ON THE VAN DER WAALS ENERGY OF TWO HALF-SPACES AT SMALL SEPARATIONS

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It is shown that the introduction of a cut-off wavenumber corresponding to electron–hole excitations leads to a 10% reduction of the Lifshitz van der Waals attraction of low density metallic half-spaces at small separations.

In order to explain experiments on the adhesion of solid particles in terms of van der Waals forces, the Lifshitz formula [1] has been assumed to be valid for extremely small separations [2] (about 4 Å). In this range, however, the lattice structure and the overlap of the electrons are expected to become important. The influence of these effects, in a most qualitative way, can be incorporated into the theory of the non-retarded van der Waals forces, as derived, e.g., by van Kampen et al. [3] by introducing a cut-off wavenumber $K_c$ into the surface excitation spectrum. For wavenumbers greater than $K_c$, the surface excitations decay into electron–hole pairs. As is shown below the cut-off leads to a modification of the $1/d^2$ law at small separations. This indicates that in this range phenomena different from surface excitations, presumably direct overlap [4], are of non-negligible influence. (Effects of the lattice structure and size effects [5] are not considered in the present paper.) The quantity $K_c$, unfortunately, is not as well defined as for bulk plasmons, except for the case $K a \ll 1$, where $K$ is the surface plasmon wavenumber and $a$ is the thickness of the surface profile [6].

Now, following the simple treatment of van Kampen [3, 7] we obtain for the van der Waals energy $U$ of two metallic half-spaces separated by a gap of width $d$

$$U(d) = \frac{\hbar \omega_p}{4\pi\sqrt{2}d^2} \sum_{K} \left[ \omega^+(K) - 2^{-1/2} \omega_p \right] + \left[ \omega^-(K) - 2^{-1/2} \omega_p \right],$$

(1)

with

$$\omega^\pm = 2^{-1/2} \omega_p \left( 1 \pm e^{-K a} \right)^{1/2},$$

(2)

where $\omega_p$ is the plasma frequency.

Insertion of (2) into (1) leads to

$$U(d) = \frac{\hbar \omega_p}{4\pi\sqrt{2}d^2} K_c d \int_0^d dx \left[ 2 - (1 + e^{-x})^{1/2} - (1 - e^{-x})^{1/2} \right],$$

(3)

which has the following limits

$$U(d \gg K_c^{-1}) = \frac{\hbar \omega_p}{64\pi\sqrt{2}d^2} \left[ 1.1 - (1 + 2K_c d) e^{-2K_c d} \right]$$

(4)

$$U(d \ll K_c^{-1}) = \frac{\hbar \omega_p}{4\pi\sqrt{2}} \left[ \frac{1}{2} K_c^2 (2 - \sqrt{2}) - \frac{3}{2} K_c^{1/2} d^{1/2} \right].$$

(5)

Eq. (4) is obtained by expanding the integrand of (3) with respect to $e^{-x}$. The error appearing for small values of $x$ is approximately compensated by the fac-

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tor $x$ in front of the bracket.

For $K_c$ we suggest an expression which has been used in the calculation of surface energies [8]

$$K_c = \frac{\omega_p}{v_F \sqrt{a_B^2}} = \left( \frac{2}{3\pi^2} \right)^{1/6} \frac{1}{a_B \sqrt{r_s}},$$

(6)

where $v_F$ is the Fermi velocity, $a_B$ is Bohr’s radius and $r_s$ is defined by $n^{-1} = 4\pi (a_B r_s)^3/3$. Using this value for $K_c$ in eq. (4) leads to a reduction of Lifshitz’s results [1] which was derived for $K_c \to \infty$. In the range of separations $d$ studied in ref. [2] this reduction is about 10% for low density metals ($r_s \approx 6$) and about 1% for high density metals ($r_s \approx 2$).

The limit (5) has little physical relevance for our problem. Nevertheless, it shows that the introduction of a finite $K_c$ leads to a value of $U(d)$ that remains finite for $d \to 0$ [8].

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