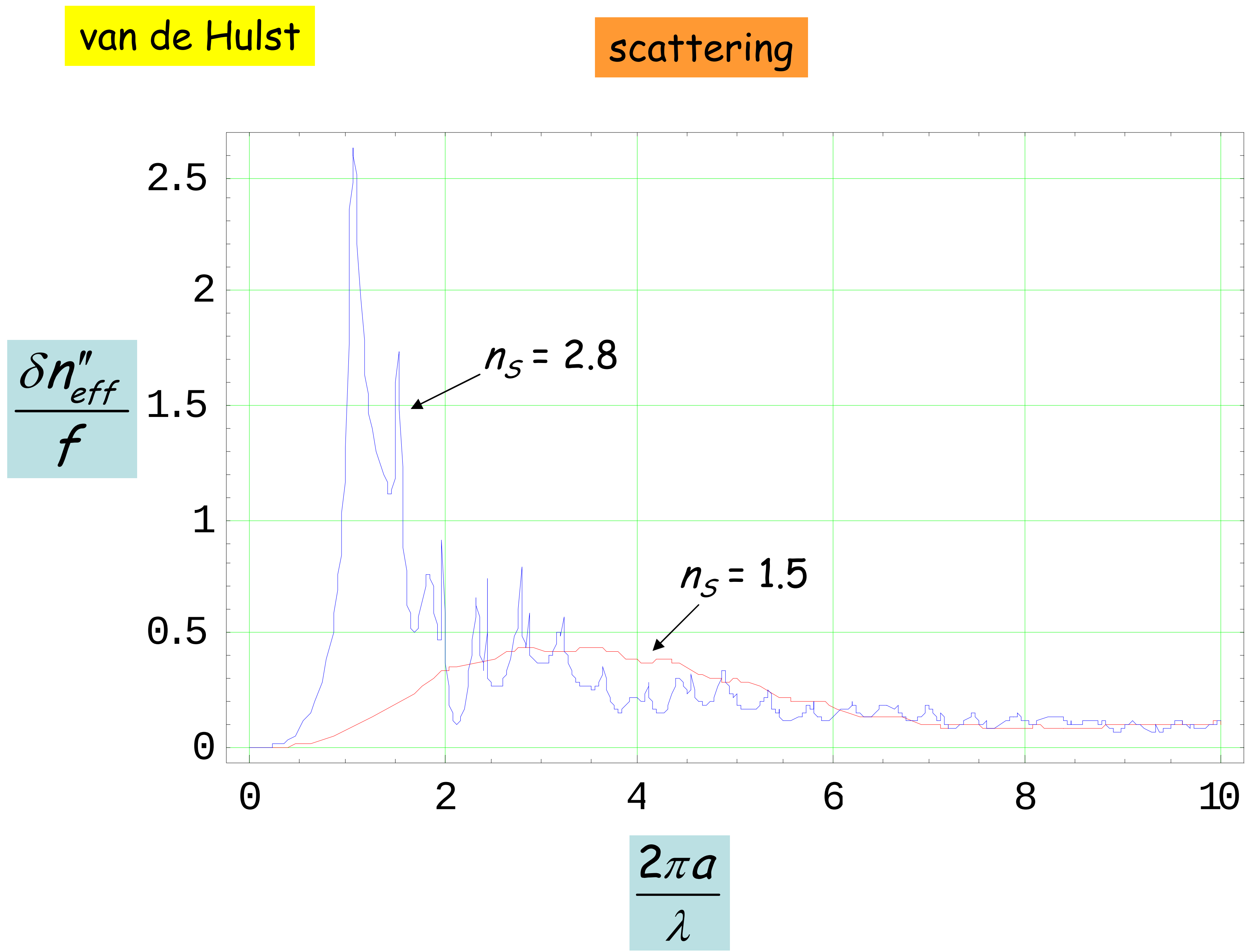
límite diluido

Scattering matrix

$$\begin{pmatrix} E_{\parallel}^{s} \\ E_{\perp}^{s} \end{pmatrix} = \frac{e^{ikr}}{-ikr}\begin{pmatrix} S_2 & 0 \\ 0 & S_1 \end{pmatrix}\begin{pmatrix} E_{\parallel}^{inc} \\ E_{\perp}^{inc} \end{pmatrix}$$

sphere

$$S_1(0) = S_2(0) = S(0)$$


van de Hulst
scattering
$\frac{\delta n''_{eff}}{f}$
$n_s = 2.8$
$n_s = 1.5$
0
0.5
1
1.5
2
2.5
0
2
4
6
8
10
$\frac{2\pi a}{\lambda}$

Temptation

Reflection amplitude

Use n_{eff} in Fresnel´s relations

Isotropic effective medium model (IEMM)

$$r^{TE} = \frac{\cos\theta_i - \sqrt{n_{eff}^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{n_{eff}^2 - \sin^2\theta_i}}$$

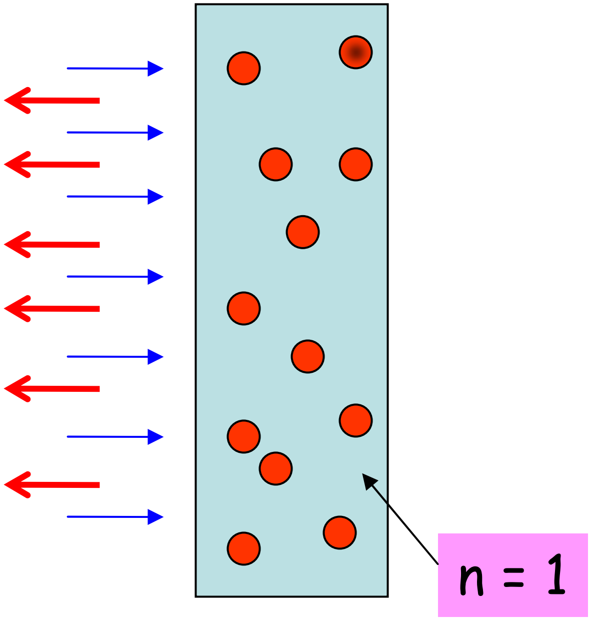
$$\frac{n_{eff}}{n_S} = 1 + i\gamma S(0)$$

$$\gamma = \frac{3}{2}\frac{f}{x^3}$$

C Bohren

J. Atmos Sci. 43, 468 (85)

Normal incidence



transmission $n_{eff} = 1 + i\gamma S(0)$

reflection $n_{eff} = 1 + i\gamma S_1(\pi)$

$$\varepsilon_{eff} = 1 + i\gamma\left[S(0) + S_1(\pi)\right]$$

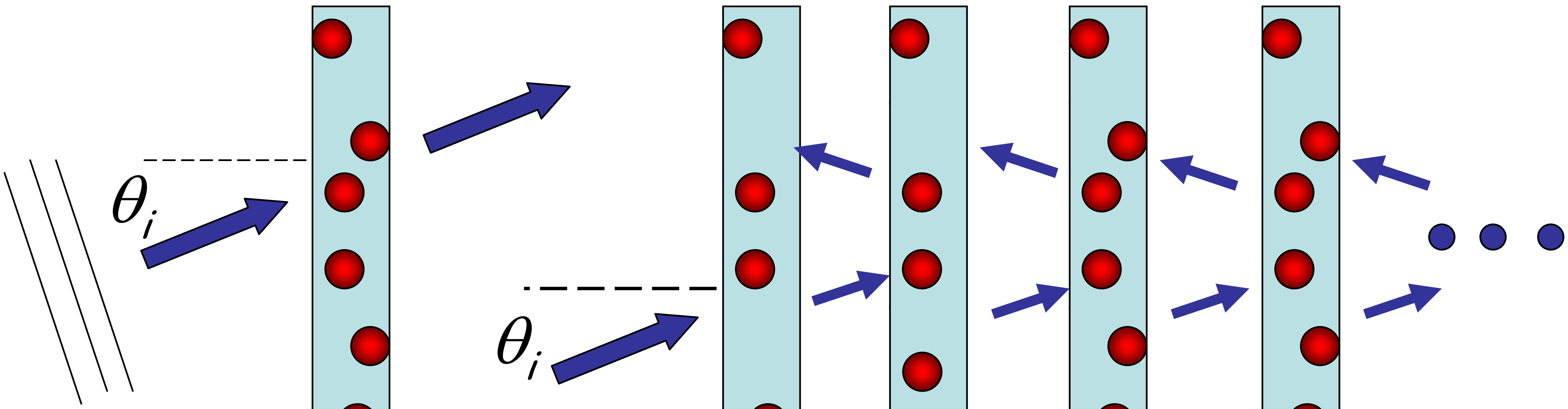
$$\mu_{eff} = 1 + i\gamma\left[S(0) - S_1(\pi)\right]$$

$$r = \frac{\sqrt{\mu} - \sqrt{\varepsilon}}{\sqrt{\mu} + \sqrt{\varepsilon}}$$

…It might be expected that a composite medium is nonmagnetic if its components are, but this is not correct… which was recognized as long ago as 1909 by Gans and Happel…

Coherent scattering approach

Multiple scattering



Single thin slab

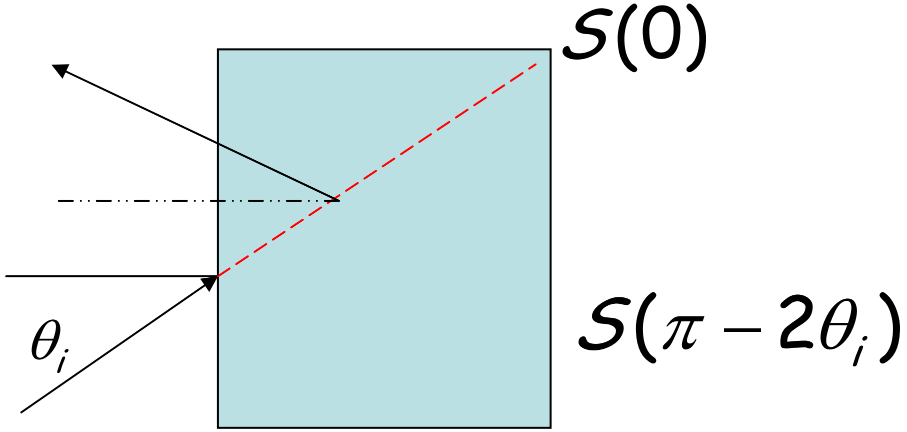
transfer equations

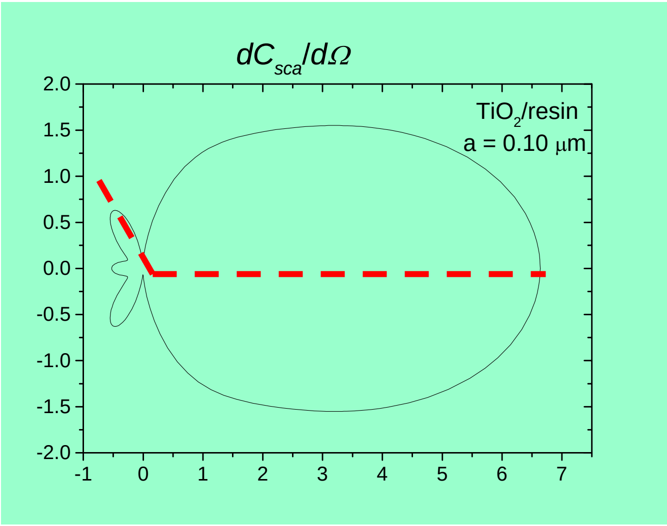
RG Barrera and A García-Valenzuela
JOSA **20**,296 (2003)
A García-Valenzuela and RG Barrera
JQSRT 79-80, 627 (2003)

Reflection amplitude

$$\gamma = \frac{3f}{2x^3}$$

$$r_{hs}^{TE} = \frac{\gamma S_1(\pi - 2\theta_i)/\cos\theta_i}{i(\cos\theta_i + [\cos^2\theta_i + 2i\gamma S(0)]^{1/2}) - \gamma S(0)/\cos\theta_i}$$

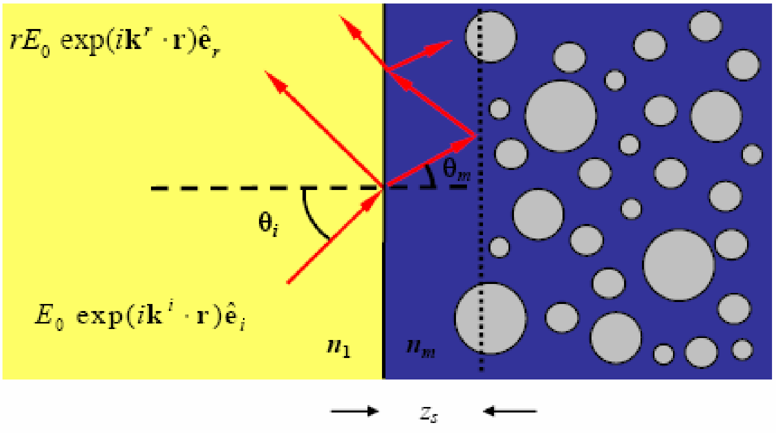




Validation

Theory

Extension to
include the matrix



Experiment

Internal reflection
Configuration

great sensitivity

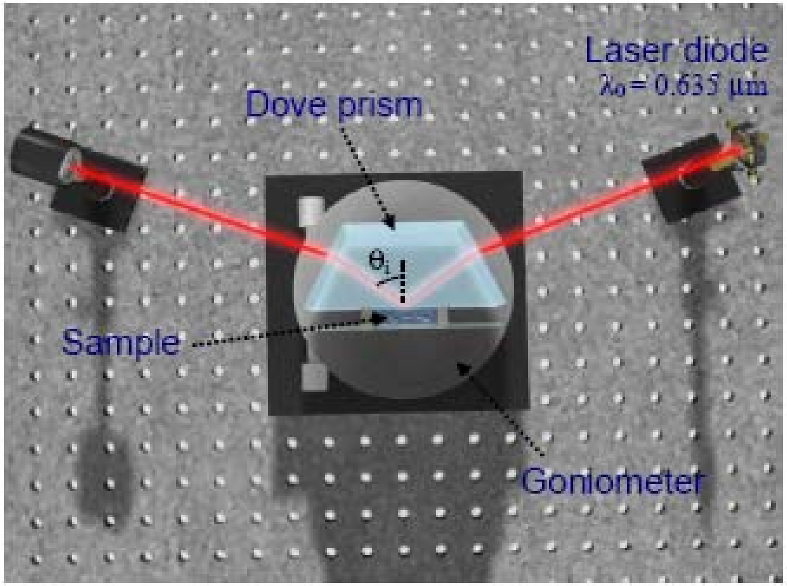
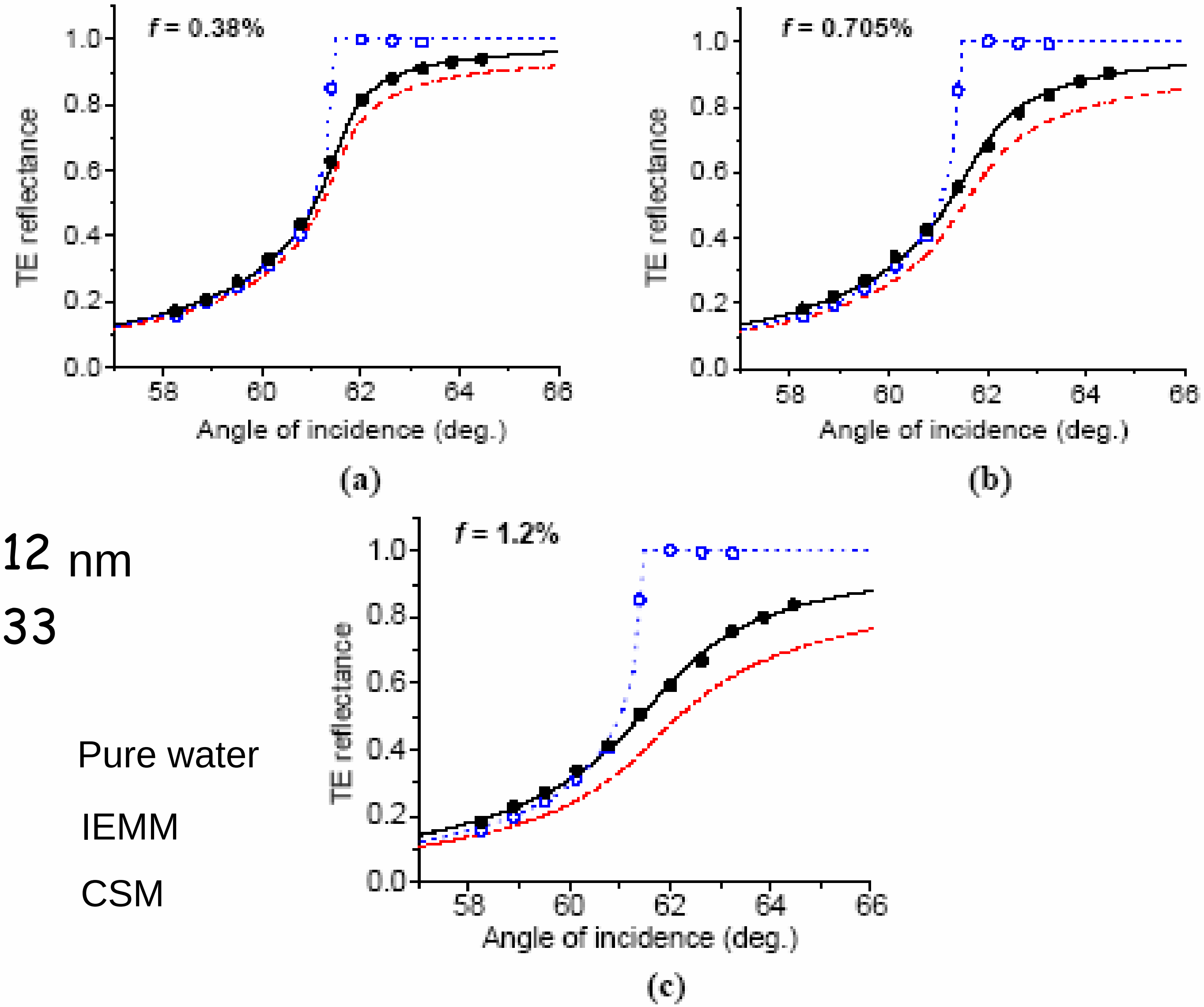


Fig.3. Schematic of the experimental setup.

A García-Valenzuela, RG Barrera,
C. Sánchez-Pérez, A. Reyes-Coronado,
E Méndez, Optics Express, **13**, 6723 (2005)

Comparison

TiO_2 / water



$a_0 = 112$ nm

$\sigma = 1.33$

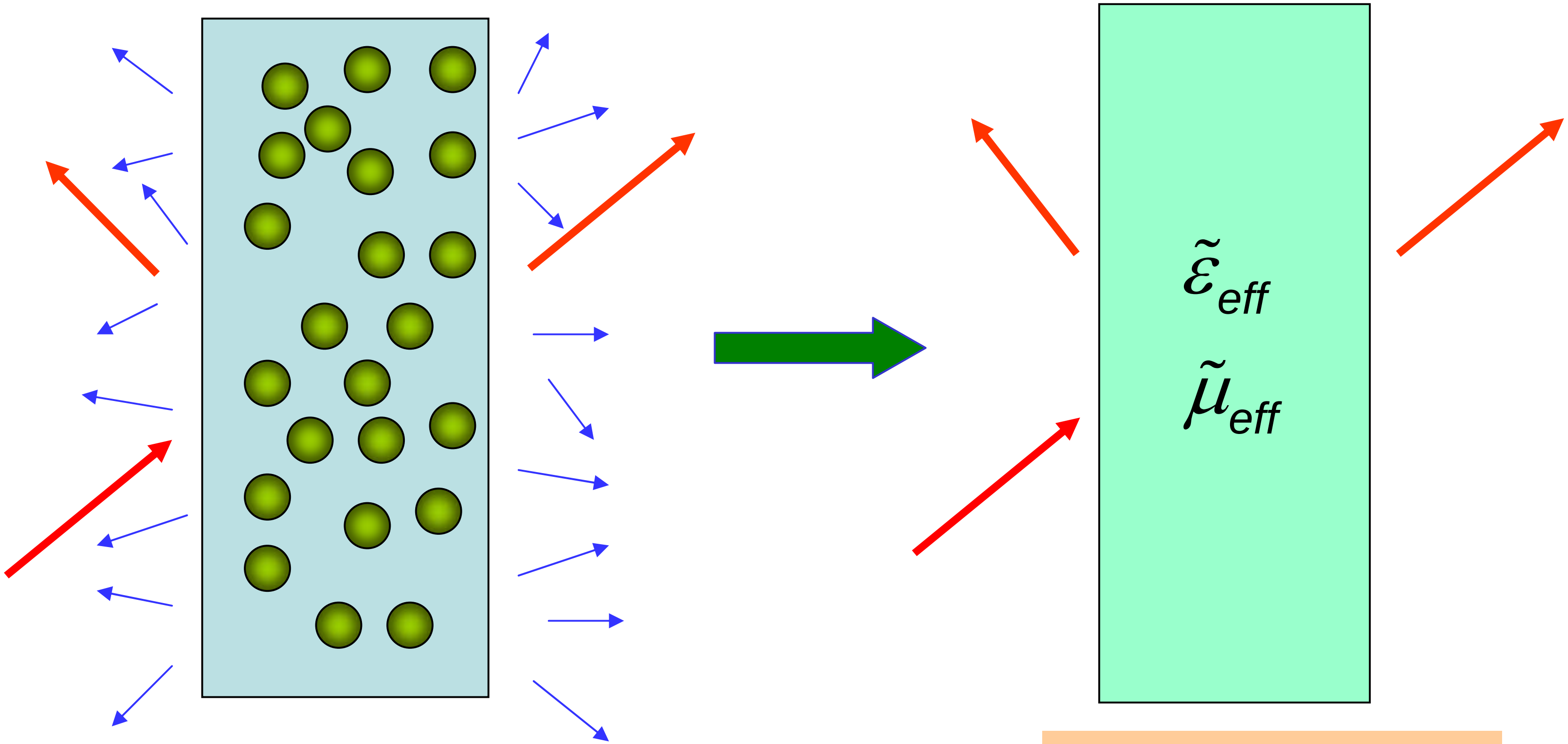
Pure water

IEMM

CSM

Problem

Can one describe the behavior of the coherent beam with an effective medium?



colloidal system

effective medium (incomplete)

Effective medium

Assume $\varepsilon_{eff}(\omega)$ $\mu_{eff}(\omega)$ then use r^{CSM} *to* determine them

$$\mu_{eff}^{TE}(\theta_i) = 1 + \frac{i\gamma S_-^{(1)}}{\cos^2\theta_i}$$

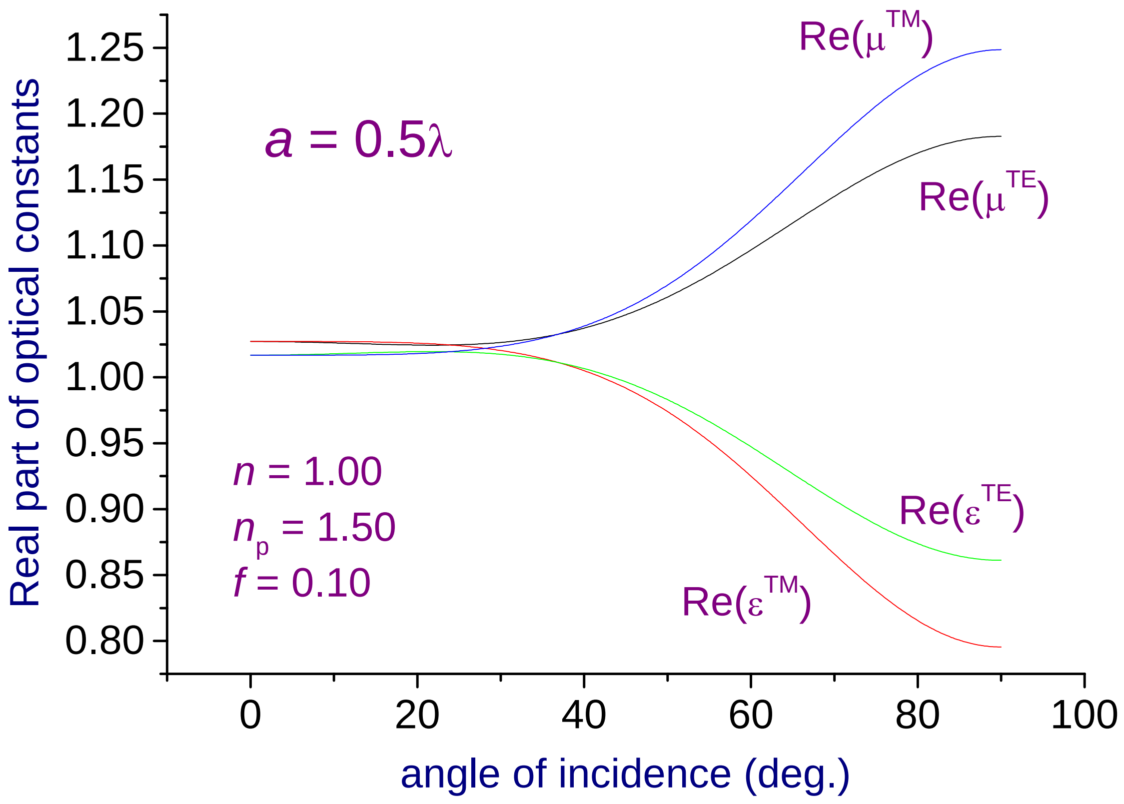
MAGNETIC

$$\varepsilon_{eff}^{TE}(\theta_i) = 1 + i\gamma\left(2S_+^{(1)}(\theta_i) - S_-^{(1)}(\theta_i)\tan^2\theta_i\right)$$

$$S_+^{(1)} = \frac{1}{2}\left[S(0) + S_1(\pi - 2\theta_i)\right]$$

$$S_-^{(1)} = S(0) - S_1(\pi - 2\theta_i)$$

Index of refraction



$$n_{eff}(\omega) = \sqrt{\varepsilon_{eff}^{TE}(\theta_i, \omega)\mu_{eff}^{TE}(\theta_i, \omega)} \approx 1 + i\gamma S(0)$$

Van de Hulst

Limiting cases

small particles

$$S(0) = S_1(\pi - 2\theta_i) \rightarrow \mu_{eff}^{TE} = 1$$

NON-MAGNETIC

normal incidence

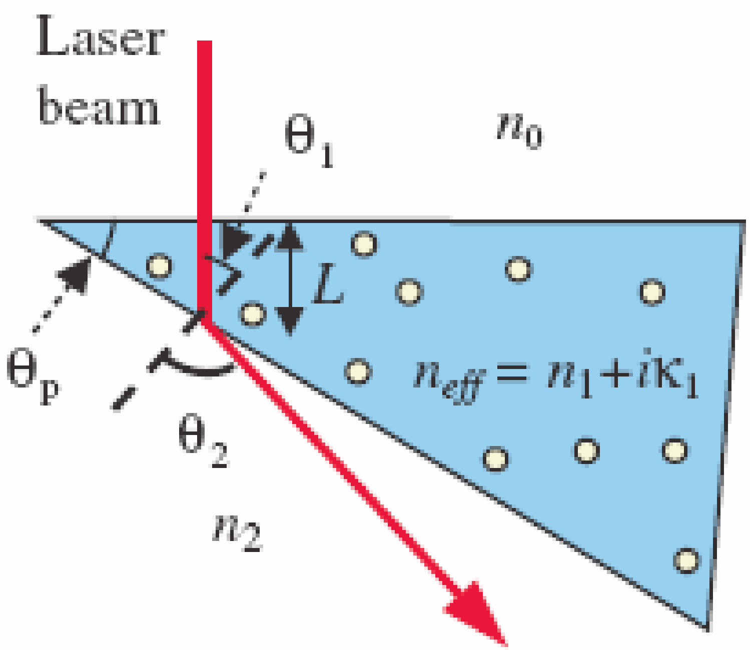
$$\theta_i = 0 \rightarrow$$

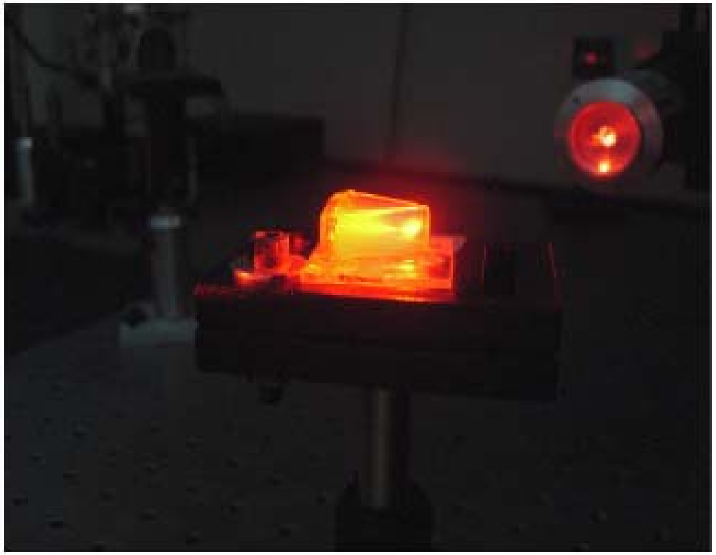
$$\varepsilon_{eff} = 1 + i\gamma\left[S(0) + S_1(\pi)\right]$$

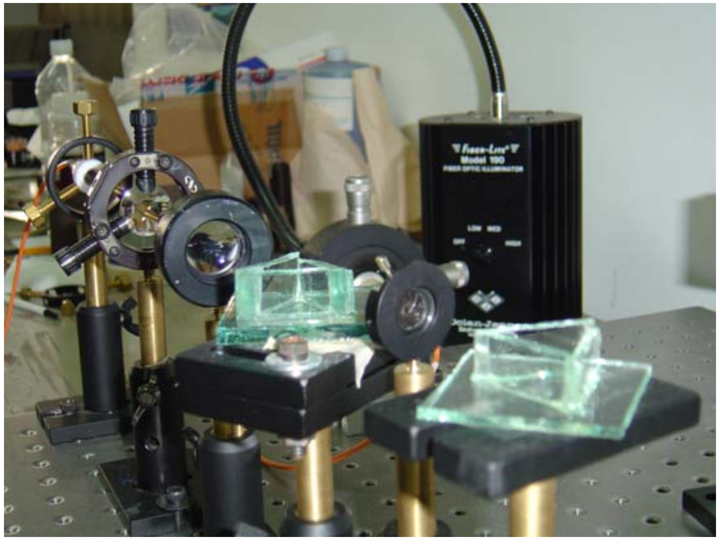
$$\mu_{eff} = 1 + i\gamma\left[S(0) - S_1(\pi)\right]$$

BOHREN

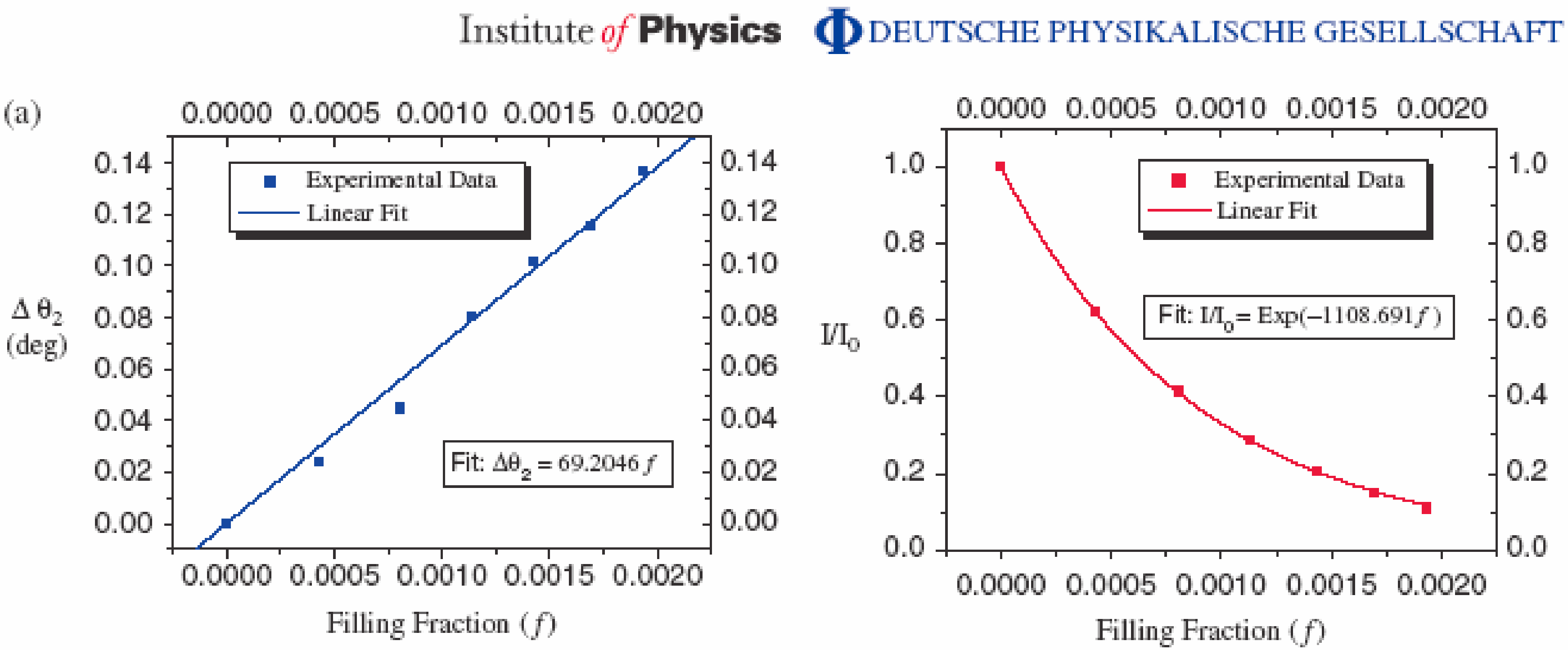
Refraction







Results



A. Reyes-Coronado, A. García-Valenzuela.
C. Sánchez-Pérez and RG Barrera
New Journal of Physics **7** (2005) 89

Comparison

Table 1. Retrieved and nominal values of experimental parameters.

Particle size	Retrieved values	Nominal values
Small spheres	$a = 0.1076\ \mu m$	$a = 0.111 \pm 0.005\ \mu m$
	$n_{sphere} = 1.566$	$n_{sphere} = 1.588$
	$\theta_1 = 47.955°$	$\theta_1 = 48.1 \pm 0.22°$
	$L = 2.039$ mm	$L = 1.9 \pm 0.25$ mm
Medium spheres	$a = 0.155\ \mu m$	$a = 0.155 \pm 0.007\ \mu m$
	$n_{sphere} = 1.588$	$n_{sphere} = 1.588$
	$\theta_1 = 48.175°$	$\theta_1 = 48.1 \pm 0.22°$
	$L = 2.05$ mm	$L = 2 \pm 0.25$ mm
Large spheres	$a = 0.247\ \mu m$	$a = 0.24 \pm 0.01\ \mu m$
	$n_{sphere} = 1.55$	$n_{sphere} = 1.588$
	$\theta_1 = 48.337°$	$\theta_1 = 48.1 \pm 0.22°$
	$L = 1.65$ mm	$L = 1.9 \pm 0.25$ mm

Again

Question

Can one describe the behavior of the coherent beam with an effective medium?

Answer

YES, but

The effective medium is magnétic

The effective medium is non local

$\mu_{eff} \neq \mu_0$

$\tilde{\varepsilon}_{eff}(p,\omega) \quad \tilde{\mu}_{eff}(p,\omega)$

spatial dispersion

Non local response

susceptibilidad

$$\tilde{\varepsilon} = \frac{\varepsilon}{\varepsilon_0} = 1 + \chi$$

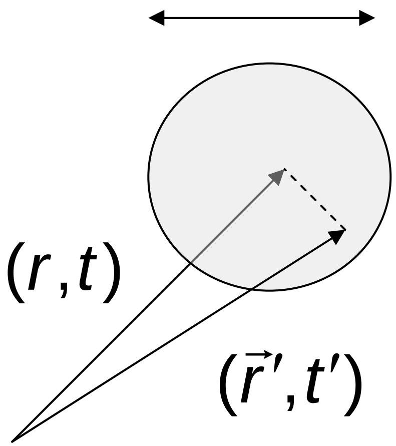
local

$$\vec{P} = \chi(\omega)\vec{E}$$

$$\vec{J} = -i\omega\vec{P} = -i\omega\chi(\omega)\vec{E}$$

non-local

$$\vec{P}(\vec{r};\omega) = \int \overleftrightarrow{\chi}(\vec{r} - \vec{r}\,';\omega)\cdot\vec{E}(\vec{r}\,';\omega)d^3r'$$



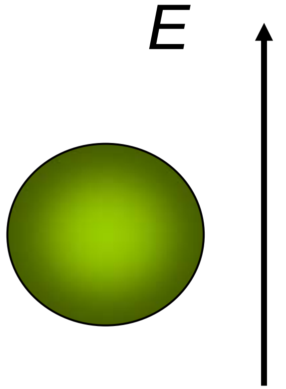
TF

$$\vec{P}(\vec{p},\omega) = \overleftrightarrow{\chi}(\vec{p},\omega)\cdot\vec{E}(\vec{p},\omega)$$

The T matrix

Single sphere

$$J^{ind}(\vec{r}) = \frac{1}{i\omega\mu_0}\int \overleftrightarrow{T}(\vec{r},\vec{r}\,')\cdot\vec{E}(\vec{r}\,')\,d^3r'$$



A collection of spheres

$$J_p^{ind}(\vec{r}) = \frac{1}{i\omega\mu_0}\int \overleftrightarrow{T}(\vec{r}-\vec{r}_p;\vec{r}\,'-\vec{r}_p)\cdot\vec{E}_p^{E}(\vec{r}\,')\,d^3r'$$

DRIVING FIELD

Effective-field approximation

$$\vec{E}_p^E(\vec{r}) \approx \left\langle \vec{E} \right\rangle (\vec{r})$$

dilute limit

TAKING A CONFIGURATIONAL AVERAGE

$$\left\langle \vec{J}^{ind} \right\rangle (\vec{r};\omega) = \frac{1}{i\omega\mu_0} \frac{N}{V} \int d^3r' \left\langle \overleftrightarrow{T} \right\rangle (|\vec{r}-\vec{r}'|) \cdot \left\langle \vec{E} \right\rangle (\vec{r}';\omega)$$

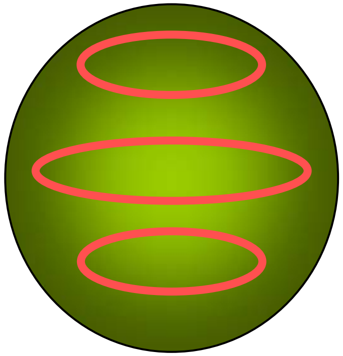
Nonlocal response

$$\frac{N}{V} \left\langle \overleftrightarrow{T} \right\rangle (|\vec{r}-\vec{r}'|) = \left\langle \sum_{p=1}^{N} \overleftrightarrow{T}(\vec{r}-\vec{r}_p, \vec{r}'-\vec{r}_p;\omega) \right\rangle$$

Effective medium

$$\frac{\varepsilon_{eff}(p,\omega)}{\varepsilon_0} = 1 + \frac{n_0}{k_0^2} T^L(p,\omega)$$

$$\frac{\mu_{eff}(p,\omega)}{\mu_0} = \frac{1}{1 - \frac{n_0}{p^2}\left(T^T(p,\omega) - T^L(p,\omega)\right)}$$



Magnetic response

Amperian magnetism

$$\vec{J}^{ind}(\vec{p};\omega) = \frac{1}{i\omega\mu_0} \overleftrightarrow{T}(\vec{p},\vec{p};\omega) \cdot \vec{E}_0$$

single, isolated sphere

Results

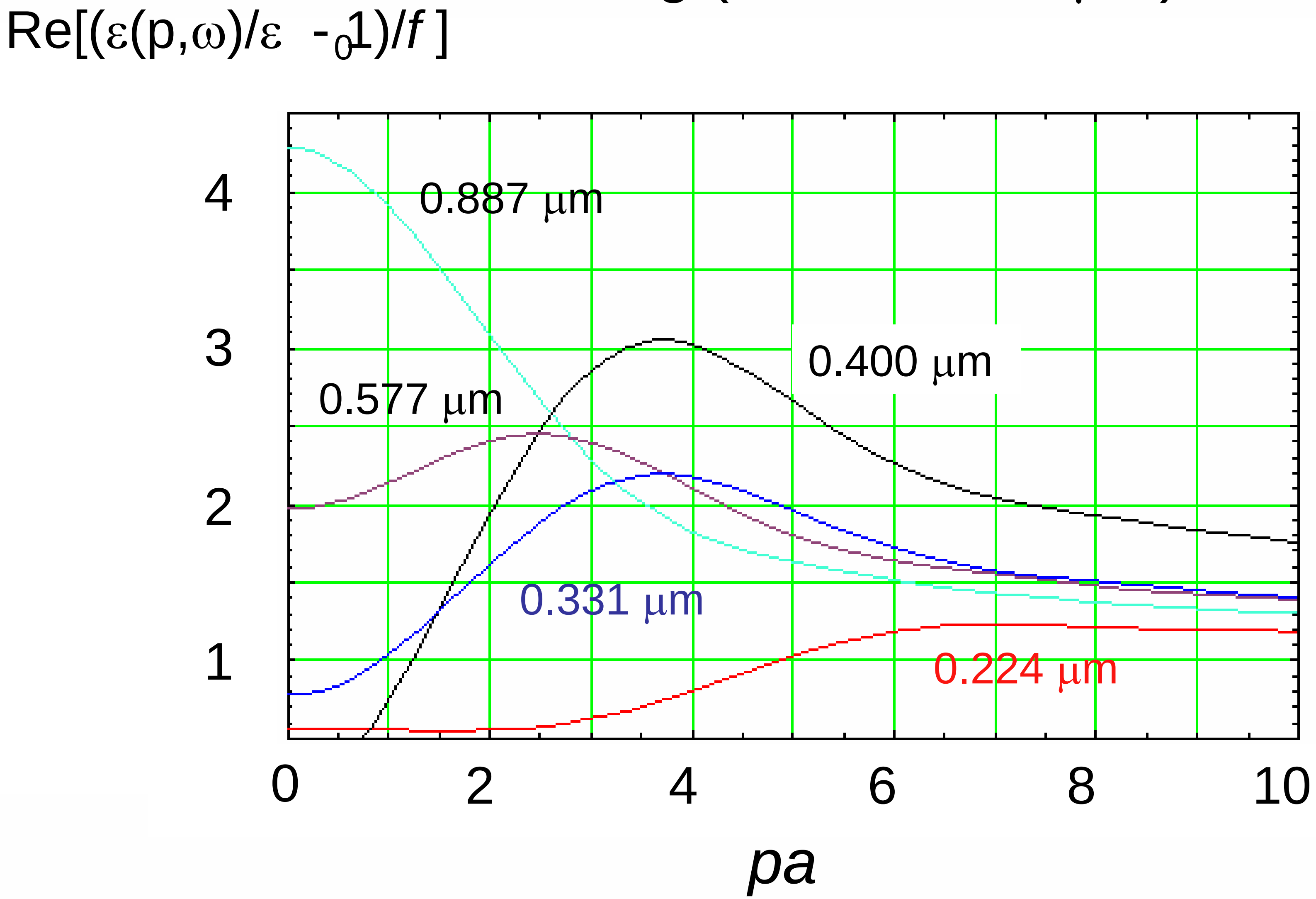
longitudinal

$$T^{L}(p,\omega)=\frac{4\pi}{3}x_0^2 a\varsigma\left[1+\chi_S\sum_{n=1}^{\infty}3n(n+1)(2n+1)d_n^L\frac{j_n(x_S)}{x_S}\frac{j_n(x_i)}{x_i}\right]$$

transverse

$$T^{T}(p,\omega)=\frac{4\pi}{3}x_0^2 a\chi_S(1-\xi)$$
$$+2\pi x_0^2 a\chi_S\xi\sum_{n=1}^{\infty}(2n+1)\left\{c_n I_2(n,n)+d_n\left[\frac{n+1}{x_i}I_1(n,n-1)+\frac{n}{x_i}I_1(n+1,n)-I_2(n+1,n-1)\right]\right\}$$

Ag (radius = 0.1 μm)



Electromagnetic modes

transverse

$$p^2 = \frac{\omega^2}{c^2}\varepsilon_{eff}(p,\omega)\mu_{eff}(p,\omega) \Rightarrow \omega^T(p)$$

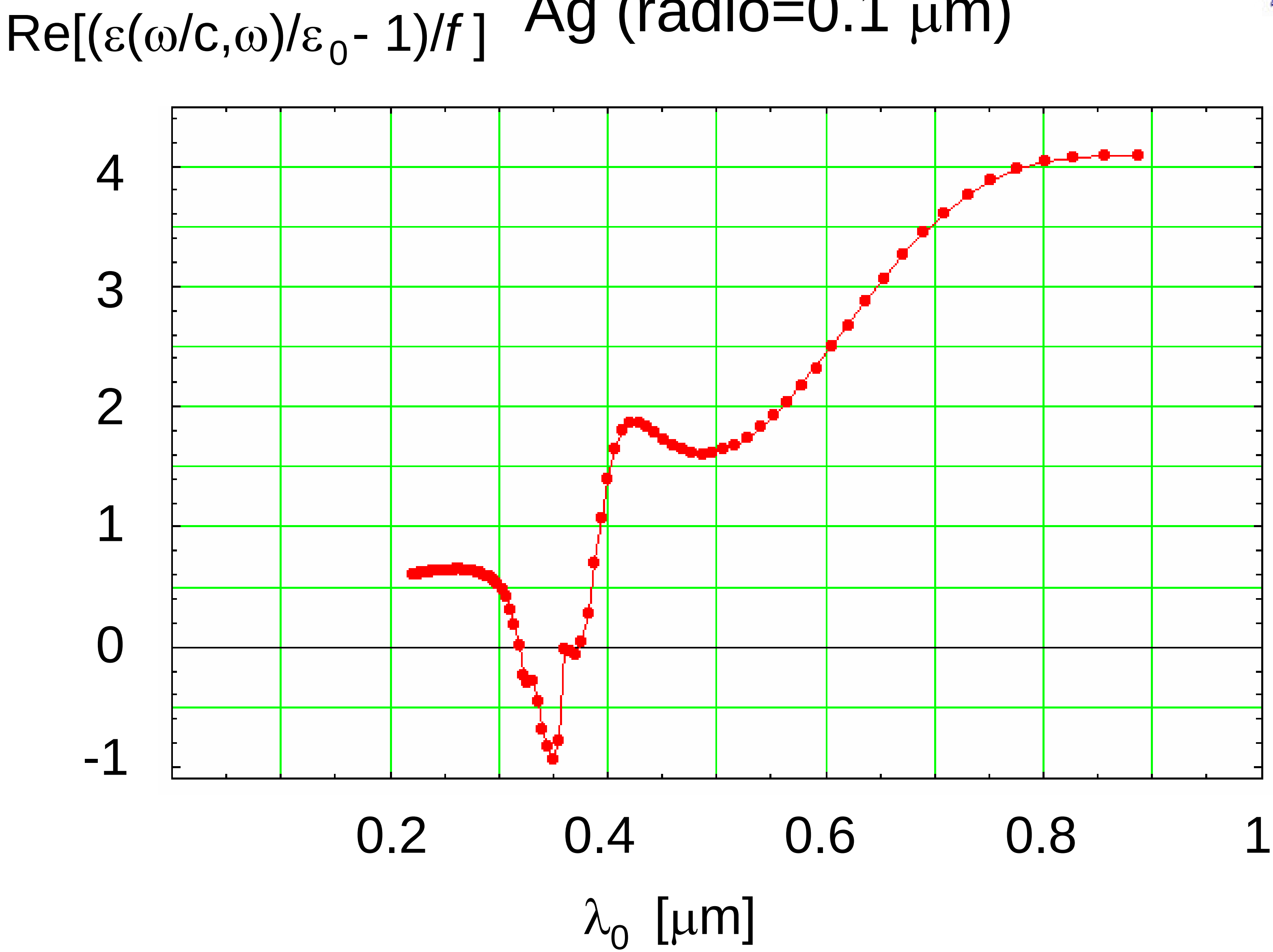
longitudinal

$$\varepsilon_{eff}(p,\omega) = 0 \Rightarrow \omega^L(p)$$

Index of refraction

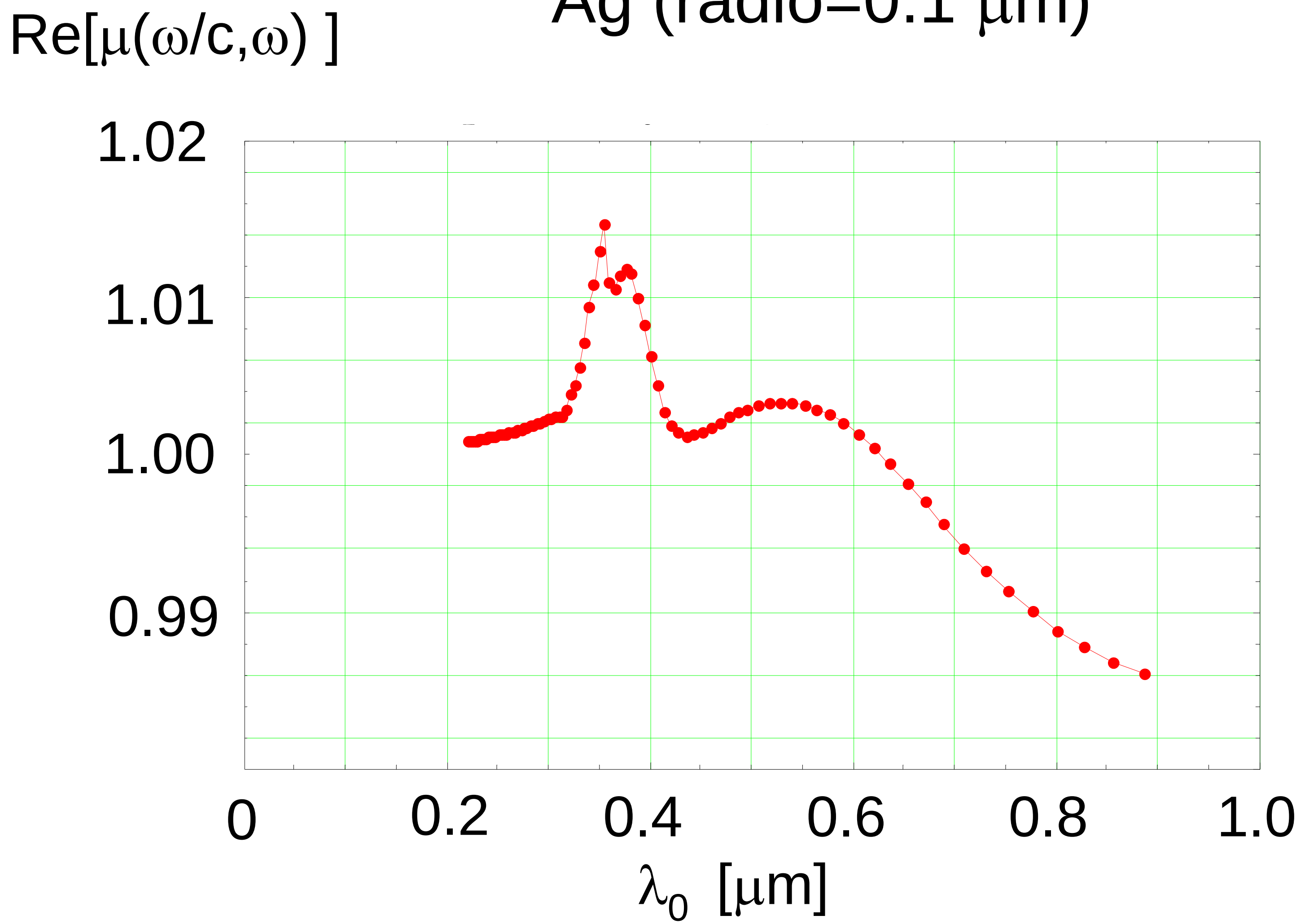
$$n^2{}_{eff}(\omega) \approx \varepsilon(p = \frac{\omega}{c},\omega)\mu(p = \frac{\omega}{c},\omega) \Rightarrow \text{Van de Hulst}$$

Ag (radio=0.1 μm)



f = 0.005

Ag (radio=0.1 μm)



Actual research

We are working on the non local properties of the electromagnetic response of colloidal systems and We are also performing refraction and reflection experiments in order to design an instrument to measure the particle-size distribution of colloidal particles using the information stored in the coherent beam.

We are building up a group on optical propertiers at an industrial lab of the paint industry, where we are applying some of the results obtained in our basic-research studies.