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Salient Features of Electric and Magnetic Multipole Transition Probabilities of Hydrogen-Like Systems

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Abstract

Several aspects of E1, M1, E2, M2, E3 and M3 transition probabilities, lifetimes and branching fractions for hydrogen-like atoms evaluated with point-nucleus Dirac eigenfunctions for $Z = 1$–118 are discussed.

1. Introduction

Nonrelativistic radiative transition probabilities for hydrogen-like atoms have been known for a long time [1–3]. In the relativistic regime, ground work was laid by Bethe and Salpeter [2]. More recently, Pal'chikov [4] underlined Z-dependency, gave detailed analytical formulas for electric and magnetic dipolar and quadrupolar transitions, and provided a small sample of numerical results. The latter have been expanded up to more than one million transition probabilities by the two of us [5] including most transitions conceivably needed in physical and astrophysical research. The results were presented as visually attractive tables [6] in the style of the National Standard Reference Data Series [3]. A corresponding critical evaluation is given elsewhere [5].

Distinct assets of our tabulation include up to E3 and M3 transition probabilities, and the necessary consolidation of different multipole data when more than one is present in a given transition. For example, $\mathrm{M2} + \mathrm{E3}$ transitions show many cases where it does make a difference relative to either M2 or E3 alone. The wide scope of the results, embracing large principal quantum number n for small ℓ values (up to $n = 25$), large ℓ values (up to $\ell = 25$), and all nuclides $Z = 1$–118, allows many studies. In this paper we examine Z-dependency and other aspects of hydrogenic radiative transition probabilities in an effort to reveal the major spectroscopic features that may be of interest to general readers and to spectroscopists in particular, especially those working in inner-shell transitions.

In Section 2 we give the basic formulas for transition probabilities, define the chosen model and discuss its limitations. Our results can be inspected using as a guide the Z-dependent behavior, discussed in Section 3. Section 4 is devoted to the discussion of predominance of a given multipole when other multipoles share the same transition. It can be asked which absorption channels can possibly be detected in a given experiment, particulary in connection with direct observation of high-order electric and magnetic multipole transitions as treated in Section 5. In Section 6, through lifetimes and branching ratios, we study the emission behavior of H-like states giving for the first time a complete scheme of Z-dependent decay modes. Finally, we present the lifetimes of $2s_{1/2}$ states in Section 7 and conclusions in Section 8.

2. Basic formulas and model

The radiative emission transition probability $A_{ki}^{L(\mathrm{E,M})}$ for an electric (E) or magnetic (M) multipole of order L [7],

$$A_{ki}^{L(\mathrm{E,M})} = 2\frac{(\Delta E)^2}{c^3}\frac{g_i}{g_k}f_{ik}^{L(\mathrm{E,M})}, \tag{1}$$

is given in terms of the oscillator strength f_{ik} as:

$$f_{ik}^{L(\mathrm{E,M})} = \frac{\pi c^3}{(2L+1)(\Delta E)^2}| < \Psi_i \| \mathrm{O}^{L(\mathrm{E,M})} \| \Psi_k > |^2. \tag{2}$$

Different from the energy-independent nonrelativistic expression for f_{ik}, in Eq. (2) there is an explicit dependence on ΔE which is approximately canceled by an implicit dependence on this quantity through the transition operator $\mathrm{O}^{L(\mathrm{E,M})}$ [7,8], resulting in a smooth and small Z-dependence [4] for most but not all transitions.

The reduced transition matrix element involves CI expansion coefficients, angular factors and radial integrals [8], separating into different L multipole electric and magnetic terms: E1, M1, E2, M2, E3, M3, etc. We disregard a complete expansion which includes toroidal moments [9,10]. Nuclear motion, isotopic effects and other related details are discussed elsewhere [5].

H-like atoms are modeled after the point-nucleus Dirac Hamiltonian. We have neglected finite nuclear size and Lamb shift corrections which become noticeable for large nuclear charge. Their influence propagates mainly through transition energies, thus affecting only those transitions involving closely spaced levels.

The eigenfunctions of the one-electron Dirac Hamiltonian and its transition matrix elements have been evaluated using very accurate variational approximations with a general purpose computer program [5]. Computational details may be found in Ref. [5].

3. Z-dependent behavior

Z-dependent behavior of transition probabilities may be analytically approximated from the properties of Dirac's eigenfunctions [4] by taking the leading term in an expansion of A_{ki} in powers of Z. This approach is rather

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accurate for electric dipole transitions E1 with low principal quantum numbers, becoming less accurate for all others.

More reliably, Z-dependency can also be studied by numerical inspection of selected results, as done here:

$$A_{ki}(Z) = A_{ki}(Z = 1)Z^{N \pm a}, \qquad (3)$$

where N is the dominant power and a is a small correction which depends on the transition. Some of these results for specific transitions have already been given in Ref. [4]. The complete picture up to electric and magnetic octupoles is exhibited in Table I. The powers shown in the fourth column of Table I should be slightly modified according to Eq. (3) as exemplified in the fifth column.

Electric L multipole transitions with $\Delta n \neq 0$ follow a $Z^{(2L+2)}$ dependence, while for $\Delta n = 0$ it jumps to $Z^{(6L+4)}$. In both cases, the dominant power coincides with the non-relativistic Z-dependence. With increasing values of the principal quantum number in *both* states, several Z-powers besides the leading power do contribute significantly to the transition probabilities yielding a larger correction a, for example, $a = 0.7$ for the E1 $10s_{1/2} \rightarrow 25p_{3/2}$ transition.

4. Multipolar predominance

Given the very high powers of Z involved in Table I, one can ask whether or not one multipole systematically prevails over all others for all Z when several multipole modes (decay or absorption) are allowed in a given transition.

After examining all possibilities we can say that, without exceptions, Z-dependence in any specific transition does not change the predominance of a given multipole over all others. However, one can investigate in which cases the weaker multipole partners become significant. This is summarized in Table II where general rules for multipole predominance are given.

Within the complete set of transitions with change of parity, E1 transitions with $\Delta n = 0$ go as Z^{10} while the corresponding M2 ones go roughly as Z^{18} rising the expectation of a significant change in the relative role of the M2 modes along Z. Nevertheless, the data in Table II confirm what has been generally assumed: in E1 + M2 or E1 + M2 + E3 transitions, E1 always predominates for all Z. One needs to go to $Z = 99$ to reach a one percent contribution of M2, observed only in intense transitions such as $1s_{1/2} \rightarrow 2p_{3/2}$.

Table I. *Approximate Z-dependent behavior of hydrogen-like one-photon emission rates and correction a, Eq. (3), for representative transitions.*

Multipole	Δn	$\Delta\ell$	Z-dependence	Correction a
E1	$\neq 0$	1	Z^4	0.01
E1	0	1	Z^{10}	0.07
E2	$\neq 0$	0,2	Z^6	0.08
E2	0	0,2	Z^{16}	0.45
E3	$\neq 0$	1,3	Z^8	0.17
E3	0	1,3	Z^{22}	0.72
M1	$\neq 0$	0,2	Z^{10}	0.24
M1	0	0	Z^{12}	0.28
M1	0	2	Z^{16}	0.37
M2	$\neq 0$	1	Z^8	0.06
M2	$\neq 0$	3	Z^{12}	0.05
M2	0	1	Z^{18}	0.63
M2	0	3	Z^{22}	0.78
M3	$\neq 0$	0,2	Z^{10}	0.04
M3	$\neq 0$	4	Z^{14}	0.19
M3	0	2	Z^{24}	0.75
M3	0	4	Z^{28}	0.88

Table II. *Predominance of a given multipole when several multipole decay or absorption modes are allowed:* A ≫ B *indicates that* B *is less than 1% of* A*;* A > B*, that* B *may be a few percent of* A*;* A ≥ B*, that* B *may be larger than a few percent of* A.

Multipoles	Condition[a]
E1 ≫ M2	$Z \leq 80$
E1 > M2	$Z > 80$
E1 ≫ E3	all Z
M2 ≥ E3	all Z, $\Delta\ell = 1$
E3 ≫ M2	all Z, $\Delta\ell = 3$
M1 > E2	$\Delta n = 0$ and $\Delta\ell = 0$
E2 ≫ M1	otherwise
E2 ≫ M3	all Z

[a] See text for Z-dependence.

The results for M2 + E3 transitions are rather interesting: in $\Delta\ell = 1$ and $\Delta j = 2$ transitions M2 are stronger than E3 but, surprisingly, in the most intense line corresponding to $2p_{1/2} \rightarrow 3d_{5/2}$, both multipoles are of the same order of magnitude: the contribution of E3 is 24% for $Z = 1$ and rises slowly up to 32% for $Z = 118$, since the Z-dependence is Z^8 in both cases. Therefore, in $\Delta\ell = 1, \Delta j = 2$ atomic transitions with change of parity, in general, accurate lifetimes involving M2 channels must always be accompanied by E3, a fact that had not been noticed before.

Alternatively, in $\Delta\ell = 3$ transitions, such as recently studied for Yb^+ [11], apart from known E3 contributions there are also M2 channels, however, M2 + E3 is essentially E3 for all Z.

In the complete set of transitions without change of parity, the M1 + E2 and M1 + E2 + M3 modes are, in general, dominated by the E2 channel. There is one (unimportant) exception: in transitions with $\Delta n = 0$ and $\Delta\ell = 0$, the Z^{16} dependence of E2 is not sufficiently strong to catch up with M1, which exhibits a weaker Z^{12} dependence.

Other $\Delta\ell = 0$ transitions, but now with $\Delta n \neq 0$, such as the M1 + E2 transition $2p_{1/2} \rightarrow 3p_{3/2}$, have a negligible M1 contribution for $Z = 1$, increasing from 0.1% ($Z = 47$) to 5% for $Z = 92$. Likewise, in the M1 + E2 + M3 transition $2p_{3/2} \rightarrow 3p_{3/2}$, the M1 contribution reaches 2% for $Z = 92$. The (E2,M3) pair behaves similarly as (E1,M2): when existing, E2 + M3 coincides with E2 for low Z. For $Z = 92$, a 1% contribution of M3 is attained in the $1s_{1/2} \rightarrow 3d_{5/2}$ transition probability.

5. Absorption from the ground state

The easiest absorption experiment is from the ground state. A complete picture of the actual outcome of such an experiment has certainly been asked for repeatedly, however, no explicit answer has yet been given. In Table III we give A_{ki} values from the $1s_{1/2}$ state of atomic hydrogen. Only E1, E1 + M2, M2 + E3, E3, M1, M1 + E2, E2 + M3, and M3 modes are allowed. E1 and E2 transitions are detectable, of course. Assuming that observation can be achieved for $A_{ki} \geq 1$ (now or in the near future, depending

Table III. *Transition probabilities* A_{ki} *(in* s^{-1}*) for absorption from the 1S ground state (*$1s_{1/2}$*) of atomic hydrogen;* $M_H = 1.00727647$, $Ry(\infty) = 109737.31568549\ cm^{-1}$, $Ry(H) = 109677.58340737\ cm^{-1}$, $c = 137.03599976$. *The notation employed for the levels, configurations or orbitals is* $n\ell$ *for* $j = \ell - 1/2$ *and* nL *for* $j = \ell + 1/2$, *viz., 2p for* $2p_{1/2}$ *and 2P for* $2p_{3/2}$. *Physically distinct transitions are numbered in the first column. The asterisks mark unnumbered transitions whose* A_{ki} *values must be consolidated with their corresponding multipolar partners.*

No.	transition	λ(Å)	E_k(cm^{-1})	A_{ki}	Multipole(s)
1	1S–2S	1215.66962039	82259.19141	0.24946E − 05	M1
2	1S–3S	1025.72010424	97492.48317	0.11087E − 05	M1
3	1S–4S	972.53445551	102824.12045	0.53028E − 06	M1
4	1S–5S	949.74066867	105291.90051	0.28705E − 06	M1
5	1S–6S	937.80108953	106632.42037	0.17111E − 06	M1
6	1S–7S	930.74589408	107440.71033	0.10968E − 06	M1
7	1S–2p	1215.66962039	82259.19141	6.2649E + 08	E1
8	1S–3p	1025.72010424	97492.48317	1.6725E + 08	E1
9	1S–4p	972.53445551	102824.12045	0.68186E + 08	E1
10	1S–5p	949.74066867	105291.90051	0.34375E + 08	E1
11	1S–6p	937.80108953	106632.42037	0.19728E + 08	E1
12	1S–7p	930.74589408	107440.71033	0.12362E + 08	E1
	1S–2P	1215.66422563	82259.55646	6.2648E + 08*	E1
	1S–3P	1025.71896628	97492.59133	1.6725E + 08*	E1
	1S–4P	972.53402393	102824.16609	0.68186E + 08*	E1
	1S–5P	949.74045794	105291.92388	0.34375E + 08*	E1
	1S–6P	937.80097063	106632.43389	0.19728E + 08*	E1
	1S–7P	930.74582032	107440.71885	0.12362E + 08*	E1
	1S–2P	1215.66422563	82259.55646	0.46843E − 01*	M2
	1S–3P	1025.71896628	97492.59133	0.17566E − 01*	M2
	1S–4P	972.53402393	02824.16609	0.79661E − 02*	M2
	1S–5P	949.74045794	105291.92388	0.42111E − 02*	M2
	1S–6P	937.80097063	106632.43389	0.24787E − 02*	M2
	1S–7P	930.74582032	107440.71885	0.15768E − 02*	M2
13	1S–2P	1215.66422563	82259.55646	6.2648E + 08	E1 + M2
14	1S–3P	1025.71896628	97492.59133	1.6725E + 08	E1 + M2
15	1S–4P	972.53402393	102824.16609	0.68186E + 08	E1 + M2
16	1S–5P	949.74045794	105291.92388	0.34375E + 08	E1 + M2
17	1S–6P	937.80097063	106632.43389	0.19728E + 08	E1 + M2
18	1S–7P	930.74582032	107440.71885	0.12362E + 08	E1 + M2
	1S–3d	1025.71896628	97492.59133	0.69290E − 08*	M1
	1S–4d	972.53402393	102824.16609	0.43210E − 08*	M1
	1S–5d	949.74045794	105291.92388	0.25793E − 08*	M1
	1S–6d	937.80097063	106632.43389	0.16136E − 08*	M1
	1S–7d	930.74582032	107440.71885	0.10633E − 08*	M1
	1S–3d	1025.71896628	97492.59133	593.75*	E2
	1S–4d	972.53402393	102824.16609	326.79*	E2
	1S–5d	949.74045794	105291.92388	184.51*	E2
	1S–6d	937.80097063	106632.43389	112.06*	E2
	1S–7d	930.74582032	107440.71885	72.545*	E2
19	1S–3d	1025.71896628	97492.59133	593.75	M1 + E2
20	1S–4d	972.53402393	102824.16609	326.79	M1 + E2
21	1S–5d	949.74045794	105291.92388	184.51	M1 + E2
22	1S–6d	937.80097063	106632.43389	112.06	M1 + E2
23	1S–7d	930.74582032	107440.71885	72.545	M1 + E2
	1S–3D	1025.71858697	97492.62738	593.74*	E2
	1S–4D	972.53388007	102824.18129	326.78*	E2
	1S–5D	949.74038770	105291.93166	184.51*	E2
	1S–6D	937.80093099	106632.43839	112.06*	E2
	1S–7D	930.74579574	107440.72169	72.543*	E2
	1S–3D	1025.71858697	97492.62738	0.73908E − 07*	M3
	1S–4D	972.53388007	102824.18129	0.45248E − 07*	M3
	1S–5D	949.74038770	105291.93166	0.26790E − 07*	M3
	1S–6D	937.80093099	106632.43839	0.16687E − 07*	M3
	1S–7D	930.74579574	107440.72169	0.10967E − 07*	M3
24	1S–3D	1025.71858697	97492.62738	593.74	E2 + M3
25	1S–4D	972.53388007	102824.18129	326.78	E2 + M3
26	1S–5D	949.74038770	105291.93166	184.51	E2 + M3

Table III. *Continued*

No.	transition	λ(Å)	E_k(cm^{-1})	A_{ki}	Multipole(s)
27	1S–6D	937.80093099	106632.43839	112.06	E2 + M3
28	1S–7D	930.74579574	107440.72169	72.543	E2 + M3
	1S–4f	972.53388007	102824.18129	0.16268E − 14*	M2
	1S–5f	949.74038770	105291.93166	0.14216E − 14*	M2
	1S–6f	937.80093099	106632.43839	0.10425E − 14*	M2
	1S–7f	930.74579574	107440.72169	0.74798E − 15*	M2
	1S–4f	972.53388007	102824.18129	0.31074E − 03*	E3
	1S–5f	949.74038770	105291.93166	0.25666E − 03*	E3
	1S–6f	937.80093099	106632.43839	0.18267E − 03*	E3
	1S–7f	930.74579574	107440.72169	0.12871E − 03*	E3
29	1S–4f	972.53388007	102824.18129	0.31074E − 03	M2 + E3
30	1S–5f	949.74038770	105291.93166	0.25666E − 03	M2 + E3
31	1S–6f	937.80093099	106632.43839	0.18267E − 03	M2 + E3
32	1S–7f	930.74579574	107440.72169	0.12871E − 03	M2 + E3
33	1S–4F	972.53380815	102824.18890	0.31073E − 03	E3
34	1S–5F	949.74035258	105291.93556	0.25666E − 03	E3
35	1S–6F	937.80091118	106632.44065	0.18266E − 03	E3
36	1S–7F	930.74578344	107440.72310	0.12871E − 03	E3
37	1S–5g	949.74035258	105291.93556	0.29323E − 21	M3
38	1S–6g	937.80091118	106632.44065	0.33220E − 21	M3
39	1S–7g	930.74578344	107440.72310	0.28774E − 21	M3

on wavelength range), E3 transitions will be detectable beginning at $Z = 3$, M1 at $Z = 4$, and M3 will be observable at $Z = 35$. Instead, M2 transitions will always be dominated by either E1 or E3, and for this reason its independent detection can be ruled out. Since the Z-dependence of E3, M1, and M3 transitions are Z^8, Z^{10} and Z^{10}, respectively, their detectability increases sharply according to the quoted Z powers.

6. Emission behavior

The discussion will be centered on lifetimes, branching fractions and Z-dependent branching fractions for each excited state. For any Z, lifetimes span six orders of magnitude, from the highest state [$n = 26$, $\ell = 25$] down to the $2p_{1/2}$ state. Lifetimes for a sample of states of He$^+$ are given in Table IV, including very high ℓ states, from $\ell = 22$ (orbital $23j_{43/2}$) until $\ell = 25$ (orbital $26j_{51/2}$). For low Z, such as the example of Table IV, they follow a Z^{-4} law, a consequence of being dominated by E1 transitions.

The lifetime of the emblematic metastable $2s_{1/2}$ state can be used as a reference to compare with other relatively long-lived states. It is larger than all others with $\ell \leq 25$ until $Z = 49$, when the Z^{-10} dependency of the M1 mode lifetime becomes small enough to make $2s_{1/2}$ comparatively less stable.

Transitions without change of parity have little effect on lifetimes since the E1 decay mode dominates over all others through the entire range of Z values, except for the decay of the $2s_{1/2}$ state, dominated by two photon (2E1) decay at low Z and M1 decay at high Z (the crossing point is at $Z = 41$, see also Section. 7).

Departure from the nonrelativistic Z^{-4} law originates from the use of relativistic wave functions as well as from

Table IV. *Lifetimes (ns) of selected states of hydrogenic-like* He *and* Z^{-4} *behavior.*

State	τ (He)	$(\tau(\mathrm{H}) \cdot Z^{-4} - \tau(\mathrm{He})) \cdot 100/\tau(\mathrm{He})$
$26J_{51/2}$	0.66691D + 05	0.041
$26j_{49/2}$	0.66691D + 05	0.041
$26E_{49/2}$	0.61539D + 05	0.041
$26e_{47/2}$	0.61539D + 05	0.041
$26C_{47/2}$	0.56575D + 05	0.041
$26c_{45/2}$	0.56575D + 05	0.041
$25E_{49/2}$	0.54728D + 05	0.041
$25e_{47/2}$	0.54728D + 05	0.041
$25C_{47/2}$	0.50331D + 05	0.041
$25c_{45/2}$	0.50331D + 05	0.041
$25B_{45/2}$	0.46100D + 05	0.041
$25b_{43/2}$	0.46100D + 05	0.041
$5g_{9/2}$	0.14685D + 02	0.042
$5g_{7/2}$	0.14685D + 02	0.042
$5f_{7/2}$	0.87662D + 01	0.043
$5f_{5/2}$	0.87660D + 01	0.045
$5d_{5/2}$	0.43525D + 01	0.045
$5d_{3/2}$	0.43521D + 01	0.052
$5p_{3/2}$	0.14862D + 01	0.039
$5p_{1/2}$	0.14863D + 01	0.038
$5s_{1/2}$	0.22002D + 02	0.069
$4f_{7/2}$	0.45310D + 01	0.042
$4f_{5/2}$	0.45309D + 01	0.044
$4d_{5/2}$	0.22583D + 01	0.044
$4d_{3/2}$	0.22581D + 01	0.051
$4P_{3/2}$	0.76877D + 00	0.039
$4p_{1/2}$	0.76878D + 00	0.039
$4s_{1/2}$	0.14150D + 02	0.070
$3d_{5/2}$	0.96630D + 00	0.043
$3d_{3/2}$	0.96623D + 00	0.049
$3p_{3/2}$	0.32934D + 00	0.039
$3p_{1/2}$	0.32934D + 00	0.040
$3s_{1/2}$	0.98910D + 01	0.073
$2p_{3/2}$	0.99726D − 01	0.037
$2p_{1/2}$	0.99719D − 01	0.043
$2s_{1/2}$	0.18989D + 07[a]	

[a] Including 2E1 (two photon) decay.

Table V. *Branching fractions (×1000) for a selection of initial states of atomic hydrogen. The possible terminal states and their respectively branching fractions are given in the same row.*

Upper state	Terminal states													
26J	25E	1000												
26j	25e	999	25E	1										
26E	25C	852	24C	148										
26e	25c	851	24c	148	25C	1								
26C	25B	723	24B	236	23B	41								
26c	25b	722	24b	236	23b	41	25B	1						
9L	8K	1000												
9l	8k	992	8K	8										
9K	8I	606	7I	394										
9k	8i	599	7i	390	8I	7	7I	4						
9I	7H	374	8H	350	6H	275	8K	1						
9i	7h	368	8h	345	6h	271	7H	6	8H	5	6H	4	8k	1
5G	4F	1000												
5g	4f	964	4F	36										
5F	3D	637	4D	363										
5f	3d	595	4d	338	3D	42	4D	24						
5D	2P	657	3P	236	4P	104	4F	3						
5d	2p	547	3p	197	2P	109	4p	86	3P	39	4P	17	4f	4
5P	1S	818	2S	118	3S	39	4S	18	4D	4	3D	3		
5p	1S	818	2S	118	3S	39	4S	18	3d	4	4d	4		
5S	2P	303	3P	212	4P	152	2p	151	3p	106	4p	76		
4F	3D	1000												
4f	3d	933	3D	67										
4D	2P	746	3P	254										
4d	2p	621	3p	212	2P	124	3P	42						
4P	1S	839	2S	119	3S	38	3D	4						
4p	1S	839	2S	119	3S	38	3d	4						
4S	2P	389	3P	277	2p	195	3p	139						
3D	2P	1000												
3d	2p	833	2P	167										
3P	1S	882	2S	118										
3p	1S	882	2S	118										
3S	2P	667	2p	333										
2P	1S	1000												
2p	1S	1000												
2S	1S	1000	2E1	1S	0	M1								

other multipole transitions, particularly the M1 transitions affecting the lifetimes of the $ns_{1/2}$ states.

In Tables V and VI we show branching fractions for a selection of states with $Z = 1$ and 47, respectively. The latter is the highest Z for which accurate experimental studies on one-electron systems have been carried out [12]. The major paths consist of E1 transitions with $\Delta j = -1$, except for the decay of $np_{1/2}$ states which consists of E1 with $\Delta j = 0$. In going from $Z = 1$ to $Z = 47$, the tenuous emergence of E2 and M1 transitions causes additional branches, with its branching fractions between a few per thousand and a few percent, respectively.

7. Lifetimes of $2s_{1/2}$ states

In Table VII we present (M1 + 2E1) lifetimes of $2s_{1/2}$ states and compare with literature data. The small difference between the experimental value and the theoretical ones for $Z = 47$ will increase slightly after introduction of nuclear volume effects and quantum electrodynamic corrections, since that will diminish the energy difference $\Delta E(1s_{1/2} - 2s_{1/2})$ and thus an even smaller transition probability will be obtained. A complete calculation will require QED corrections to the wave functions themselves so far unknown.

8. Conclusions

Salient features of H-like spectra have been discussed, including: (*i*) a complete picture of Z-dependency of transition probabilities, (*ii*) comparable contributions of M2 and E3 multipoles to many A_{ki} values for all Z, (*iii*) possible detection of E3, M1 and M3 absorption modes from the ground state beginning at relatively small values of Z: E3 at $Z = 3$, M1 at $Z = 4$ and M3 at $Z = 35$, (*iv*) branching fractions, providing a comprehensive account of emission behavior.

The complete tables are available for consultation [6] and can even be used for estimating transition probabilities in N-electron systems: given any two levels of an atomic system and a particular multipole transition, a rough estimate of its value can be obtained by assigning a pertinent effective charge. For example, consider the experimentally studied [23] M2 radiative transition probability of $1s2s2p\ ^4P_{5/2} \rightarrow 1s^2 2s^2\ S_{1/2}$ for Ar^{15+}. The above transition can be approximated as an M2 $2p \rightarrow 1s$ transition of a single electron under an effective charge of

Table VI. *Branching fractions (times 1000) for a selection of states of hydrogen-like silver.*

Upper state	Terminal states													
26J	25E	1000												
26j	25e	999	25E	1										
26E	25C	852	24C	148										
26e	25c	851	24c	148	25C	1								
26C	25B	723	24B	236	23B	41								
26c	25b	722	24b	236	23b	41	25B	1						
9L	8K	998	7I	2										
9l	8k	990	8K	8	7i	2								
9K	8I	604	7I	393	6H	2								
9k	8i	597	7i	389	8I	7	7I	4	6h	2				
9I	7H	372	8H	348	6H	274	5G	3	6G	1	8K	1		
9i	7h	367	8h	343	6h	270	7H	6	8H	5	6H	4	5g	3
	6g	1	8k	1										
5G	4F	993	3D	6	4D	1								
5g	4f	958	4F	35	3d	5	3D	1						
5F	3D	629	4D	358	2P	12								
5f	3d	592	4d	331	3D	41	4D	24	2p	10	2P	3		
5D	2P	639	3P	231	4P	100	1S	5	4F	3				
5d	2p	554	3p	188	2P	98	4p	77	3P	37	1S	25	4P	16
	4f	4												
5P	1S	817	2S	119	3S	38	4S	16	4D	5	3D	4	2P	1
5p	1S	799	2S	128	3S	42	4S	19	4d	6	3d	5		
5S	2P	326	3P	223	4P	159	2p	133	3p	92	4p	65	1S	
4F	3D	989	2P	10	3P	1								
4f	3d	924	3D	65	2p	7	2P	2	3p	1				
4D	2P	728	3P	248	1S	23	2S	1						
4d	2p	626	3p	196	2P	113	3P	40	1S	23	2S	1		
4P	1S	840	2S	119	3S	36	3D	4						
4p	1S	823	2S	129	3S	41	3d	6						
4S	2P	417	3P	293	2p	169	3p	119	1S	1				
3D	2P	980	1S	18	2S	2								
3d	2p	824	2P	156	1S	18	2S	2						
3P	1S	884	2S	116										
3p	1S	870	2S	130										
3S	2P	714	2p	285	1S	2								
2P	1S	1000												
2p	1S	1000												
2S	1S	644	M1	1S	356	2E1								

Table VII. *Comparison of calculated and experimental lifetimes (ns) for $2s_{1/2}$ H-like states.*

Z	Expt.	Previous work	This work
2	1904000[a]	1899000[b]	1899000
8	452[c]	464[d]	464
9	237[c]	228[d]	228
16	7.3[e]	7.16[d]	7.161
18	3.487[f]	3.497[g,h]	3.4979
28	0.2171[i]	0.21545[h]	0.2157
36	0.0368[j]	0.03699[g]	0.03703
47	0.0449[k]	0.04305[h]	0.04304

[a] Ref.[13], [b] Ref.[14], [c] Ref.[15], [d] Ref.[16], [e] Ref.[17], [f] Ref.[18], [g] Ref.[19], [h] Ref.[20], [i] Ref.[21], [j] Ref.[22], [k] Ref.[12].

17 (18 minus the shielding of a 1s spectator electron) and an extra 2s spectator electron with no shielding role. Our result [6], $A_{ki} = 3.28 \times 10^8\ \mathrm{s}^{-1}$ is to be compared with three values from the literature: 3.14×10^8 [24], 3.18×10^8 [25], and 3.16×10^8 [26]. In general, the tabulated data can be used to make similar estimates in the study of inner shell transitions [27].

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